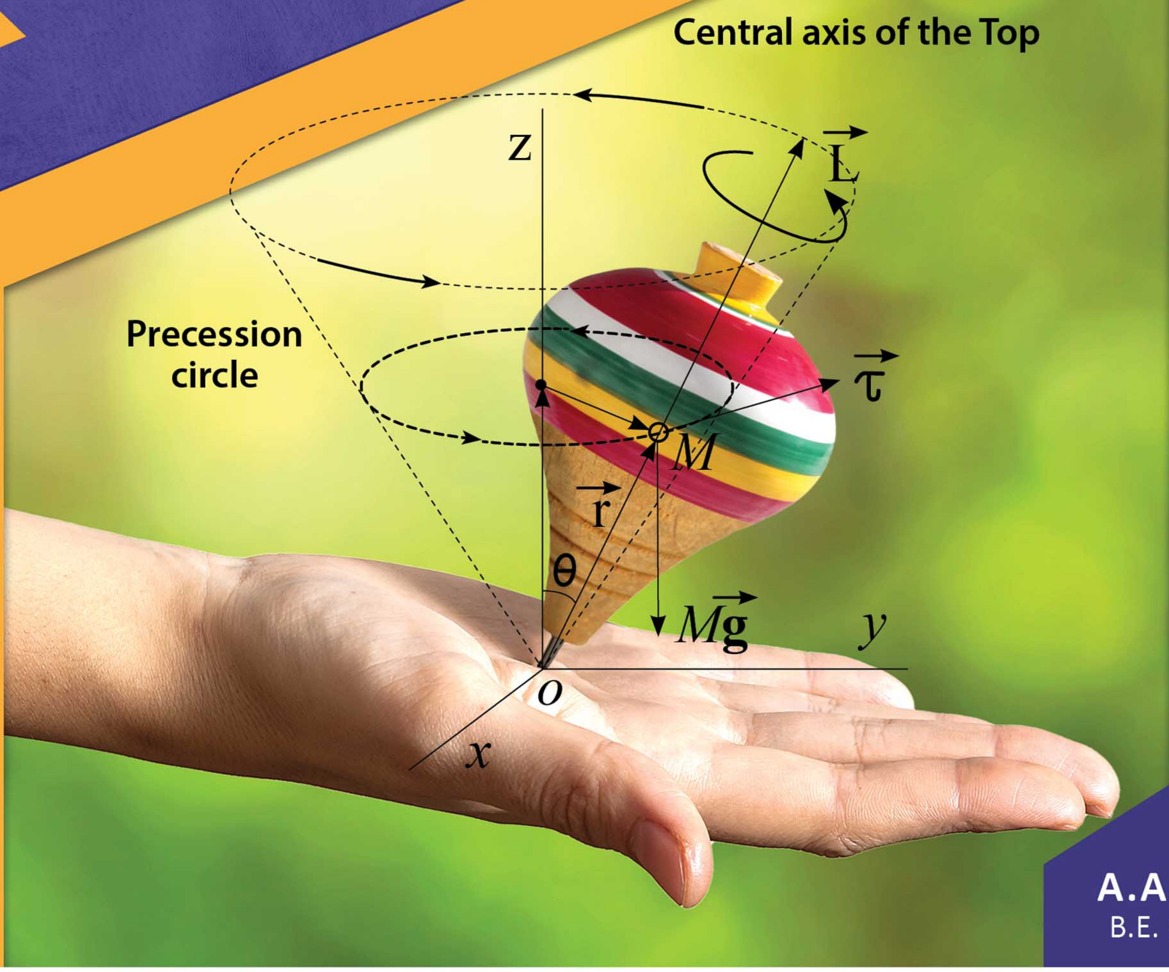
SAMPLE CONTENT

Basic concepts of PHYSICS* Simplified

*(Only Mechanics for Jr. College, Std. XI, XII)



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Basic concepts of

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PREFACE

Mechanics is a branch of Physics that is considered to be the **most difficult** subject by many students. The challenge gets more formidable since a major portion of total marks in Physics exams are allotted to questions on Mechanics.

The first important step for excelling in mechanics is to obtain a very good **understanding of its basic concepts.** Hence, it is crucial to study this subject by following a systematic approach. Difficult topics of Mechanics can also become easier once its concepts are clearly understood.

These are the main considerations that directed the format of this book. Students will notice that the focus throughout this book is on presenting basic concepts of mechanics with a maximum degree of clarity. Each topic is explained with the help of suitable diagrams and graphs, in a language that is simple and easy to understand. Therefore, it is expected that students will derive maximum benefits by making use of this presentation.

But, simplifying a difficult subject has its own challenges. It is very easy to slip-up and misrepresent essential aspects of theory in attempting to simplify it. Avoiding such pitfalls and realizing this unique goal was indeed challenging, but was facilitated by many friends who also share the same passion. I want to acknowledge and thank these friends for their contributions for improving the text of this book.

I am deeply grateful to Dr. K G Bhole, MSc, DHE, PhD, ex- HOD Physics- Vaze college & coordinator IGNOU, who checked the entire manuscript and provided valuable feedback and several important suggestions. Mr. Avinash Marathe, BE (*my engineering batchmate*, ex-*BARC and TCS executive, who retired from Tech-Mahindra as Vice President*), and Prof. O A Ramdasi, MSc, MPhil and professor of Physics at Ruia college, extended all the help for editing the draft and gave extremely valuable suggestions for improving the draft. I thank them both for their insights and suggestions. I would also like to thank Dr. D. G. Bagul, PhD (Nuclear Physics), ex-sr. Scientist BARC, Prof. E G Ghatpande, MSc, ex- prof of Physics at Vaze college and Mr. A B More, who gave valuable inputs to improve the draft.

Dr. Prakash G Dixit, MSc, MPhil, PhD, Head of dept. of Statistics and Vice principal at Modern college, Dr. A V Deodhar and Mr. J V Kadhe, extended all the help to introduce me to several leading faculties in Physics for editing the book. I am grateful for their help.

Incidentally, the choice of a spinning toy-top on the cover page was relatively easy because understanding mechanics is somewhat similar to trying to spin the toy-top on your palm. Initially, it is extremely difficult to catch the spinning toy-top over the palm of your hand. But once you master the technique, it turns out to be an exciting skill and great fun.

A A Tilak

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About the Book

Mechanics is a science of motion, the oldest amongst all the physical sciences. All the types of motions, from the path of a cricket ball hit by a batsman, to the path of a 'Mangal-yaan' space probe sent to Mars, come within the scope of its study. Analysis of motions of different vehicles, trains, airplanes, and satellites (*as well as analysis of collisions and interactions between atomic particles*), are part of various topics of mechanics. We experience most of these motions as part of our daily life. Therefore, understanding various facets of these motions makes the study of mechanics an interesting experience.

The study of mechanics can be broadly divided into two segments. **Dynamics** relates to the study of forces and properties of moving objects. And **Kinematics** is a part of mechanics that uses equations to describe and represent the motion of objects.

With this book, we are embarking on the study of different types of motions. We observe that different objects can also rotate, vibrate or oscillate when they move. A cricket ball can be made to spin or swing when it is bowled. An arrow vibrates as it follows its trajectory to its target. A car can swing sideways due to high-velocity winds as it moves forward.

These composite motions are quite difficult to analyze at the beginner's level as we start our study of mechanics. The complications described above need not be considered in the motion of an **ideal object** called **'particle'**. Mathematically, a particle is treated as a point or an object without any dimensions. Hence complex motion considerations are not required to be considered when we study the motion of a particle.

