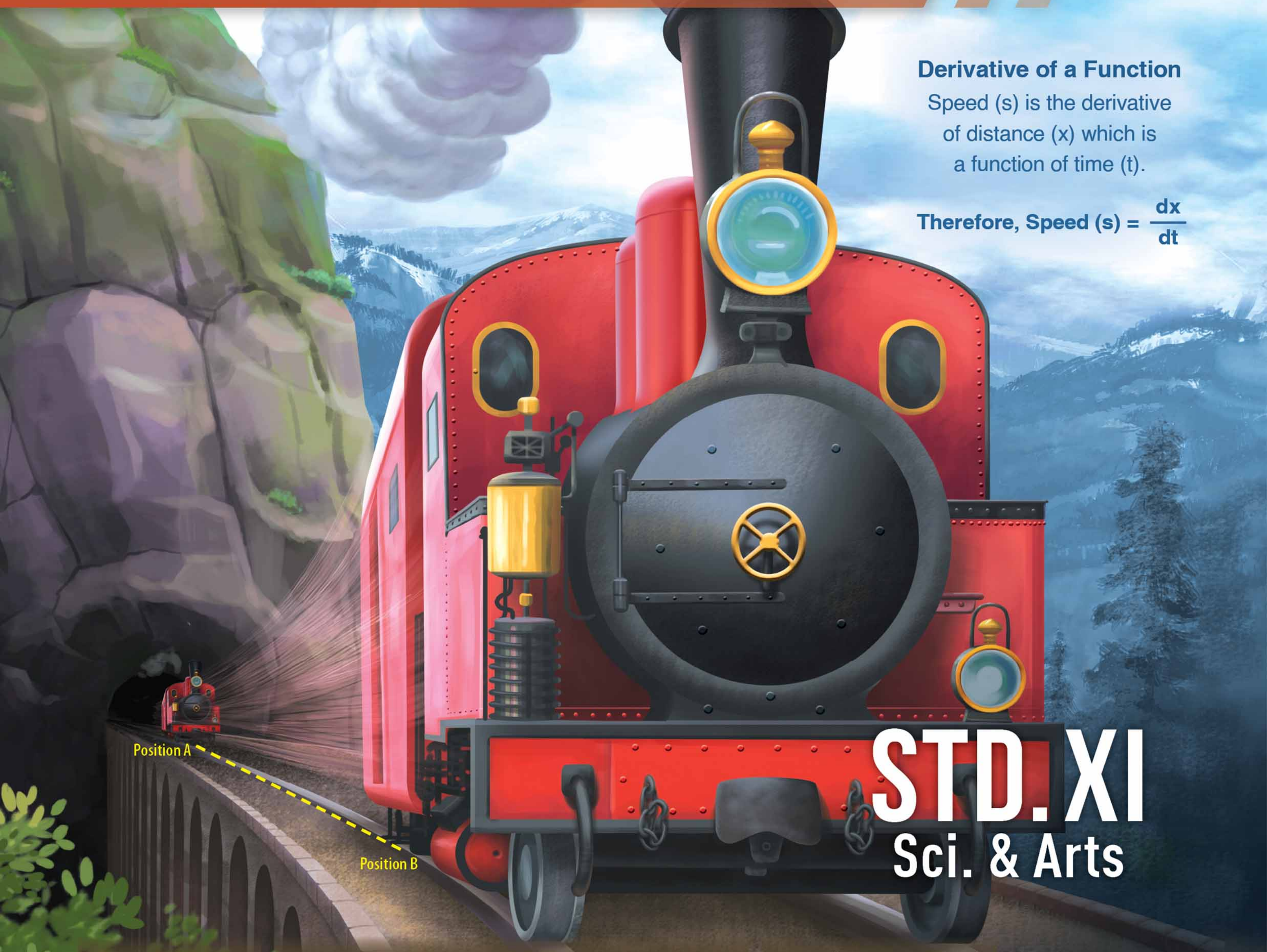
Perfect

MATHEMATICS & STATISTICS Part - II



Derivative of a Function

Speed (s) is the derivative of distance (x) which is a function of time (t).

Therefore, Speed (s) = $\frac{dx}{dt}$

STD. XI
Sci. & Arts

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Written as per the latest textbook prescribed by the Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune

PERFECT

MATHEMATICS - II

Std. XI Sci. & Arts

Salient Features

- Written as per the latest textbook
- Exhaustive coverage of entire syllabus
- Precise theory for every topic
- Covers answers to all exercises and miscellaneous exercises given in the textbook.
- All derivations and theorems covered
- Includes additional problems for practice and MCQs
- Illustrative examples for selective problems
- Topic Test for every chapter
- Recap of important formulae at the end of the book
- Tentative marks allocation for all problems
- Smart Check to enable easy rechecking of solutions
- 'Competitive Corner' presents questions from prominent Competitive Examinations
- Inclusion of **QR Codes** for students to access the 'Solutions' for the Topic Tests.

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PREFACE

"The only way to learn Mathematics is to do Mathematics" – Paul Halmos

'Mathematics – II : Std. XI' forms a part of **'Target Perfect Notes'** prepared as per the **Latest Textbook**. It is a complete and thorough guide critically analysed and extensively drafted to boost the students' confidence.

The book provides **answers to all textbook questions** included in exercises as well as miscellaneous exercises. Apart from these questions, we have provided **ample questions for additional practice** to students based on every exercise of the textbook. Only the final answer has been provided for such additional practice questions.

Precise theory has been provided at the required places for better understanding of concepts. Further, all **derivations and theorems have been covered** wherever required. A **recap of all important formulae** has been provided at the end of the book for quick revision. We have newly introduced **'competitive corner'** in this book wherein we have included questions from prominent competitive exams. It will help students to get an idea about the type of questions that are asked in Competitive Exams. We all know that there are certain sums that can be solved by multiple methods. Besides, there are also other ways to check your answer in Maths. **'Smart Check'** has been included to help you understand how you can check the correctness of your answer. 'One Mark Questions' have been covered along with their answers.
Every chapter contains a 'Topic Test'. This test stands as a testimony to the fact that the child has understood the chapter thoroughly.
We have provided **QR Codes** for students to access 'Solutions' for the given Topic Tests.

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you. Pls write to us on: mail@targetpublications.org

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

Publisher

Edition: Fourth

Disclaimer

KEY FEATURES

Smart Check

Smart Check is a technique to verify the answers. This is our attempt to cross-check the accuracy of the answer. Smart check is indicated by ✓ symbol

One Mark Questions

These questions require very short solutions with direct application of mathematical concepts.

Topic Test

Topic Test covers questions from the chapter for self-evaluation purpose. This is our attempt to provide the students with revision and help them assess their knowledge of chapter.

Competitive Corner

Competitive Corner presents questions from prominent [JEE (Main), MHT CET] competitive exams based entirely on the syllabus covered in the chapter. This is our attempt to introduce students to MCQs asked in competitive exams.

Important Formulae

Important Formulae given at the end of the book include all of the key formulae in the chapter. This is our attempt to offer students a handy tool to solve problems and ace the last minute revision.

QR Codes

QR code provides:
The solutions of the Topic tests

Additional Problems for Practice

In this section we have provided ample practice problems for students. Solved examples from textbook are indicated by “+”.

CONTENTS

[Reference: Maharashtra State Board of Secondary and Higher Secondary Education, Pune - 04**]**

Solved examples from textbook are indicated by "+".

Smart check is indicated by ✓ symbol.