However, in the real world, there is no such thing as an object without dimensions. But the concept of a **particle** is very useful because real objects often behave as particles to a very good degree of approximation. (*In fact, any object can be treated as a particle of the same weight situated at the center of gravity of the object*). Hence, for the sake of simplification, initially, we will confine our analysis of motions to that of a **particle-like object**. (*However, in this book, we will continue to use the term* ***object*** *instead of a* ***particle-like object*** *since it is convenient to use the word* ***object*** *rather than* ***particle-like object*** *every time. Besides this, one can easily relate to different motions of an* ***object*** *rather than that of a* ***particle***).

We will initially assume that there is no resistance due to friction or air resistance to the movement of an object, to further simplify our analysis.

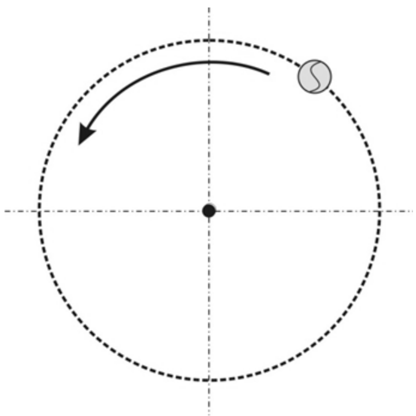
Motion-related variables like displacement, velocity, and acceleration of an object are all vector quantities. However, we can limit our analysis of the motion of an object to one axis to avoid using vectors. One axis movement of such an object has only two directions –right and left or up and down or clockwise and anti-clockwise– available to it for its movement. Hence, we can indicate these directions with plus or minus signs and avoid the use of vectors in the beginning. This 'one axis motion' is considered **initially** in this book to simplify the analysis of different motions.

(*It would be a lot easier for students to take up the study of vectors at a later stage after understanding the concepts of different motions. Vectors play an important role in finding solutions to more complex problems* *when you need to analyze the motion of an object in three dimensions. Vector analysis is extensively covered later in chapter no. 8 on **Scalars and vectors***).

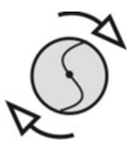
Let us first get familiarized with different types of motions with the help of an example of a tennis ball.

1. Linear motion.

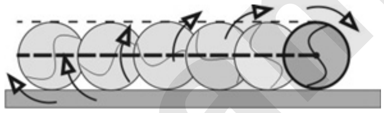
2. Circular motion.



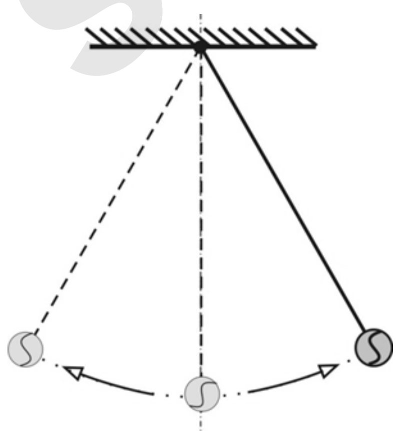
3. Rotational motion.



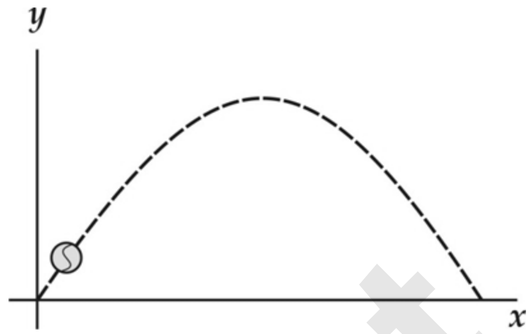
4. Rolling motion.



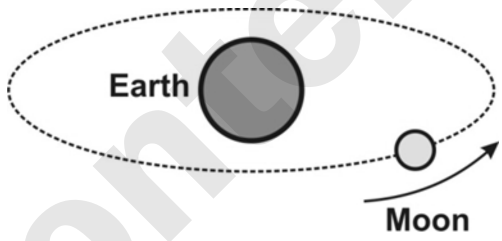
5. Oscillatory motion.



6. Projectile motion.



7. Orbital motion.



Our objective is to analyze every motion and then develop mathematical equations for each motion. Throughout this book, a major emphasis is given to building a strong foundation of basic concepts of different motions. Diagrams and graphs are an important part of any scientific presentation. Students will notice that the text in every chapter of this book is generously supported with diagrams and graphs to make that topic easy to understand. Students are encouraged to devote adequate time to read every chapter carefully and get a firm grip on the basic concepts of each motion. They will derive maximum benefits if the core ideas and basic concepts of different motions are thoroughly understood.

The depth of our understanding of basic concepts of mechanics is severely tested when we sit down to solve its application examples (*which are generally not easy*). This is because one needs to use basic concepts of mechanics along with analytical skills to visualize and evolve a precise roadmap leading to the answer to solve any problem.

It is important to first study and understand **“how to follow a systematic approach for solving these problems”.** Therefore, more than 200 application examples along with their solutions are included in this book. **A detailed analysis of the logic and the line of thinking that is used to evolve a roadmap to every solution is also provided**. **Solution to every problem is further simplified by providing a free-body diagram of the forces acting on an object along with a sketch or a diagram as a visual illustration of the problem.** Students will benefit immensely by studying the analysis of these solutions.

(*Another equally effective but slightly difficult way is to first solve few examples from every chapter of this book without looking at their solutions. You can cross-check your method and your answers later by comparing them with the solutions given in the book. You may further improve your analytical skills by solving more problems either from old exam papers or other Physics books*).

Newton’s three laws of motion are the foundation of modern mechanics. These three laws can be used to translate different motions into their corresponding mathematical expressions. Let us analyze and understand these three laws in the first chapter before moving on to the analysis of different types of motions.

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Note:- *It is expected that the reader of this book has a basic knowledge of Calculus. It is easier to explain acceleration as the derivative of velocity, and velocity as the integral of acceleration by using the language of Calculus*. (***Calculus was developed by Newton to explain and define the equations of motion***).

01 Newton's Laws of Motion

Introduction

Analysis of planetary motions has engaged the interest of many scientists/astronomers for more than two thousand years. Several astronomers like Ptolemy, Aryabhata, Bhaskaracharya, Copernicus, Galileo, Brahe, and Kepler have made significant contributions towards the understanding of different physical characteristics of planetary motions.

But it was Newton who stated three **laws of motion** to accurately represent different types of motions with mathematical equations. He was able to precisely describe all the types of motions, right from the motion of a falling apple to the orbital motion of planets, based on his three laws of motion and the law of universal gravitation. His complete work was published in the classic book 'Principia Mathematica' in 1687. Newton's laws of motion have triggered breakthroughs in many areas of science and laid the foundation for unprecedented technological advances made during the last 300 years.

Therefore, a comprehensive understanding of Newton's laws of motion is an important part of the study for every student of Physics.

1.1 Newton's first law of motion:

'Every object persists in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed'. (*Translated from Newton's 'Principia Mathematica'*).

We can rephrase Newton's first law of motion into a simpler language as follows:-
'An object at rest continues to remain at rest, or if it is in motion, continues to move at the same constant speed in the same direction, unless it is acted upon by a net external force'.

For our analysis, let us simplify Newton's first law by splitting it into three parts as follows.

First part: An object at rest continues to remain at rest.

Second part: An object in motion continues to move at a constant speed in the same direction.

Third part: Speed of an object, either at rest or moving at a constant speed, changes only when a net external force acts on the object.

All of us can readily agree with the first part of this law as it is in line with our everyday experience. We have always observed that any object lying on a flat surface does not start moving on its own.

The property of an 'object at rest' to continue to remain at rest is called its 'Inertia'.

We know from our experience that some objects have more Inertia than others. We can observe that a cricket ball has more Inertia than a table tennis ball. Also, a big truck has more Inertia than a small car. We may state that heavier things have more Inertia.

The second part of the law does not match our usual experience. We have always observed that whenever we throw a cricket ball, it does fall to the ground and eventually comes to a standstill after rolling on for some time. However, as per Newton's first law, the ball is supposed to continue to move indefinitely with a constant speed in the direction of our throw.

There are a few reasons why this does not happen. The cricket ball first slows down due to air resistance to its motion. Then it falls to the ground due to gravity and eventually stops moving due to its friction with the ground. We can theoretically argue that the cricket ball would keep on moving indefinitely with constant speed if there is no air resistance, no gravity, and no friction.

For several centuries, astronomers and scientists had observed that all the planets in our solar system were moving around the sun in their respective orbits with different speeds. However, the most **surprising thing** they observed was that each planet was moving in its orbit with a **constant speed** (*though this constant speed was different for each planet*) even when observed repeatedly over many years.

Newton, with his tremendous insight and abstraction, must have visualized by looking at the orbits of these planets (*where there is no air resistance, no gravity, and no friction*), that in a similar environment, any object in motion would also keep moving indefinitely at a **constant speed** like the planets. He may have also visualized that this **constant speed** of a moving object can change only if a **net external force** is applied to it.

Newton captured all these observations into his 'first law of motion' and thus laid the foundation of classical physics.

We can now broaden the earlier definition of **Inertia** of **'object at rest'** by including **'object in motion'** in its description.

The property of an 'object at rest' to continue to remain at rest, or an 'object in motion' to continue to remain moving at the same constant velocity is called its Inertia.

Because of its heavy emphasis on Inertia, Newton's first law of motion is also called the **law of Inertia**.

We can also think of Newton's first law as a law relating to an object maintaining its **'status quo'** in its existing condition, whether it is 'of rest' or 'of motion'.

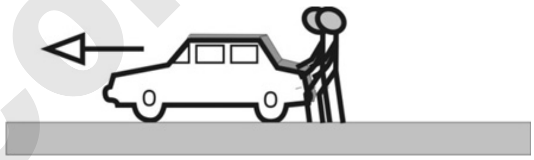
The Inertia of an object is measured by its mass. And mass can be determined by measuring how difficult it is to move an object. (*Twice the mass will need twice the effort to move the object*). The more mass an object has, the harder it is to move it, as it has more Inertia.

The **unit of measurement** of **the mass** of an object in **SI units** is **kilogram** or **kg.**

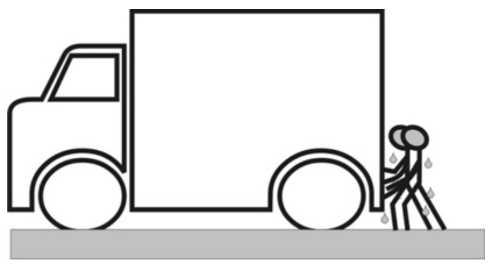
We can easily move a chair as it has a smaller mass. But it is difficult to move a steel cupboard as it has a bigger mass. As stated earlier, heavier things have a bigger mass or bigger Inertia.

Let us see a couple of examples to bring more clarity to the concept of Inertia.

Case 1:-
With some reasonable effort, two people can move a small car by pushing it.

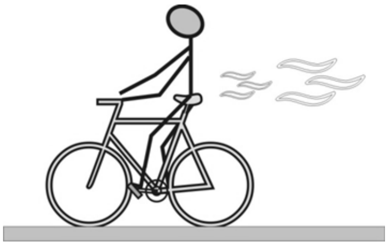


But two people cannot move a big truck, no matter how hard they push it. You may need more than five people to push and move a big truck. This is because a truck has a higher mass or a higher weight or a higher Inertia than a small car.



Case 2:-
On a bicycle, we can reach fairly high speeds of up to **15** to **20 km/hour** on a flat road if we pedal the bicycle fast enough. After that, even if we suddenly stop pedaling, the bicycle keeps on moving at the almost same speed for quite some distance due to its **Inertia** (*mass of a bicycle and a rider*). This is an example that explains the **'third part'** of Newton's first law. (*Speed of an object moving at a constant*

speed, will change only when a net external force acts on the object). This bicycle moving at a constant speed can be stopped by applying a **net external opposing force** on the bicycle in the form of brakes.



1.1.1 Net external force:

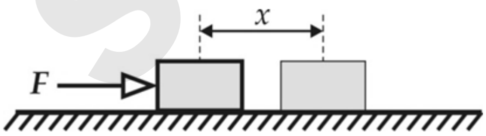
Let us bring some more clarity to the term 'Net external force'.

Two external forces of the same magnitude acting on the same object in opposite directions will nullify each other and hence will not result in any **net external force**.

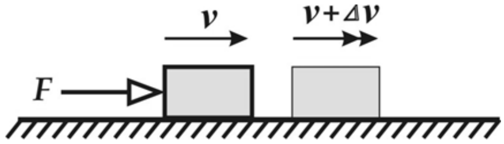
Also, if we are sitting inside a car, and try to push the car from **inside**, the car does not move as there is **no external force** acting on the car. The car can be moved only if we push it by standing outside of the car as **net external force** gets applied to the car only when we push it from outside.

We will see two examples of Newton's first law about how the **status quo** position of an object can be changed by applying a **net external force.**

Example 1:
An object at rest can be moved from its position of rest by applying **net external force** $\boldsymbol{F}$.



Example 2:
Let us assume that an object is initially moving at a constant velocity $\boldsymbol{v}$. The object will accelerate and will start moving at a new constant velocity ($\boldsymbol{v}+\Delta\boldsymbol{v}$) if a net external force $\boldsymbol{F}$ is applied to the object in the direction of motion.



1.2 Newton's second law of motion:

Newton's first law states that an object moving at a **constant velocity** will continue to move at the same **constant velocity** provided there is no **net external force** acting on it.

Thus, Newton's **first law of motion** refers to a simple case where **net external force** acting on an object is **zero**.
However, Newton's **second law of motion** refers to a situation when there is a net **external force** acting on an object.

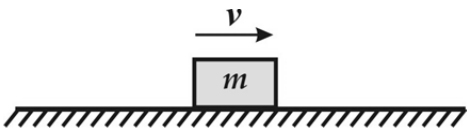
Newton's second law of motion states that **'the rate of change of linear momentum of an object is directly proportional to the net external force applied to the object'**. (*The change in momentum takes place in the direction of the net force*).

Let us take a pause here to first understand the concept of **Momentum** before analyzing Newton's second law.

Brick has a larger mass than that of a cricket ball. We can catch a cricket ball thrown from the top of a three-storied building. But we cannot catch a brick thrown from the top of a three-storied building, because the force with which it will strike our hand is far more than that of a cricket ball. Therefore, the mass $\boldsymbol{m}$ of an object plays an important part to decide the **amount of force** with which a moving object strikes our hand.