1 Complex Numbers

SYLLABUS

- A complex number (C.N.)
- Algebra of C.N.
- Geometrical Representation of C.N.
- Polar and Exponential form of C.N.
- De Moivre's Theorem.

LET'S STUDY

Introduction

A linear equation in x is in the form $ax + b = 0$ having a real root $\frac{-b}{a}$. Solution of a quadratic equation is obtained by factorization.

But every quadratic equation is not factorizable such as $x^2 + 1 = 0$.

Now, $x^2 + 1$ has no factors in the set of real numbers. Also, $x^2 = -1$ is not possible in the set of real numbers, as squares of real numbers are non-negative.

Inspite of the facts mentioned, the solution set of equation $x^2 + 1 = 0$ is $x = \pm\sqrt{-1}$, where $\sqrt{-1}$ is called imaginary unit and it is denoted by i.

i.e., $i = \sqrt{-1}$

$\therefore \quad i^2 = -1$

In general, $x = \pm\sqrt{a}\,i$ is the solution of equation $x^2 + a = 0$, where a is a positive real number.

Thus i is an imaginary number.

Now, consider the equation $x^2 - 6x + 13 = 0$.

$\therefore \quad x^2 - 6x + 9 = -4$

$\therefore \quad (x-3)^2 = 4i^2$

$\therefore \quad x - 3 = \pm 2i$

$\therefore \quad x = 3 \pm 2i$

$\therefore \quad x = 3 + 2i$ or $x = 3 - 2i$

Hence the equation $x^2 - 6x + 13 = 0$ has two solutions $3 + 2i$ and $3 - 2i$, which are not real numbers. These numbers are called *complex numbers.*

Complex Numbers

Imaginary number:

A number of form bi, where $b \in R$, $b \neq 0$, $i = \sqrt{-1}$ is called an imaginary number.

Example:

$\sqrt{-36} = 6i, 3i, -\frac{4}{9}i$ etc.

Note:

The number i satisfies following properties,

i. $i \times 0 = 0$

ii. If $a \in R$, then $\sqrt{-a^2} = \sqrt{i^2a^2} = \pm ia$.

iii. If $a, b \in R$, and $ai = bi$, then $a = b$.

Complex number:

Definition:

A number of the type a + ib or a + bi, where a and b are real numbers and $i = \sqrt{-1}$ is called a complex number.

i. In a complex number a + ib, a is called the real part and b is called the imaginary part of the complex number a + ib.

ii. Note that real part and imaginary part of complex number are real numbers.

The complex number is denoted by z.

$\therefore \quad z = a + ib$

where real part denoted by Re(z) or R(z) and Imaginary part denoted by Im(z) or I(z)

$\therefore \quad z = \text{Re}(z) = R(z) = a$

$\therefore \quad \text{Im}(z) = I(z) = b$

Example:

If $z = 2 + 3i$ is a complex number, then

$\text{Re}(z) = 2$ and $\text{Im}(z) = 3$

Note:

i. A complex number whose real part is zero is called a purely imaginary number. Such a number is of the form $z = 0 + ib = ib$.

ii. A complex number whose imaginary part is zero is a real number.

$z = a + 0i = a$, is a real number.

iii. A complex number whose both real and imaginary parts are zero is the zero complex number. $0 = 0 + 0i$.

iv. The set R of real numbers is a subset of the set C of complex numbers.

v. The real part and imaginary part cannot be combined to form single term. E.g. $5 + 2i \neq 3i$.

Algebra of complex numbers

1. Equality of two complex numbers:

Two complex numbers $z_1 = a + bi$ and $z_2 = c + id$ are said to be equal if their corresponding real and imaginary number parts are equal.

Two complex numbers a + ib and c + id are said to be equal if a = c and b = d

i.e., a + ib = c + id, if a = c and b = d

2. Conjugate of a complex number

If a + ib is a complex number, then a – ib is the conjugate complex number of a + ib. If z = a + ib then its conjugate complex number is denoted by $\bar{z}$.

∴ $\bar{z} = a - ib$

Example:

Complex numbers	Conjugate complex numbers
$3 + 2i$	$3 - 2i$
$4 - \sqrt{5}\,i$	$4 + \sqrt{5}\,i$
$2i - 3$	$-3 - 2i$
$\cos\theta + i\sin\theta$	$\cos\theta - i\sin\theta$

Properties of conjugate of a complex number

i. $\overline{(\bar{z})} = z$

ii. $z + \bar{z} = 2\,\text{Re}(z)$

iii. $z - \bar{z} = 2i.\,\text{Im}(z)$

iv. $z = \bar{z}$

∴ z is real

v. Let $z \neq 0$.

$\bar{z} + z = 0$

∴ z is purely imaginary.

vi. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

vii. $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

viii. $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$

ix. $\overline{(z_1 z_2 z_3 \ldots z_n)} = \bar{z}_1 \cdot \bar{z}_2 \cdot \bar{z}_3 \ldots \bar{z}_n$

x. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq 0$

3. Addition of complex numbers:

If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are two complex numbers, then their sum is $z_1 + z_2$ and is defined as $z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2)$

$= (a_1 + a_2) + i(b_1 + b_2)$

∴ $\text{Re}(z_1 + z_2) = \text{Re}(z_1) + \text{Re}(z_2)$

and $\text{Im}(z_1 + z_2) = \text{Im}(z_1) + \text{Im}(z_2)$

Thus $z_1 + z_2$ is a complex number.

Example:

i. $(3 - 7i) + (5 + 3i) = (3 + 5) + i(-7 + 3)$

$= 8 - 4i$

ii. $(-2 + 5i) + (3 - 7i) = (-2 + 3) + i(5 - 7)$

$= 1 - 2i$

Properties of addition:

If z_1, z_2, z_3 are complex numbers, then

i. $z_1 + z_2 = z_2 + z_1$ (commutative)

ii. $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$ (associative)

iii. $z_1 + 0 = 0 + z_1 = z_1$ (identity)

iv. $z_1 + \bar{z}_1 = 2\text{Re}(z_1)$

v. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

4. Subtraction of complex numbers:

If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are two complex numbers, then their subtraction is $z_1 - z_2$ and is defined as

$z_1 - z_2 = (a_1 + ib_1) - (a_2 + ib_2)$

$= (a_1 - a_2) + i(b_1 - b_2)$

Thus $z_1 - z_2$ is a complex number.

Example:

i. $(4 + i) - (2 - 3i) = (4 - 2) + (1 + 3)i$

$= 2 + 4i$

ii. $(5 + 13i) - (4 + 7i) = (5 - 4) + (13 - 7)\,i$

$= 1 + 6i$

5. Scalar multiplication

If z = a + ib is any complex number, then for every real number k, define kz = ka + i(kb)

Example:

i. If z = 7 + 3i, then

5z = 5(7 + 3i) = 35 + 15i

ii. $z_1 = 3 - 4i$ and $z_2 = 10 - 9i$, then

$2z_1 + 5z_2 = 2(3 - 4i) + 5(10 - 9i)$

$= 6 - 8i + 50 - 45i$

$= 56 - 53i$

Note:

0.z = 0(a + ib) = 0 + 0i = 0

6. Multiplication of complex numbers:

If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are any two complex numbers, then their product is $z_1.z_2$ and is defined as

$z_1.z_2 = (a_1 + ib_1)(a_2 + ib_2)$

$= a_1a_2 + i(a_1b_2) + i(b_1a_2) + i^2(b_1b_2)$

$= a_1a_2 + i(a_1b_2 + b_1a_2) - b_1b_2$ …[∵ $i^2 = -1$]

$= (a_1a_2 - b_1b_2) + i(a_1b_2 + b_1a_2)$

Thus product $z_1.z_2$ is a complex number.