But we cannot catch the same cricket ball thrown from the top of a twenty-storied building. The force with which it strikes our hand is far more now since it's moving at a far higher speed. Hence, the speed of an object or its velocity $\boldsymbol{v}$ plays an equally important part to decide the **amount of force** with which a moving object strikes our hand.

Therefore, taken together, the product of mass $\boldsymbol{m}$ and velocity $\boldsymbol{v}$ becomes an **important new parameter** called **linear momentum** $\boldsymbol{p}$, which influences the magnitude of the **resultant force** of an object in motion. The momentum $\boldsymbol{p}$ of an object in motion at any instant of time is defined as the product of mass $\boldsymbol{m}$ of an object, and velocity $\boldsymbol{v}$ at which the object is moving at that instant of time.



The mathematical expression of linear momentum $\boldsymbol{p}$ of an object is as follows.

$$\boldsymbol{p} = \boldsymbol{mv}$$

1.2.1 Derivative method:

Mathematical expression for Newton's second law of motion can be derived by using derivative method as follows.

The rate of change of momentum can be expressed as $\frac{dp}{dt}$ in the language of calculus.

As per Newton's second law, the **rate of change of momentum** $\frac{dp}{dt}$ **is proportional to the net external force** $\boldsymbol{F}$. This can be written mathematically as follows.

$$F \propto \frac{dp}{dt}$$

$$\therefore \quad F \propto \frac{d(mv)}{dt}$$

$$\therefore \quad F = K\frac{d(mv)}{dt} \quad \ldots\ldots\ldots (1)$$

Where $\boldsymbol{K}$ is the constant of proportionality. For a given object, the mass $\boldsymbol{m}$ of an object is constant. Hence, we can rewrite (1) as follows.

$$F = K\,m\frac{dv}{dt} \quad \ldots\ldots\ldots\ldots (2)$$

$\frac{dv}{dt}$ is the **rate of change in velocity** of an object, which is the **acceleration** $\boldsymbol{a}$ of an object. Hence, we can rewrite (2) as follows.

$$F = K\,ma \quad \ldots\ldots\ldots\ldots (3)$$

The unit of measure for **force** $\boldsymbol{F}$ was decided based upon Newton's second law. **One unit force** was defined as the force required for accelerating **1 kg** mass by **1 m/sec^2**. Hence the value of constant $\boldsymbol{K}$ automatically evolved as equal to **1**. We can now rewrite equation (3) as follows.

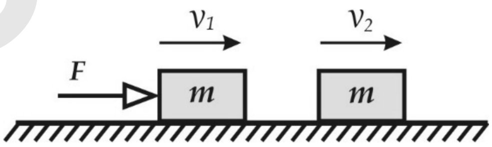
$$F = ma \quad \ldots\ldots\ldots\ldots (4)$$

This is the mathematical expression of Newton's second law.

1.2.2 Algebraic method:

Mathematical expression for Newton's second law of motion can also be derived by using Algebraic method as follows.

Let us assume that an object of mass $\boldsymbol{m}$ is moving with a velocity of $\boldsymbol{v_1}$. Therefore, the initial momentum of the object is $\boldsymbol{p_1 = mv_1}$. External force of magnitude $\boldsymbol{F}$ is exerted on the object in the direction of its motion for time $\boldsymbol{t}$ to increase its velocity to $\boldsymbol{v_2}$. Therefore, the momentum of the object gets increased to $\boldsymbol{p_2 = mv_2}$.



We observe that the change in the magnitude of momentum of the object due to the application of force $\boldsymbol{F}$ is equal to $(\boldsymbol{p_2} - \boldsymbol{p_1})$. And the rate of change in momentum of the object is equal to $\frac{p_2 - p_1}{t}$.

Newton's second law of motion states that **'the rate of change of linear momentum of an object is directly proportional to the net external force applied to the object'.**

$$\therefore \quad F = \frac{p_2 - p_1}{t}$$

We can simplify this momentum equation by substituting values of $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ which we have calculated earlier.

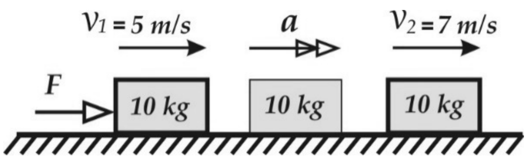
$$\therefore \quad F = \frac{mv_2 - mv_1}{t}$$

$$= m\left(\frac{v_2 - v_1}{t}\right)$$

$$\therefore \quad F = ma \quad \ldots\ldots\ldots\ldots (4)$$

This is the same mathematical expression of Newton's second law which we have calculated earlier by using derivative method.

Example :
Let us assume that an object having mass of **10 kg** is moving at a constant velocity of **5 m/s**. A net external force F is applied to that object for 1 sec to increase its velocity to **7 m/s**. Our objective is to find out the magnitude of the force F.



Let us first calculate initial and final momentums of the object by using initial and final velocities of the object as follows.

$$p_1 = mv_1$$
$$= 10 \times 5$$
$$= 50 \text{ kgm/s}$$

And $p_2 = mv_2$
$$= 10 \times 7$$
$$= 70 \text{ kgm/s}$$

We will now use following momentum equation, representing the Newton's second law to calculate the magnitude of force F.

$$\therefore \quad F = \frac{p_2 - p_1}{t}$$
$$= \frac{70 - 50}{1}$$
$$= 20 \text{ newton}$$

We will once again calculate the magnitude of force F by using equation $F = ma$ of Newton's second law.

We observe that we will have to first calculate the acceleration a of the object as follows.

$$a = \frac{v_2 - v_1}{t}$$
$$= \frac{7 - 5}{1}$$
$$= 2 \text{ m/sec}^2$$

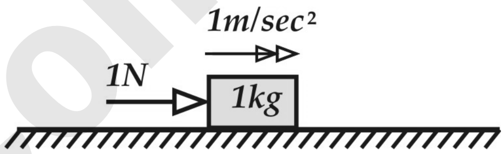
Therefore, $F = ma$
$$= 10 \times 2$$
$$= 20 \text{ newton}$$

(Note:- The magnitude of force F calculated by both the methods will be the same).

The unit of measure for force, **'newton'** (Abbreviation **N**) was derived using this mathematical expression of Newton's second law of motion, $F = ma$.

(In **SI** units, the unit for measurement of force **'newton'** was chosen to honor **Isaac Newton**, for introducing the concept of force. The unit of measure for mass is **kg**, and the unit of measure for acceleration a is **m/sec/sec** or **m/sec^2**. Since the value of constant K is equal to one, therefore, the unit of measure of force is **1 N = 1 kg m/sec^2**).

1 newton force is defined as the force required to accelerate the object of **1 kg** mass by **1 m/sec^2**.



$F = ma$ OR Force = Mass × Acceleration

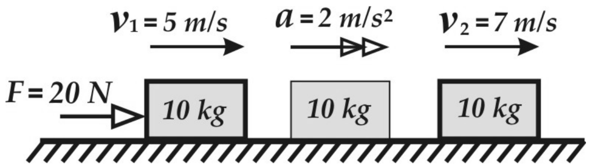
Newton's second law gives us the definition of force. Newton's second law also tells us how much an object of a certain **mass** will **accelerate** when a **net external force** of a certain magnitude is applied to it. It gives us a quantitative expression linking **net external force F to the mass m and acceleration a** of an object.

(*When $F = 0$ then $a = 0$. This means the object is not experiencing any acceleration when there is no external force acting on it. In other words, it means the object will continue to remain at rest or continue to move at the same velocity as long as it is not acted upon by an external force. This is the statement of Newton's first law. Hence, Newton's first law is consistent with the second law, as a special case*).

Newton's second law also gives us a specific expression for determining how the **acceleration** (*and velocity*) of an object **change** under the influence of **net external force.** ($F = ma$).

We observe that acceleration $\boldsymbol{a}$ is directly proportional to the net force $\boldsymbol{F}$, and is inversely proportional to the mass $\boldsymbol{m}$. For example, if the net force $\boldsymbol{F}$ was doubled, the acceleration $\boldsymbol{a}$ of the object will also be twice as large. However, if the mass $\boldsymbol{m}$ of the object was doubled, the acceleration would be half as large. We will see a few examples to bring further clarity to these statements.

Example 1:



Let us assume that a mass of **10 kg** is moving at a constant velocity of **5 m/s**. A net external force of **20 newtons** is applied to that mass for **1 sec**. Our objective is to find out the acceleration of the object, and the new constant velocity with which the object will move as a result of applying this net force for 1 sec.

$$\boldsymbol{F} = \boldsymbol{ma}$$

$$\therefore \quad \mathbf{20} = \mathbf{10} \times \boldsymbol{a}$$

$$\therefore \quad \boldsymbol{a} = \mathbf{2\ m/sec^2}$$

Hence the acceleration of the object will be equal to 2 m/sec^2.

We also know that acceleration $\boldsymbol{a} = \dfrac{\boldsymbol{v_2} - \boldsymbol{v_1}}{\boldsymbol{t}}$

In this expression, $\boldsymbol{v_2}$ is the final velocity, and $\boldsymbol{v_1}$ is the initial velocity of the object. And $\boldsymbol{t}$ is the time of acceleration, which is equal to **1 sec** in this case. Hence, by substituting these values we will get the following expression.

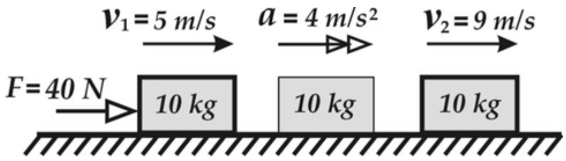
$$\mathbf{2} = \frac{\boldsymbol{v_2} - \mathbf{5}}{\mathbf{1}}$$

$$\therefore \quad \boldsymbol{v_2} = \mathbf{7\ m/s}$$

Hence the 10 kg object will be moving with a new constant velocity of 7 m/s after applying the external force of 20 N for 1 sec.

Example 2:

Let us find out acceleration if we apply a net external force of **40 newtons** for **1 sec** to the same mass of **10 kg** moving at a constant velocity of **5 m/s**.



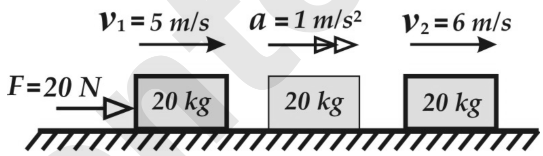
$$\boldsymbol{F} = \boldsymbol{ma}$$

$$\therefore \quad \mathbf{40} = \mathbf{10} \times \boldsymbol{a}$$

$$\therefore \quad \boldsymbol{a} = \mathbf{4\ m/sec^2}$$

Hence the acceleration of the object will be equal to 4 m/sec^2. We observe that, as we increased the force to twice the original value, the acceleration also increased twofold in the direction of motion.

Example 3:



Let us find out acceleration if we apply a net external force of **20 newtons** for **1 sec** to a mass of **20 kg** moving at a constant velocity of **5 m/s**.

$$\boldsymbol{F} = \boldsymbol{ma}$$

$$\therefore \quad \mathbf{20} = \mathbf{20} \times \boldsymbol{a}$$

$$\therefore \quad \boldsymbol{a} = \mathbf{1\ m/sec^2}$$

Hence the acceleration of the object will be equal to 1 m/sec^2. We observe that, as we increased the mass, acceleration decreased.

1.3 Difference between mass and weight of an object:

Before moving on to Newton's third law, we will again pause to examine the difference between the mass and weight of an object.

It is important to understand that mass and weight are two entirely different parameters. Let us consider an object of mass $\boldsymbol{m}$ that is released near the surface of the earth. It will fall downward with free-fall acceleration of $\mathbf{g} = \mathbf{9.8\ m/s^2}$ due to Earth's gravitational attraction force $\boldsymbol{F}$. We can use Newton's second law ($\boldsymbol{F} = \boldsymbol{ma} = \boldsymbol{mg}$), to calculate this gravitational force $\boldsymbol{F}$ acting on the object, which produces this acceleration $\mathbf{g}$.

Let us take a look at a different situation where we hold the object in our hands instead of releasing it. The acceleration of the object is zero. Therefore, the net downward force acting on it is also zero as per Newton's second law. But we have not **switched off** the Earth's gravitational force of attraction for the object. This force is still acting on the object and it is again equal to $\boldsymbol{F} = \boldsymbol{mg}$. Our hands must exert an **upward force** on the object which is equal in magnitude to the downward force $\boldsymbol{mg}$ so that the net force acting on the object is zero when we hold the object in our hands. (*These two forces are not action-reaction pair related to Newton's third law, as they are acting on the same object*). We can feel the presence of this **upward force** by the tension in our hand muscles. In this way, we are experiencing Earth's gravitational force of attraction for the object.

The downward force of the Earth's gravity acting on the **mass** of an object is the **weight** of that object. The force of the Earth's gravity acting on an object is the same whether the object is falling or is at rest. We can measure the weight of an object directly if we place it on a platform scale/bathroom scale. The display of the scale indicates the magnitude of the force that the object is exerting on the surface of the scale. This force is in the downward direction and has a magnitude of $\boldsymbol{mg}$.

∴ **Weight of an object = mass of an object × g**

$$\boldsymbol{W} = \boldsymbol{mg} \quad \ldots\ldots\ (5)$$

Weight is measured in **force units** such as **newton**, (*or kg m/s*^{2}). [*However, the mass is measured in kg (kilograms)*].

Instead of saying that the weight of a person is 60 kg, we should be saying that the mass of the person is 60 kg, and the weight of the person is (60 X 9.8 =) 588 newton. **But we hardly refer to the weight of an object in terms of newton**. We tend to consider 60 kg mass as 60 kg weight because we have been measuring every object in terms of its weight (*which is its mass*) for hundreds of years even before Newton defined his three laws of motion. This practice continues as before, and it is accepted by everyone that masses of all the objects are referred to as their weights on earth. Hence, all the weighing scales are calibrated to read weights of objects in kilogram (*mass units*), rather than in newton (*weight units or force units*).

Therefore, every time we come across an 'Example' where the weight of an object is given in terms of kilogram (*i.e., say equal to 5 kg*)**, then we should consider it as the mass of that object for its linear motion calculations.** (*Therefore, for calculations involving free fall of an object under gravitational acceleration* **g**, *where the weight of an object is given as 5 kg, we should consider it as a mass of an object, and use equation* $\boldsymbol{F} = \boldsymbol{ma} = \boldsymbol{mg}$ *to get gravitational force acting on an object under free fall.*
∴ $\boldsymbol{F} = \boldsymbol{mg} = 5 \times 9.8 = 49\ N$).