Example:

i. $(1 + i)(2 - 3i) = 2 + 2i - 3i - 3i^2$

$= 2 - i + 3$ …[∵ $i^2 = -1$]

$= 5 - i$

ii. $(2 + i)(2 - i) = (2)^2 - (i^2) = 4 + 1 = 5$

Properties of multiplication:

i. $z_1.z_2 = z_2.z_1$ (commutative)

ii. $(z_1.z_2).z_3 = z_1.(z_2.z_3)$ (associative)

iii. $(z_1.1) = 1.z_1 = z_1$ (identity)

iv. $\overline{(z_1.z_2)} = \overline{z_1}\,.\,\overline{z_2}$

v. If $z = a + ib$, then $z.\overline{z} = a^2 + b^2$

TRY THIS

1. Verify: $z + \overline{z} = 2\text{Re}(z)$ *(Textbook page no. 3)*

Solution:

Let $z = a + bi$

$\therefore \overline{z} = a - ib$

$z + \overline{z} = a + bi + a - ib$

$= 2a$, which is a real part of z

$\therefore z + \overline{z} = 2\text{Re}(z)$

2. Verify: $z - \overline{z} = 2\text{Im}(z)$ *(Textbook page no. 3)*

Solution:

Let $z = a + bi$

$\therefore \overline{z} = a - ib$

$z - \overline{z} = a + ib - a + ib$

$= 2ib$, which is a imaginary part of z

$\therefore z - \overline{z} = 2\text{Im}(z)$

3. Verify: $\overline{(z_1 \cdot z_2)} = \overline{z_1} \cdot \overline{z_2}$ *(Textbook page no. 3)*

Solution:

Let $z_1 = a + ib$ and $z_2 = c + id$

$\therefore \overline{z_1} = a - ib$ and $\overline{z_2} = c - id$

$z_1.\ z_2 = (a + ib)(c + id)$

$= ac + adi + bci - bd$

$= (ac - bd) + (ad + bc)\,i$

$\therefore \overline{z_1 \cdot z_2} = (ac - bd) - (ad + bc)\,i$ ...(i)

$\overline{z_1} \cdot \overline{z_2} = (a - ib)(c - id)$

$= ac - adi - bci - bd$

$= (ac - bd) - (ad + bc)\,i$ ...(ii)

From (i) and (ii), we get

$\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

7. **Powers of i:**

Consider i^n, where n is a positive integer and $n > 4$.

Now divide n by 4 and let the quotient be m and the remainder obtained be 'r'.

$n = 4m + r$, where $0 \le r < 4$

$\therefore i^n = i^{(4m + r)}$

$\therefore i^n = (i^4)^m.i^r$

$\therefore i^n = i^r$ ....[$\because i^4 = 1$]

Example:

$i^{82} = (i^4)^{20}.i^2 = (1)^{20}.i^2 = -1$

In general,

$\left.\begin{array}{ll} i^{4n} = 1, & i^{4n+1} = i \\ i^{4n+2} = -1, & i^{4n+3} = -i \end{array}\right\}$ where $n \in N$

8. **Division of complex numbers:**

Let $a + ib$ and $c + id$ be any two complex numbers, where $c + id$ is non-zero, then division is defined as

$$\frac{a+ib}{c+id} = \frac{a+ib}{c+id} \times \frac{c-id}{c-id}$$

$$= \frac{ac - adi + cbi - i^2bd}{c^2 - (id)^2}$$

$$= \frac{ac - adi + cbi - (-1)bd}{c^2 - i^2d^2}$$

$$= \frac{ac - adi + cbi + bd}{c^2 - (-1)d^2}$$

$$= \frac{ac + bd + (bc - ad)i}{c^2 + d^2}$$

$$= \frac{ac + bd}{c^2 + d^2} + i\,\frac{bc - ad}{c^2 + d^2}$$

Example:

If $z_1 = 2 + 3i$ and $z_2 = 1 + 2i$, then

$$\frac{z_1}{z_2} = \frac{2+3i}{1+2i} \times \frac{1-2i}{1-2i} = \frac{8}{5} - \frac{1}{5}i$$

Properties of division:

i. $\frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{-1} = -i$

ii. $\frac{1}{a+ib} = \frac{a-ib}{a^2+b^2}$

REMEMBER THIS

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = i^2\, i = (-1)\, i = -i$

$i^4 = (i^2)^2 = (-1)^2 = 1$

$i^5 = i^4.\ i = (1)^4 \times i = i$

$i^6 = (i^2)^3 = (-1)^3 = -1$ and so on

$\frac{1}{i} = -i$

EXERCISE 1.1

1. **Simplify:** **[2 Marks Each]**

i. $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

ii. $4\sqrt{-4} + 5\sqrt{-9} - 3\sqrt{-16}$

Solution:

i. $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

$= \sqrt{16 \times -1} + 3\sqrt{25 \times -1} + \sqrt{36 \times -1} - \sqrt{625 \times -1}$

$= 4i + 3(5i) + 6i - 25i$
$= 25i - 25i$
$= 0$

ii. $4\sqrt{-4} + 5\sqrt{-9} - 3\sqrt{-16}$
$= 4\sqrt{4\times-1} + 5\sqrt{9\times-1} - 3\sqrt{16\times-1}$
$= 4(2i) + 5(3i) - 3(4i)$
$= 8i + 15i - 12i$
$= 11i$

2. Write the conjugates of the following complex numbers: **[1 Mark Each]**

i. $3 + i$ **ii.** $3 - i$
iii. $-\sqrt{5} - \sqrt{7}\,i$ **iv.** $-\sqrt{-5}$
v. $5i$ **vi.** $\sqrt{5} - i$
vii. $\sqrt{2} + \sqrt{3}\,i$ **viii.** $\cos\theta + i\sin\theta$

Solution:

i. Conjugate of $(3 + i)$ is $(3 - i)$.

ii. Conjugate of $(3 - i)$ is $(3 + i)$.

iii. Conjugate of $\left(-\sqrt{5} - \sqrt{7}i\right)$ is $\left(-\sqrt{5} + \sqrt{7}i\right)$.

iv. $-\sqrt{-5} = -\sqrt{5}\times\sqrt{-1} = -\sqrt{5}\,i$.
Conjugate of $\left(-\sqrt{-5}\right)$ is $\sqrt{5}\,i$.

v. Conjugate of $(5i)$ is $(-5i)$.

vi. Conjugate of $\left(\sqrt{5} - i\right)$ is $\left(\sqrt{5} + i\right)$.

vii. Conjugate of $(\sqrt{2} + \sqrt{3}\,i)$ is $(\sqrt{2} - \sqrt{3}\,i)$.

viii. Conjugate of $(\cos\theta + i\sin\theta)$ is $(\cos\theta - i\sin\theta)$.