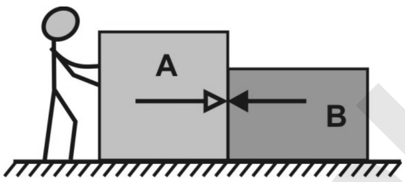
Everything is fine with this arrangement as long as we are somewhere on earth where gravitational acceleration is **g** = 9.8 m/s^2. But gravitational acceleration on the moon is $\mathbf{g}_m$= 1.63 m/s^2. Therefore, a person weighing 60 kg on earth, will weigh only 10 kg on the Moon. But the mass of the person will be still 60 kg on the moon even though he weighs 10 kg on the moon. We will get wrong results in motion calculations on the moon if we consider the mass of the person as 10 kg because he weighs 10 kg on the moon. **We observe that the mass of an object remains the same on any planet**. Therefore, the same amount of force applied on the object of the same mass will produce the same acceleration at any location on earth or any other planet. (*But the weight of the same object will be different on different planets corresponding to different free-fall accelerations on those planets*).

You may sometimes see an expression like **1 kg mass = 9.8 newton weight**. This strictly cannot be correct as the units of mass and weight are different. At best it can be taken to mean that at a location anywhere on Earth (*where* **g** = ***9.8 kg m/s²***), the weight of an object of mass of **1 kg** will be equal to **9.8 newtons or 9.8 N**. On the surface of the moon, the same equation will be written as **1 kg mass = 1.63 N weight**. (*The weight of an object of mass of 1 kg will be equal to 0.163 kg on the Moon*).

1.4 Newton's third law of motion:

Newton's **first law of motion** refers to the simple case where **net external force** acting on an object is **zero**.
Newton's **second law of motion** refers to how an object's **linear momentum** changes when there is a net **external force** acting on it.

Newton's **third law of motion** refers to the fact that when **object A** exerts an **external force** (*action force*) on **object B**, then **object B exerts an external force** (*reaction force*) of the same magnitude instantly on **object A** in **opposite direction**.



Newton's third law of motion states that **'For every action, there is always an equal and opposite reaction'**.

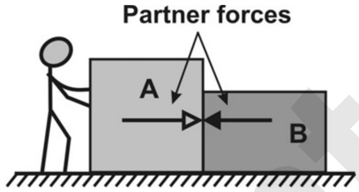
This means that you cannot just have **object A** exerting force on **object B**, without that **object B** also exerting an equal force in opposite direction on **object A**.

There is a simpler way of stating Newton's third law as follows.

Forces always occur in pairs. Force exerted on an **object A** by **B** is equal and opposite of the force exerted on **object B** by **A**. The force exerted on **B** by **A** and the force exerted on **A** by **B act at the same instant**.

(*This law is* ***not the case of cause and effect****, as the 'effect' inherently has a delay element associated with it. As stated earlier, the force* exerted *on object* ***B*** *by* ***A*** *and the force* exerted *on object* ***A*** *by* ***B act at the same instant***).

These **two equal and opposite forces acting on two different objects** are termed **force pairs** or **partner forces**. This is shown in the following figure.



As we can see, forces of action and reaction are acting on **A** and **B**, which are two different objects.

$$F_{BA} = -F_{AB}$$

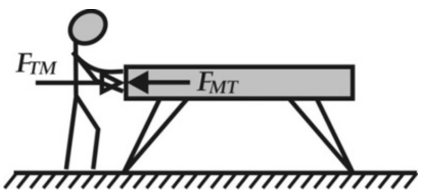
(***Force*** *exerted on object* **B** *by* **A)**

= – **(*Force*** *exerted on object* **A** *by* **B)**

Here '**– ve sign**' indicates force in opposite direction.
Therefore, when we want to analyze the motion of **any one object** (*of either* **A** *or* **B**), only one of the two forces, which is acting on that object needs to be considered. **It is not correct to add these two partner forces and conclude that the net force acting on that object is zero, as these partner forces are not acting on the same object**.

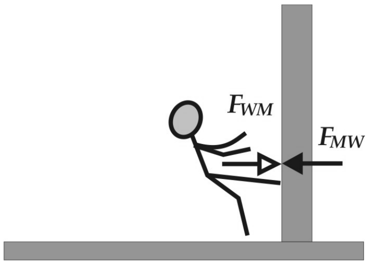
Example 1:

Let us take an example of a man pushing a table by exerting force on the table. Force $\boldsymbol{F_{TM}}$ is exerted by the man on the table. (*Following convention is followed to represent different forces of action and reaction. The first subscript letter '$_T$' in $\boldsymbol{F_{TM}}$ represents that the force is exerted on the table, and the second subscript letter '$_M$' in $\boldsymbol{F_{TM}}$ represents that that force is exerted by the man*). As per Newton's third law, the table also exerts an equal and opposite force $\boldsymbol{F_{MT}}$ on the man as soon as the man exerts a force $\boldsymbol{F_{TM}}$ on the table.



We can observe that **action force F_{TM}** is exerted by the man on the table first. Then equal **reaction force F_{MT}** is instantly exerted by the table in opposite direction on the man. We can see that these two forces are exerted on **two different objects**.

Example 2:



Let us take an example of a man kicking a rigid wall in 'Bruce Lee' style fast-flying kick. As soon as the man applies a force F_{WM} on the wall by hitting it with a flying kick, the wall produces an equal and opposite force F_{MW} instantaneously on the man. That is why that man experiences a force that tries to throw him away from the wall. This can be explained as per Newton's third law, which states that for every **action** there is an equal and opposite **reaction**.

We observe that the speed with which the force was applied to the wall in the second example was far greater as compared to the table in the first example. However, equal and opposite **reaction force** in both the examples was **instantly applied** as soon as the **action force** was applied.

1.5 Newton's three laws of motion:

After analyzing and understanding Newton's three laws, we are now in a position to observe a common thread running through these three laws.

The first law is about the condition of an object when it is not acted upon by any **external force**.

The second law is about the change in the condition of an object when it is acted upon by an **external force**.

The third law states that when an object applies an **external force** on the second object, the second object instantly exerts an **equal and opposite force** on the first object.

Thus, we can see that the common thread that runs through Newton's three laws is **'net external force'**.

We have so far used Newton's laws of motion to analyze and calculate the effect of force on the objects in linear motion. But we still know very little about the nature of these forces. **Understanding different types of forces is important for analyzing different motions.**

1.6 Different types of forces:

In Physics, a force is a push or a pull on an object. A force exerted on the object can cause the object to accelerate. At the most fundamental level, there are four types of forces in nature.

1. Gravitational force
2. Electromagnetic force
3. Weak nuclear force
4. Strong nuclear force

However, our analysis of mechanical systems involves only two types of forces, Gravitational force, and Electromagnetic force.

Gravitational force is the result of Earth's attraction for objects, which gives the objects their weight, as seen earlier in section 1.3. We need to consider only the earth's gravitational attraction force on other objects for our calculations, as the gravitational attraction between 'any other two objects' is very weak and hence can be neglected.