3. Find a and b if **[2 Marks Each]**

i. $a + 2b + 2ai = 4 + 6i$
ii. $(a - b) + (a + b)i = a + 5i$
iii. $(a + b)(2 + i) = b + 1 + (10 + 2a)i$
iv. $abi = 3a - b + 12i$
v. $\frac{1}{a+ib} = 3 - 2i$
vi. $(a + ib)(1 + i) = 2 + i$

Solution:

i. $a + 2b + 2ai = 4 + 6i$
Equating real and imaginary parts, we get
$a + 2b = 4$ …(i)
$2a = 6$ …(ii)
$\therefore$ $a = 3$
Substituting, $a = 3$ in (i), we get
$3 + 2b = 4$
$\therefore$ $b = \frac{1}{2}$
$\therefore$ $a = 3$ and $b = \frac{1}{2}$

SMART CHECK

For $a = 3$ and $b = \frac{1}{2}$:
Consider, L.H.S. $= a + 2b + 2ai$
$= 3 + 2\left(\frac{1}{2}\right) + 2(3)i$
$= 4 + 6i =$ R.H.S.

ii. $(a - b) + (a + b)i = a + 5i$
Equating real and imaginary parts, we get
$a - b = a$ …(i)
$a + b = 5$ …(ii)
From (i), $b = 0$
Substituting $b = 0$ in (ii), we get
$a + 0 = 5$
$\therefore$ $a = 5$
$\therefore$ $a = 5$ and $b = 0$

iii. $(a + b)(2 + i) = b + 1 + (10 + 2a)i$
$\therefore$ $2(a + b) + (a + b)i = (b + 1) + (10 + 2a)i$
Equating real and imaginary parts, we get
$2(a + b) = b + 1$
$\therefore$ $2a + b = 1$ …(i)
and $a + b = 10 + 2a$
$-a + b = 10$ …(ii)
Subtracting equation (ii) from (i), we get
$3a = -9$
$\therefore$ $a = -3$
Substituting $a = -3$ in (ii), we get
$-(-3) + b = 10$
$\therefore$ $b = 7$
$\therefore$ $a = -3$ and $b = 7$

iv. $abi = 3a - b + 12i$
$\therefore$ $0 + abi = (3a - b) + 12i$
Equating real and imaginary parts, we get
$3a - b = 0$
$\therefore$ $3a = b$ …(i)
and $ab = 12$
$\therefore$ $b = \frac{12}{a}$ …(ii)

Substituting $b = \frac{12}{a}$ in (i), we get

$3a = \frac{12}{a}$

$\therefore$ $3a^2 = 12$

$\therefore$ $a^2 = 4$

$\therefore$ $a = \pm 2$

When $a = 2$, $b = \frac{12}{a} = \frac{12}{2} = 6$

When $a = -2$, $b = \frac{12}{a} = \frac{12}{-2} = -6$

$\therefore$ $a = 2$ and $b = 6$ or $a = -2$ and $b = -6$

---

v. $\frac{1}{a+ib} = 3 - 2i$

$\therefore$ $a + ib = \frac{1}{3-2i}$

$\therefore$ $a + ib = \frac{1}{3-2i} \times \frac{3+2i}{3+2i}$

$\therefore$ $a + ib = \frac{3+2i}{3^2 - 2^2 i^2}$

$\therefore$ $a + ib = \frac{3+2i}{9-4(-1)}$ ...[$\because i^2 = -1$]

$\therefore$ $a + ib = \frac{3+2i}{13}$

$\therefore$ $a + ib = \frac{3}{13} + \frac{2}{13}i$

Equating real and imaginary parts, we get

$\therefore$ $a = \frac{3}{13}$ and $b = \frac{2}{13}$

---

vi. $(a + ib)(1 + i) = 2 + i$

$\therefore$ $a + ai + bi + bi^2 = 2 + i$

$\therefore$ $a + (a + b)i + b(-1) = 2 + i$ ...[$\because i^2 = -1$]

$\therefore$ $(a - b) + (a + b)i = 2 + i$

Equating real and imaginary parts, we get

$a - b = 2$ ...(i)

$a + b = 1$ ...(ii)

Adding equations (i) and (ii), we get

$2a = 3$

$\therefore$ $a = \frac{3}{2}$

Substituting $a = \frac{3}{2}$ in (ii), we get

$\frac{3}{2} + b = 1$

$\therefore$ $b = 1 - \frac{3}{2} = -\frac{1}{2}$

$\therefore$ $a = \frac{3}{2}$ and $b = -\frac{1}{2}$

4. Express the following in the form of a + ib, a, b ∈ R, $i = \sqrt{-1}$. State the values of a and b:

[2 Marks Each]

i. $(1 + 2i)(-2 + i)$ **ii.** $(1 + i)(1 - i)^{-1}$

iii. $\frac{i(4+3i)}{1-i}$ **iv.** $\frac{(2+i)}{(3-i)(1+2i)}$

v. $\left(\frac{1+i}{1-i}\right)^2$ **vi.** $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$

vii. $(1 + i)^{-3}$ **viii.** $\frac{2+\sqrt{-3}}{4+\sqrt{-3}}$

ix. $(-\sqrt{5} + 2\sqrt{-4}) + (1 - \sqrt{-9}) + (2 + 3i)(2 - 3i)$

x. $(2 + 3i)(2 - 3i)$ **xi.** $\frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2}$.

Solution:

i. $(1 + 2i)(-2 + i) = -2 + i - 4i + 2i^2$

$= -2 - 3i + 2(-1)$ ...[$\because i^2 = -1$]

$\therefore$ $(1 + 2i)(-2 + i) = -4 - 3i$

$\therefore$ $a = -4$ and $b = -3$

---

ii. $(1 + i)(1 - i)^{-1} = \frac{1+i}{1-i}$

$= \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1-i^2}$

$= \frac{1+2i-1}{1-(-1)}$ ...[$\because i^2 = -1$]

$= \frac{2i}{2}$

$= i$

$\therefore$ $(1 + i)(1 - i)^{-1} = 0 + i$

$\therefore$ $a = 0$ and $b = 1$

---

iii. $\frac{i(4+3i)}{1-i} = \frac{4i+3i^2}{1-i}$

$= \frac{-3+4i}{1-i}$ ...[$\because i^2 = -1$]

$= \frac{(-3+4i)(1+i)}{(1-i)(1+i)}$

$= \frac{-3-3i+4i+4i^2}{1-i^2}$

$= \frac{-3+i+4(-1)}{1-(-1)}$ ...[$\because i^2 = -1$]

$= \frac{-7+i}{2}$

$\therefore$ $\frac{i(4+3i)}{1-i} = \frac{-7}{2} + \frac{1}{2}i$

$\therefore$ $a = \frac{-7}{2}$ and $b = \frac{1}{2}$

Page no. **6** to **42**are purposely left blank.

To see complete chapter buy **Target Notes** or **Target E-Notes**

20. If ω is the cube root of unity, then find the value of $\left(\frac{-1+i\sqrt{3}}{2}\right)^{18}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{18}$. **[3 Marks]**

Solution:

If ω is the complex cube root of unity, then

$\omega^3 = 1, \omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \left(\frac{-1-i\sqrt{3}}{2}\right)^2$

Consider, $\left(\frac{-1+i\sqrt{3}}{2}\right)^{18}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{18}$

Given Expression $= \omega^{18} + (\omega^2)^{18}$
$= \omega^{18} + \omega^{36}$
$= (\omega^3)^6 + (\omega^3)^{12}$
$= (1)^6 + (1)^{12} = 2$

ONE MARK QUESTIONS

1. Simplify: $\sqrt{-289}+4\sqrt{-169}-3\sqrt{-196}$
2. Find the distance of the point P from the origin, where the point P represents the complex number z = 3 + 4i in the plane.
3. If $z = 2 + 2\sqrt{3}$ i, find the amplitude of z.
4. If ω is a complex cube root of unity, then find the value of ω^{-39}.
5. Express $z = \frac{\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}}{\sqrt{5}}$ in the exponential form.