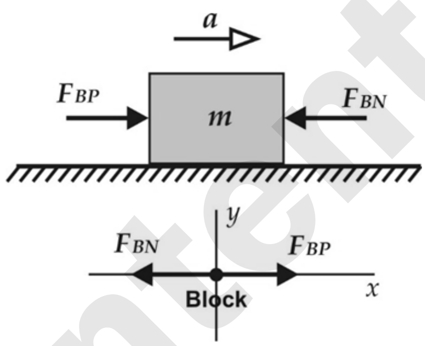
We will now examine different types of ***contact forces*** (*which are broadly regarded as* ***electromagnetic forces***) as follows. A **tension force** is a force in a stretched rope or a string, a **normal force** is created when one object pushes another object, a **frictional force** is created when one surface rubs against another surface, a **viscous force** is generated by air or fluid resistance, an **elastic force** is a property of a spring or other similar elastic materials. Microscopically, all these forces originate from the contact forces exerted by one atom on another and therefore they are all categorized as **electromagnetic forces**. However, when we deal with ordinary mechanical systems, we can ignore the microscopic basis and replace it with a single effective force of specific 'magnitude and direction' for our analysis.

We will now carry out a detailed analysis of different types of contact forces in the following sections by using a few application examples. But, let us first get familiarized with the important steps that one needs to follow for analyzing these examples.

1. Select the orientation and positive directions of the XYZ coordinate system. Components of forces in the positive direction on X, Y, and Z-axes are considered as positive.
2. Draw a **free-body diagram** for each object to show all the forces acting on the object in each axis. The object is regarded as a particle or a point object in this diagram.
3. It is important to label each force in this diagram with two subscripts so that their effect on an object can be correctly examined. (*For example, F_{AB} can indicate force on object **A** exerted by object **B***).
4. Separately add the components of the forces in each axis to get an equation of the **net force** acting on the object in that axis. You can then find one more equation for the same net force acting on the object in each axis by using Newton's second law.
5. You can equate these two equations of the net force to create a **force equation** for each axis to find the missing parameter from the problem.

We will consider two cases related to the movement of a block along the X-axis to bring more clarity to this procedure. (*The same process can be repeated for Y and Z-axis*)

Case 1: Let us consider a block of mass m that is acted upon by two external forces F_{BP} and F_{BN}. Force F_{BP} is acting on the block in a positive direction of X-axis, and force F_{BN} is acting on the block in a negative direction of the X-axis. As a result, the block moves with acceleration a in a positive direction on X-axis. A figure and a free-body diagram of the block as an illustration for case 1 are given below.



The components of the net force $\sum F_x$ acting on the block in X-axis are as follows.

$$\sum \boldsymbol{F_x} = \boldsymbol{F_{BP}} - \boldsymbol{F_{BN}}$$

We can obtain another equation for net force $\sum F_x$ acting on the block in X-axis by using Newton's second law as follows.

$$\sum \boldsymbol{F_x} = \boldsymbol{m} \times (+\,\boldsymbol{a})$$

(Note:- *Positive sign is assigned for acceleration a since the acceleration of the block is in a positive direction of the X-axis*).

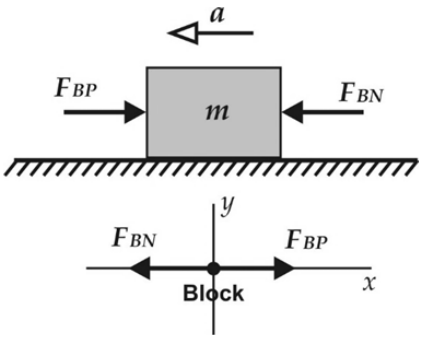
We can now equate these two equations of $\sum F_x$ to create a force equation for the block for X-axis as follows.

$$\therefore \quad \boldsymbol{F_{BP}} - \boldsymbol{F_{BN}} = \boldsymbol{m} \times \boldsymbol{a}$$

In a majority of examples, one of the parameters from F_{BP}, F_{BN}, m, or a is not given. You are required to generate a force equation and solve it with available information to find the missing parameter.

Case 2: Case 2 is exactly similar to case 1, except that the block is now moving with acceleration a on the negative side of the X-axis.

We can draw a figure and a free-body diagram of the block as an illustration for case 2 as follows.



The components of the net force $\sum F_x$ acting on the block in X-axis are as follows.

$$\sum F_x = F_{BP} - F_{BN}$$

We can obtain another equation for net force $\sum F_x$ acting on the block in X-axis by using Newton's second law as follows.

$$\sum F_x = m \times (-a)$$

(Note:- *We observe that the negative sign is assigned for acceleration a in case 2 since the acceleration of the block is in a negative direction of the X-axis*).

We can now equate these two equations of $\sum F_x$ to create a force equation for the block for X-axis as follows.

$$\therefore \quad F_{BP} - F_{BN} = -m \times a$$

As stated earlier, in a majority of examples, one of the parameters from F_{BP}, F_{BN}, m, or a is not given. You are required to generate a force equation and solve it with available information to find the missing parameter.

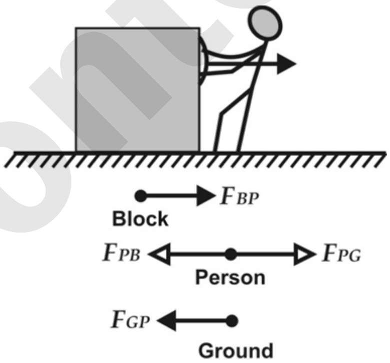
1.6.1 Tension force:

Tension force is described as the pulling force transmitted axially by the means of a string, a rope, a cable, a chain, or similar object. (*A string or rope is ideally considered as having length but being massless with zero cross section*). The object transmitting tension will exert pulling force from one end of the string on another object which is connected to the other end of the string. As per Newton's third law, these are the same action-reaction forces exerted on the ends of the string by the objects to which the ends of the string are attached. If the string curves around one or more pulleys, it will still have constant tension along its length since the pulleys are ideally considered as massless and frictionless.

We will now analyze few examples to bring further clarity to the concept of tension force.

Example 1:- The following figure shows a person **pulling a block** towards the positive side of the X-axis to accelerate it with a force F_{BP}.

Therefore, as per Newton's third law, the block will also exert a reaction force F_{PB} on the person. As we have seen earlier, **reaction force** F_{PB} is equal in magnitude and opposite in direction to **action force** F_{BP}.



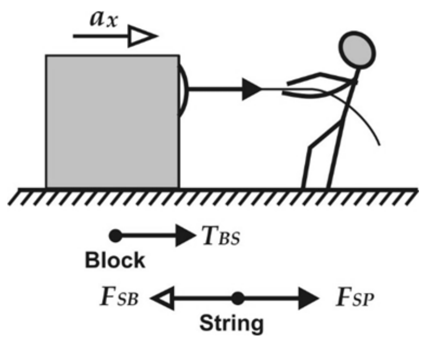
To pull the block, the person has to push forward on the ground with his legs with **action force** of F_{GP}. Therefore, **reaction force** F_{PG} of the ground acts on the person to push the person backward. The figure shows two action-reaction pairs as follows. (*Let us assume X-axis to be horizontal with its positive direction on the right-hand side*).

$$\boldsymbol{F_{BP} = -F_{PB}} \text{ . . . . . (\textbf{\textit{Block and person}})}$$
$$\boldsymbol{F_{PG} = -F_{GP}} \text{ . . . . . (\textbf{\textit{Person and ground}})}$$

The two forces acting on the person are F_{PG} and $-F_{PB}$, which are equal in magnitude and opposite in direction. **Hence these two forces will cancel each other**. Therefore, the only force F_{BP} acting on the block (*exerted by the person*) will result in moving the block towards the positive side on x axis.