ADDITIONAL PROBLEMS FOR PRACTICE

Based on Exercise 1.1

+1. Write the following complex numbers z in the form of a + ib and Re(z), Im(z) **[3 Marks Each]**

i. 2 + 4i ii. 5i
iii. 3 – 4i iv. $5+\sqrt{-16}$
v. $2+\sqrt{5}\,i$ vi. $7+\sqrt{3}$

2. Evaluate: $\sqrt{-25}+3\sqrt{-4}+2\sqrt{-9}-\sqrt{-144}$. **[2 Marks]**

3. Write the conjugates of the following complex numbers: **[1 Mark Each]**

i. 5 + 4i ii. 1 –2i
iii. $\sqrt{5}+3i$ iv. –12i
v. $-\sqrt{3}+\sqrt{2}\,i$ vi. cos 2θ – i sin 2θ

4. Express the following in the form of a + ib, where a, b ∈ R, i = $\sqrt{-1}$. State the values of a and b: **[2 Marks Each]**

i. (1 + i) (1 + 2i) ii. $2(2-i)(2+i)^{-1}$
iii. $\frac{3+2i}{-2+i}$ iv. $\frac{(1+2i)(2-3i)}{3+4i}$
v. $\frac{(1+i)(1+\sqrt{3}i)}{1-i}$ vi. $\left(\frac{1+i}{1-i}\right)^2$

+5. i. Write $(1+2i)(1+3i)(2+i)^{-1}$ in the form a + ib. **[2 Marks]**
ii. If a and b are real and $(i^4+3i)a+(i-1)b+5i^3=0$, find a and b. **[2 Marks]**
iii. If $x + 2i + 15i^6y = 7x + i^3(y + 4)$, find $x + y$, given that $x, y \in R$. **[2 Marks]**

+6. Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^3 = i$ **[3 Marks]**

7. Show that $\frac{3+2i}{2-5i}+\frac{3-2i}{2+5i}$ is real. **[3 Marks]**

8. Find the values of x and y which satisfy the following equations ($x, y \in R$) :
i. $\frac{x-1}{3+i}+\frac{y-1}{3-i}=i$ **[3 Marks]**
ii. $(1-i)x+(1+i)y=1-3i$ **[2 Marks]**
iii. If $(3x-7)-2i^3y=-5y+(5+x)i$, find $x+y$. **[3 Marks]**

9. i. Evaluate: $i^{18}+\frac{1}{i^{24}}$ **[2 Marks]**
ii. Prove that $(1+i)^3\times\left(1+\frac{1}{i}\right)^3=8$. **[2 Marks]**
iii. Evaluate: $\frac{i^{258}+i^{256}+i^{254}+i^{252}+i^{250}}{i^{248}+i^{246}+i^{244}+i^{242}+i^{240}}$ **[3 Marks]**

10. Find the value of $(1+2i^5)+(1+3i)(2+i)^{-1}$. **[3 Marks]**

Based on Exercise 1.2

+1. Find the square root of **[3 Marks Each]**
i. 6 + 8i ii. 3 – 4i

2. Find the square roots of the following complex number: **[3 Marks Each]**
i. – i ii. – 15 – 8i
iii. 5 + 12i iv. $1-4\sqrt{5}\,i$

+3. Solve: **[3 Marks Each]**
i. $x^2+x+1=0$
ii. $x^2-\left(2\sqrt{3}+3i\right)x+6\sqrt{3}\,i=0$

4. Solve the following quadratic equations: **[2 Marks Each]**

i. $2x^2 + x + 1 = 0$

ii. $5x^2 - 6x + 2 = 0$

5. Solve the following quadratic equations: **[2 Marks Each]**

i. $x^2 + 10ix - 21 = 0$

ii. $x^2 + 12ix - 36 = 0$

6. Solve the following quadratic equations: **[4 Marks Each]**

i. $x^2 - (7 - i)x + (18 - i) = 0$

ii. $2x^2 - (3 + 7i)x - (3 - 9i) = 0$

+7. Find the value of: **[3 Marks Each]**

i. $x^3 - x^2 + 2x + 10$ when $x = 1 + \sqrt{3}\,i$

ii. $x^4 + 9x^3 + 35x^2 - x + 64$ if $x = -5 + 2\sqrt{-4}$

8. Find the value of **[3 Marks Each]**

i. $x^3 + 7x^2 - x + 16$, if $x = 1 + 2i$

ii. $2x^3 + 2x^2 - 7x + 72$, if $x = \frac{3-5i}{2}$

iii. $x^4 + 9x^3 + 35x^2 - x + 4$, if $x = -5 + 2\sqrt{-4}$.

Based on Exercise 1.3

+1. i. If z = 1 + 3i, find the modulus and amplitude of z. **[2 Marks]**

ii. Find the modulus, argument of the complex number – 7 + 24i. **[2 Marks]**

2. Find the modulus and amplitude for each of the following complex numbers: **[2 Marks Each]**

i. 3i ii. 1 + 3i

iii. 3 + 4i iv. $-3\sqrt{2} + 3\sqrt{2}\,i$

+3. i. Represent the complex numbers $z = 1 + i$, $\bar{z} = 1 - i$, $-\bar{z} = -1 + i$, $-z = -1 - i$ in Argand's diagram and hence find their arguments from the figure. **[4 Marks]**

ii. Represent the following complex numbers in the polar form and in the exponential form **[3 Marks Each]**

a. $4 + 4\sqrt{3}\,i$ b. -2

c. $3i$ d. $-\sqrt{3} + i$

4. Represent the following complex numbers in polar form:

i. $1 - i$ **[2 Marks]** ii. $5i$ **[2 Marks]**

iii. $\sqrt{3} + i$ **[2 Marks]** iv. $\frac{1+3i}{1-2i}$ **[4 Marks]**

+5. i. Express $z = \sqrt{2}.e^{\frac{3\pi}{4}i}$ in the a + ib form. **[2 Marks]**

ii. a. Express (i) $3.e^{\frac{5\pi}{12}i} \times 4.e^{\frac{\pi}{12}i}$ **[3 Marks]**

b. $\dfrac{\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)}{2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)}$ in a + ib form **[3 Marks]**

6. Experess the following numbers in the form $x + iy$:

i. $\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$ **[2 Marks]**

ii. $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ **[1 Mark]**

Based on Exercise 1.4

+1. i. If ω is a complex cube root of unity, then prove that **[2 Marks Each]**

a. $\frac{1}{\omega} + \frac{1}{\omega^2} = -1$

b. $(1 + \omega^2)^3 = -1$

c. $(1 - \omega + \omega^2)^3 = -8$

ii. If ω is a complex cube root of unity, then show that **[2 Marks Each]**

a. $(1 - \omega + \omega^2)^5 = (1 + \omega - \omega^2)^5 = 32$

b. $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^5) = 9$

2. Find the values of **[1 Mark Each]**

i. ω^{30} ii. ω^{-84}

3. If ω is the complex cube root of unit *y*, show that **[2 Marks Each]**

i. $(3 - \omega^2)(3 - \omega) = 13$

ii. $(1 - \omega + \omega^2)^5 = 32$

4. If ω is the complex cube root of unity, find the values of : **[2 Marks Each]**

i. $(1 - \omega + \omega^2)^6$

ii. $(1 + \omega)^2 (1 + \omega^2)^2 - (1 + \omega^2)^3$

5. If ω is the complex cube root of unity, show that : **[2 Marks Each]**

i. $(1 + \omega - \omega^2)(1 - \omega + \omega^2) = 4$

ii. $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^5) = 9$

6. If ω is the complex cube root of unity, show that $1 + \omega^n + \omega^{2n} = 3$, if n is a multiple of 3. **[4 Marks]**

7. If ω denotes the complex cube root of unity, prove the following:
$(\omega^2 + \omega - 1)^3 = -8$ **[3 Marks]**

8. If ω is the complex cube root of unity, then prove that
$(2 + 5\omega + 2\omega^2)^6 = (2 + 2\omega + 5\omega^2)^6 = 729$
[4 Marks]

9. If $z = \lambda + 3 + i\sqrt{5-\lambda^2}$, then find the locus of point p representing z, in cartesion form.
[3 Marks]

10. If $|z| = 2$, then find the curve on which the complex numbers $-1 + 5z$ lie. **[3 Marks]**

11. If $|z^2 - 1| = |z|^2 + 1$, then find the cartesian equation of locus of z. **[3 Marks]**

12. Find the locus of z satisfying
$\log_{\frac{1}{3}}|z+1| > \log_{\frac{1}{3}}|z-1|$ **[4 Marks]**

13. Simplify $\dfrac{(\cos 3\theta + i\sin 3\theta)^5 (\cos 4\theta - i\sin 4\theta)^4}{(\cos 5\theta - i\sin 5\theta)^3 (\cos 6\theta + i\sin 6\theta)^2}$
[3 Marks]

14. Prove that
$(1 + \cos\theta + i\sin\theta)^n + (1 - \cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\dfrac{\theta}{2}\cos\dfrac{\pi\theta}{2}$ **[3 Marks]**

15. If n is a positive integer, prove that
$\left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n = 2^{n+1}\cos\dfrac{n\pi}{6}$ **[3 Marks]**

Based on Miscellaneous Exercise – 1

1. Simplify the following and express in the form of a + ib:

i. $-2 + \sqrt{-3}$ **[1 Mark]**

ii. $(1 + 2i)^{-3}$ **[2 Marks]**

iii. $\dfrac{(2+5i)}{i(3-2i)}$ **[2 Marks]**

iv. $\dfrac{(2+i)^3}{2+3i}$ **[2 Marks]**

v. $\left(1+\dfrac{2}{i}\right)\left(1+\dfrac{3}{i}\right)(2+i)^{-1}$ **[2 Marks]**

vi. $\dfrac{2+3i}{2-3i}+\dfrac{2-3i}{2+3i}$ **[2 Marks]**

2. Solve the following equations for $x, y \in R$:
[2 Marks Each]

i. $(x + iy)(2 - 3i) = 4 + i$

ii. $(3x - 2iy)(3 + 4i) = 10 + 10i$

iii. $(1 + 3i)x + (i - 1)y + 5i^3 = 0$.

3. Evaluate : **[1 Mark Each]**

i. $i^{79} + i^{21}$ ii. $(2i^2 + i + 2)^{-15}$

4. Find the modulus and amplitude of the following numbers: **[2 Marks Each]**

i. $\dfrac{1+7i}{(2-i)^2}$ ii. $\dfrac{1+3i}{1-2i}$

iii. $\dfrac{-1+i\sqrt{3}}{2}$ iv. $\dfrac{1+3i}{2-i}+\dfrac{1-2i}{2+i}$

v. $\dfrac{2-i}{1+2i}\cdot\dfrac{1+i}{1-2i}$.

5. i. Show that $\left(\dfrac{\sqrt{3}}{2}+\dfrac{i}{2}\right)^3$ is purely imaginary number **[2 Marks]**

ii. Show that $\dfrac{5-2i}{3+i}+\dfrac{5+2i}{3-i}$ is a real number
[2 Marks]

6. Let $0 < \theta < 2\pi$ and the expression $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ be purely imaginary. Find θ **[3 Marks]**

7. Find the smallest positive integer for which $\left(\dfrac{1+i}{1-i}\right)^n = 1$ **[3 Marks]**

8. Find the number of solutions to the equation $z^2 + \bar{z} = 0$ **[3 Marks]**

9. If the number $\dfrac{z-1}{z+1}$ is purely imaginary, then find the locus of z. **[3 Marks]**

10. If $x_r = \cos\left(\dfrac{\pi}{2^r}\right) + i\sin\left(\dfrac{\pi}{2^r}\right)$, then evaluate x, $x_2, x_3 \ldots$ to ∞ **[4 Marks]**

11. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then find arg $\left(\dfrac{z_1}{z_2}\right)$
[4 Marks]

12. If $z = x + iz$ and $z^{\frac{1}{3}} = a - ib$, $a, b \neq 0$ and $\dfrac{x}{a} - \dfrac{y}{b} = k(a^2 - b^2)$, then find k. **[4 Marks]**

13. Let (r, θ) denote a complex number $z = r(\cos\theta + i\sin\theta)$ If $a \equiv (1, \alpha)$, $b \equiv (1, \beta)$, $c \equiv (1, r)$ and $a + b + c = 0$, then evaluate $a^{-1} + b^{-1} + c^{-1}$ **[4 Marks]**

14. Express $\dfrac{(\cos\theta + i\sin\theta)^8}{(\sin\theta + i\cos\theta)^4}$ in $x + iy$ form.
[3 Marks]

MULTIPLE CHOICE QUESTIONS

1. If n is a positive integer, then which of the following relations is false
   (A) $i^{4n} = 1$ (B) $i^{4n-1} = i$
   (C) $i^{4n+1} = i$ (D) $i^{-4n} = 1$

2. The value of $(1 + i)^5 \times (1 - i)^5$ is
   (A) -8 (B) $8i$
   (C) 8 (D) 32

3. If $x = 3 + i$, then $x^3 - 3x^2 - 8x + 15 =$
   (A) 6 (B) 10
   (C) -18 (D) -15

4. If $(1 - i)x + (1 + i)y = 1 - 3i$, then $(x, y) =$
   (A) $(2, -1)$ (B) $(-2, 1)$
   (C) $(-2, -1)$ (D) $(2, 1)$

5. $\frac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}} =$
   (A) $-\frac{3}{2}i$ (B) $\frac{3}{2}i$
   (C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

6. If $\frac{5(-8+6i)}{(1+i)^2} = a + ib$, then (a, b) equals
   (A) (15, 20)
   (B) (20, 15)
   (C) (–15, 20)
   (D) None of these

7. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then
   (A) $a = 2, b = -1$ (B) $a = 1, b = 0$
   (C) $a = 0, b = 1$ (D) $a = -1, b = 2$

8. The conjugate of the complex number $\frac{2+5i}{4-3i}$ is
   (A) $\frac{7-26i}{25}$ (B) $\frac{-7-26i}{25}$
   (C) $\frac{-7+26i}{25}$ (D) $\frac{7+26i}{25}$

9. $\left|(1+i)\frac{(2+i)}{(3+i)}\right| =$
   (A) $-\frac{1}{2}$ (B) $\frac{1}{2}$
   (C) 1 (D) -1

10. $\arg\left(\frac{3+i}{2-i}+\frac{3-i}{2+i}\right)$ is equal to
   (A) $\frac{\pi}{2}$ (B) $-\frac{\pi}{2}$ (C) 0 (D) $\frac{\pi}{4}$

11. The amplitude of $\frac{1+\sqrt{3}i}{\sqrt{3}-i}$ is
   (A) 0 (B) $\pi/6$
   (C) $\pi/3$ (D) $\pi/2$

12. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are
   (A) $\sqrt{2}$ and $\frac{\pi}{6}$ (B) 1 and 0
   (C) 1 and $\frac{\pi}{3}$ (D) 1 and $\frac{\pi}{4}$

13. $\sqrt{-8-6i} =$
   (A) $1 \pm 3i$ (B) $\pm(1 - 3i)$
   (C) $\pm(1 + 3i)$ (D) $\pm(3 - i)$

14. The square root of $3 - 4i$ is
   (A) $\pm(2 + i)$ (B) $\pm(2 - i)$
   (C) $\pm(1 - 2i)$ (D) $\pm(1 + 2i)$

15. $(27)^{1/3} =$
   (A) 3 (B) $3, 3i, 3i^2$
   (C) $3, 3\omega, 3\omega^2$ (D) None of these

16. If ω is a complex cube root of unity, then $(x - y)(x\omega - y)(x\omega^2 - y) =$
   (A) $x^2 + y^2$ (B) $x^2 - y^2$
   (C) $x^3 - y^3$ (D) $x^3 + y^3$

17. If $1, \omega, \omega^2$ are the three cube roots of unity, then $(3 + \omega^2 + \omega^4)^6 =$
   (A) 64 (B) 729
   (C) 21 (D) 0

18. If α and β are imaginary cube roots of unity, then the value of $\alpha^4 + \beta^{28} + \frac{1}{\alpha\beta}$ is
   (A) 1 (B) -1
   (C) 0 (D) None of these

19. If ω is an imaginary cube root of unity, $(1 + \omega - \omega^2)^7$ equals
   (A) 128ω (B) -128ω
   (C) $128\omega^2$ (D) $-128\omega^2$

Time: 1 Hour | **TOPIC TEST** | **Total Marks: 20**

SECTION A

Q.1. Select and write the correct answer. **[4]**

i. The value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$ is equal to:

(A) –2 (B) 1 (C) 0 (D) –1

ii. If ω is a complex cube root of unity, then the value of $\omega^{99}+\omega^{100}+\omega^{101}$ is:

(A) –1 (B) 1 (C) 0 (D) 3

Q.2. Answer the following. **[2]**

i. Express $z = \frac{e^{i\frac{5\pi}{4}}}{\sqrt{3}}$ in polar form.

ii. Find the value of $i^{49}+i^{68}+i^{89}+i^{110}$.

SECTION B

Attempt any two of the following: **[4]**

Q.3. Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^3 = i$

Q.4. If z = 3 + 5i, then represent the z , $\bar{z}$ in Argand's diagram.

Q.5. If ω is the complex cube root of unity, show that
$(2+\omega+\omega^2)^3-(1-3\omega+\omega^2)^3 = 65$.

SECTION C

Attempt any two of the following: **[6]**

Q.6. Solve the following quadratic equation
$2x^2-\sqrt{3}x+1=0$

Q.7. Express the following in the form a + ib, a, b ∈ R, using De Moivre's theorem
$(1+i)^6$

Q.8. Find the value of x^3-x^2+x+46, if $x = 2+3i$

SECTION D

Attempt any one of the following: **[4]**

Q.9. If $x+iy=\sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2+y^2)^2=\frac{a^2+b^2}{c^2+d^2}$.

Q.10. If α and β are complex cube roots of unity, prove that $(1-\alpha)(1-\beta)(1-\alpha^2)(1-\beta^2)=9$.

ANSWERS

ONE MARK QUESTIONS

1. 27i
2. 5
3. $\frac{\pi}{3}$
4. 1
5. $\frac{1}{\sqrt{5}}e^{i\frac{7\pi}{12}}$

ADDITIONAL PROBLEMS FOR PRACTICE

Based on Exercise 1.1

1. i. 2 + 4i, 2, 4
   ii. 0 + 5i, 0, 5
   iii. 3 – 4i, 3, – 4
   iv. 5 + 4i, 5, 4
   v. $2+\sqrt{5}i$, 2, $\sqrt{5}$
   vi. $(7+\sqrt{3})+0i$, $7+\sqrt{3}$, 0

2. 5i

3. i. $5 - 4i$ ii. $1 + 2i$
iii. $\sqrt{5} - 3i$ iv. 12i
v. $-\sqrt{3} - \sqrt{2}\,i$
vi. $\cos 2\theta + i \sin 2\theta$

4. i. $a = -1, b = 3$
ii. $a = \frac{6}{5}, b = -\frac{8}{5}$
iii. $a = -\frac{4}{5}, b = -\frac{7}{5}$
iv. $a = \frac{28}{25}, b = -\frac{29}{25}$
v. $a = -\sqrt{3}, b = 1$
vi. $a = -1, b = 0$

5. i. $-1 + 3i$ ii. $a = b = \frac{5}{4}$
iii. 9

8. i. $x = -4, y = 6$ ii. $x = 2, y = -1$
iii. 1

9. i. 0 iii. -1

10. $-1 + 3i$

Based on Exercise 1.2

1. i. $\pm\sqrt{2}\,(2 + i)$ ii. $2 - i$ or $-2 + i$

2. i. $\pm\frac{1}{\sqrt{2}}(1 - i)$ ii. $\pm(1 - 4i)$
iii. $\pm(3 + 2i)$ iv. $\pm\left(\sqrt{5} - 2i\right)$

3. i. $\frac{-1+\sqrt{3}i}{2}$ and $\frac{-1-\sqrt{3}i}{2}$
ii. $2\sqrt{3}$ and 3i

4. i. $\frac{-1+\sqrt{7}i}{4}, \frac{-1-\sqrt{7}i}{4}$
ii. $\frac{3+i}{5}, \frac{3-i}{5}$

5. i. $-3i, -7i$ ii. $-6i$

6. i. $4 - 3i, 3 + 2i$ ii. $\frac{3}{2} + \frac{1}{2}i, 3i$

7. i. 6 ii. -100

8. i. $-17 + 24i$ ii. 4
iii. -160

Based on Exercise 1.3

1. i. $\sqrt{10}, \tan^{-1}(3)$
ii. $25, \pi - \tan^{-1}\left(\frac{24}{7}\right)$

2. i. $3, \frac{\pi}{2}$ ii. $\sqrt{10}, \tan^{-1} 3$
iii. $5, \tan^{-1}\left(\frac{4}{3}\right)$ iv. $6, \frac{3\pi}{4}$

3. i. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
ii. a. $z = 8\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right), z = 8e^{i\left(\frac{\pi}{3}\right)}$
b. $z = 2(\cos\pi + i\sin\pi), z = 2e^{i\pi}$
c. $z = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right), z = 3e^{i\frac{\pi}{2}}$
d. $z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$,
$z = 2e^{i\left(\frac{5\pi}{6}\right)}$

4. i. $\sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$
ii. $5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$
iii. $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
iv. $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

5. i. $-1 + i$
ii. a. 12i b. $-\frac{1}{2} - \frac{i}{2}$

6. i. $-1 - i$ ii. $\sqrt{3} + i$

Based on Exercise 1.4

2. i. 1 ii. 1

4. i. 64 ii. 2

9. $(x - 3)^2 + y^2 = 5$

10. circle $|\omega + 1| = 10$

11. $x = 0$

12. $x < 0$ or $\text{Re}(z) < 0$

13. $(\cos 2\theta + i \sin 2\theta)$

Based on Miscellaneous Exercise – 1

1. i. $-2+\sqrt{3}i$ ii. $\frac{-11}{125}+\frac{2i}{125}$
   iii. $\frac{19}{13}+\frac{4}{13}i$ iv. $\frac{37}{13}+\frac{16}{13}i$
   v. $-3-i$ vi. $-\frac{10}{13}$
2. i. $x=\frac{5}{13}, y=\frac{14}{13}$
   ii. $x=\frac{14}{15}, y=\frac{1}{5}$
   iii. $x=\frac{5}{4}, y=\frac{5}{4}$
3. i. 0 ii. i
4. i. $\sqrt{2}, \frac{3\pi}{4}$ ii. $\sqrt{2}, \frac{3\pi}{4}$
   iii. $1, \frac{2\pi}{3}$ iv. $\frac{1}{\sqrt{5}}, -\tan^{-1}(2)$
   v. $\sqrt{\frac{2}{5}}, \tan^{-1}\left(\frac{1}{3}\right)$
6. $\frac{\pi}{3}, \frac{2\pi}{3}$ 7. 4
8. 4 9. $|z|=1$
10. – 1 11. 0
12. 4 13. 0
14. $\cos 12\theta + i \sin 12\theta$

MULTIPLE CHOICE QUESTIONS

1.	(B)	2.	(D)	3.	(D)	4.	(A)
5.	(A)	6.	(A)	7.	(B)	8.	(C)
9.	(C)	10.	(C)	11.	(D)	12.	(B)
13.	(B)	14.	(B)	15.	(C)	16.	(C)
17.	(A)	18.	(C)	19.	(D)		

TOPIC TEST

1. i. (D) ii. (C)
2. i. $\frac{1}{\sqrt{3}}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$
   ii. 2i
6. $\left(\frac{\sqrt{3}+\sqrt{5}i}{4}\right)$ and $\left(\frac{\sqrt{3}-\sqrt{5}i}{4}\right)$
7. – 8i
8. 7

COMPETITIVE CORNER

1. Let ω be a complex number such that $2\omega+1=z$, where $z=\sqrt{-3}$. If
$\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega^7\end{vmatrix}=3k$, then k is equal to **[JEE (Main) 2017]**
(A) 1 (B) – z
(C) z (D) – 1

2. If α, β ∈ C are the distinct roots of the equation $x^2-x+1=0$, then $\alpha^{101}+\beta^{107}$ is equal to **[JEE (Main) 2018]**
(A) 0 (B) 1
(C) 2 (D) – 1

3. If α and β be the roots of the equation $x^2-2x+2=0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n=1$ is **[JEE (Main) 2019]**
(A) 2 (B) 5
(C) 4 (D) 3

4. Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right|=1$ and $|z|=\frac{5}{2}$. Then the value of $|z+3i|$ is **[JEE (Main) 2020]**
(A) $\sqrt{10}$ (B) $2\sqrt{3}$
(C) $\frac{15}{4}$ (D) $\frac{7}{2}$

5. If $\frac{3}{2+\cos\theta+i\sin\theta}=a+ib$, then $[(a-2)^2+b^2]$ is equally to **[MHT CET 2021]**
(A) 0 (B) 1
(C) –1 (D) 2

Answers:
1. (B) 2. (B) 3. (C) 4. (D)
5. (B)

Hints:

1. $2\omega+1=\sqrt{-3}$

∴ $2\omega+1=\sqrt{3}i$

∴ $\omega=\frac{-1}{2}+\frac{i\sqrt{3}}{2}$

$\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega^7\end{vmatrix}=3k$

∴ $\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}=3k$

$\therefore \begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = 3k$

...[Applying $R_1 \to R_1 + R_2 + R_3$]

$\therefore$ $3(-\omega^3 - \omega - \omega^4) = 3k$

$\therefore$ $3(-1 - \omega - \omega) = 3k$

$\therefore$ $k = -(1 + 2\omega) = -z$

2. $x^2 - x + 1 = 0$

$\therefore$ $x = \dfrac{1 \pm \sqrt{3}i}{2}$

α and β are the roots of the given equation.

$\alpha = \dfrac{1 + \sqrt{3}i}{2} = -\omega$

$\beta = \dfrac{1 - \sqrt{3}i}{2} = -\omega^2$

$\alpha^{101} + \beta^{107} = (-\omega)^{101} + (-\omega^2)^{107}$

$= -\left[\left(\omega^3\right)^{33} \cdot \omega^2 + \left(\omega^3\right)^{71} \cdot \omega\right]$

$= -(\omega^2 + \omega)$ ...$[\omega^3 = 1]$

$= 1$ ...$[1 + \omega + \omega^2 = 0]$

3. $x^2 - 2x + 2 = 0$

$\Rightarrow x = \dfrac{2 \pm \sqrt{4-8}}{2} = \dfrac{2 \pm 2i}{2} = 1 \pm i$

Let $\alpha = 1 + i$ and $\beta = 1 - i$

$\dfrac{\alpha}{\beta} = \dfrac{1+i}{1-i}$

$= \dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i}$

$= \dfrac{(1+i)^2}{1-i^2}$

$= \dfrac{2i}{2} = i$

$\left(\dfrac{\alpha}{\beta}\right)^n = 1$ ...[Given]

$\Rightarrow (i)^n = 1$

$\Rightarrow$ least value of $n = 4$

4. Let $z = x + iy$

$\left|\dfrac{z-i}{z+2i}\right| = 1$

$\therefore$ $|z - i| = |z + 2i|$

$\therefore$ $x^2 + (y-1)^2 = x^2 + (y+2)^2$

$\therefore$ $y = -\dfrac{1}{2}$.

$|z| = \dfrac{5}{2}$

$\therefore$ $\sqrt{x^2 + y^2} = \dfrac{5}{2}$

$\therefore$ $x^2 + \left(-\dfrac{1}{2}\right)^2 = \dfrac{25}{4}$

$\therefore$ $x^2 = 6$

$\therefore$ $x = \pm\sqrt{6}$

$\therefore$ $z = \pm\sqrt{6} - \dfrac{1}{2}i$

$\therefore$ $|z + 3i| = \sqrt{6 + \dfrac{25}{4}} = \sqrt{\dfrac{49}{4}} = \dfrac{7}{2}$

5. Given, $\dfrac{3}{2 + \cos\theta + i\sin\theta} = a + ib$

$\Rightarrow \dfrac{3\left[(2+\cos\theta) - i\sin\theta\right]}{(2+\cos\theta)^2 + \sin^2\theta} = a + ib$

$\Rightarrow \dfrac{3[2 + \cos\theta - i\sin\theta]}{5 + 4\cos\theta} = a + ib$

$\Rightarrow a = \dfrac{3(2+\cos\theta)}{5+4\cos\theta}$

and $b = -\dfrac{3\sin\theta}{5+4\cos\theta}$

$\therefore$ $(a-2)^2 + b^2 = \left(\dfrac{6+3\cos\theta}{5+4\cos\theta} - 2\right)^2 + \dfrac{9\sin^2\theta}{(5+4\cos\theta)^2}$

$= \dfrac{(-4-5\cos\theta)^2 + 9\sin^2\theta}{(5+4\cos\theta)^2}$

$= \dfrac{16 + 25\cos^2\theta + 40\cos\theta + 9\sin^2\theta}{(5+4\cos\theta)^2}$

$= \dfrac{(5+4\cos\theta)^2}{(5+4\cos\theta)^2} = 1$

Scan the given Q. R. Code in *Quill - The Padhai App* to view the solutions of the Topic Test.