We may observe that the person is the active agent that is responsible for the motion. It is the reaction force F_{PG} exerted by the ground on the person that makes the movement of the block possible. (*The person will not be able to pull the block if there was no friction between the person's shoes and the ground, as the person will not be able to exert force F_{GP} on the ground. We can see that if there is no action force F_{GP}, then the reaction force F_{PG} cannot exist, and the person's shoes will keep on slipping on the ground*).

Example 2:- Let us analyze an example of a person pulling a block to accelerate it with the help of a string, to understand the concept of **'tension force'**. Let us assume this motion along the X-axis that is horizontal with its positive direction on the right-hand side.



The person is pulling a block by pulling on a string with a force F_{SP}, to accelerate it over the frictionless surface. The force on the block is not exerted directly by a person now, but rather by a string. Let us call this force as tension force T_{BS}. Therefore, the string pulls the block with tension force T_{BS}. Hence, as per Newton's third law, the block pulls on the string with a reaction force F_{SB}, which is equal in magnitude to T_{BS} but in the opposite direction. (*We assume that the string is massless and not stretchable. Hence, the tension in the string is always in the direction of the string*).

The **net force** $\sum F_x$ **acting on the string** in the direction of the X-axis will be given by the following expression. $\sum F_x = F_{SP} - F_{SB}$

We can obtain another equation for net force $\sum F_x$ acting on the string by using Newton's second law as follows. $\sum F_x = m_{String} \times a_x$

Therefore, a force equation for the string for X-axis can be written as follows.

$$\therefore \quad F_{SP} - F_{SB} = m_{String} \times a_x$$

Since we have assumed the mass of the string m_{String} to be equal to zero, we can simplify the above expression as follows.

$$\therefore \quad F_{SP} - F_{SB} = 0 \times a_x$$

$$\therefore \quad F_{SP} = F_{SB}$$

The **net force** $\sum F_x$ **acting on the block** in the direction of the X-axis will be given by the following expression. $\sum F_x = T_{BS}$

We can obtain another equation for net force $\sum F_x$ acting on the block by using Newton's second law as follows. $\sum F_x = m_{Block} \times a_x$

Therefore, a force equation for the block for X-axis can be written as follows.

$$\therefore \quad \boldsymbol{T_{BS} = m_{Block} \times a_x} \quad \ldots\ldots\ldots (5)$$

However, the magnitude of T_{BS} and F_{SB} are the same, since they are forces of action and reaction as per Newton's third law.

$$\therefore \quad \boldsymbol{F_{SP} = T_{BS} = F_{SB}}$$

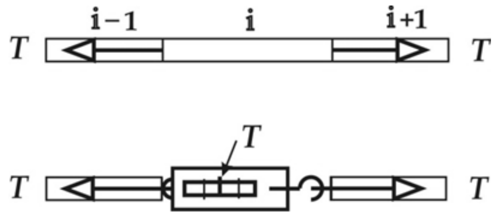
Therefore, equation (5) can be rewritten by substituting F_{SP} in place of T_{BS} as follows.

$$\therefore \quad \boldsymbol{F_{SP} = T_{BS} = m_{Block} \times a_x}$$

Thus, the thin mass-less string simply transfers the force F_{SP} applied by person from one end of the string to the other end of the string and applies it as a **tension force** T_{BS} on the block without any change in its magnitude or direction.

(Note:- *We will now highlight an* ***important characteristic of tension force*** *by taking an example where we will consider a string which is under tension T. We can assume that the string is made up of several small pieces numbered from 1 to n. Let us analyze three elements of this string, as shown below.*

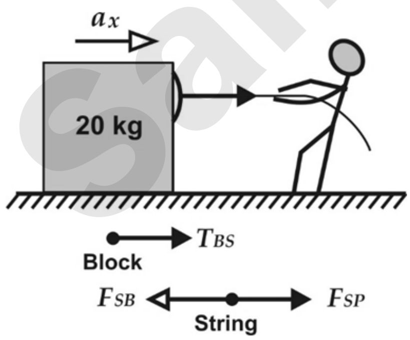
*Element ($\boldsymbol{i}$) in the string, experiences a tension T in the right direction due to element ($\boldsymbol{i}$+**1**), and an equal tension T in the left direction due to element ($\boldsymbol{i}$–**1**). If we cut the string at the position of the element ($\boldsymbol{i}$) and insert a spring* *scale between the cut ends, the spring scale would read the magnitude of the tension T in the string directly. It may be observed that the spring scale does not read the tension in string as $2T$ even though string elements from both the ends pull the scale with tension T.*



This is similar to the case where a spring scale does not read the weight of an object as ($2W$) when the object of weight (W) is hung on the spring scale, even though there is a downward force of (W) from the weight of the object, and also an upward force equal to (W) at the top end of the scale, from where it is supported).

Example 3:- A person is pulling a block of 20 kg on a horizontal frictionless surface with a string/rope. He is applying a constant force of 100 N to pull the rope. Our objective is to find tension in the rope and acceleration of the object. (*Assume that the rope is massless and very thin*).

Solution:- The person is applying a constant force F_{SP} = 100 N. We have earlier proved that $F_{SP} = T_{BS} = F_{SB}$. Hence tension in the string is equal to 100N.



We can use Newton's second law to calculate the acceleration of the block as follows.

$$F_{SP} = m_{Block} \times a_x$$

$$100 = 20 \times a_x$$

$$\therefore \quad a_x = 5 \text{ m/s}^2$$

Example 4:- A 20 kg block is hung from the ceiling of an elevator with a thin string. Our objective is to find the tension in the string in the following situations.

i. when the elevator is moving up with constant velocity.

ii. when the elevator is moving up with an acceleration of 5 m/s^2.

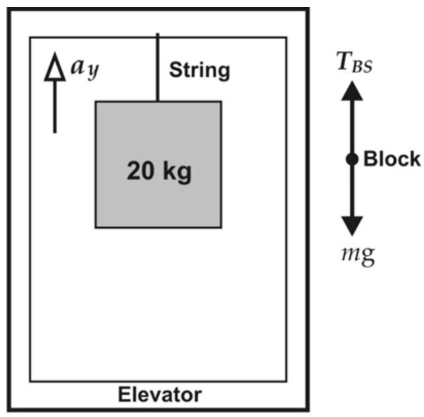
Solution:- Free-body diagram of the block shows that two forces are acting on the block. Downward force mg, due to Earth's gravity and upward force T_{BS}, due to tension in the string.

i. Since the elevator is moving up with constant velocity, its acceleration a_y= 0.

$$\therefore \quad T_{BS} = m\,(g + 0)$$
$$= 20 \times 9.8 = 196 \text{ N}$$

Therefore, the tension in the string when the elevator is moving up with constant velocity is 196 N.

(*We observe that even if the elevator is* ***moving down*** *with constant velocity, or when it is stationary, the tension in the string will be also equal to* 196 *N*).



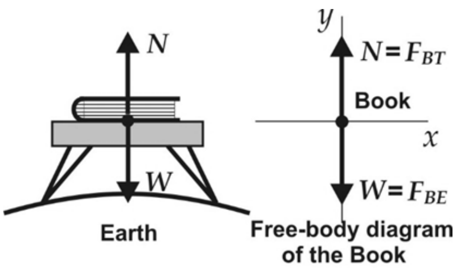
ii. Since the elevator is now moving up with the acceleration a_y= 5 m/s^2.

$$\because \quad \boldsymbol{T_{BS} = m\,(g + a_y)}$$
$$\boldsymbol{= 20 \times (9.8 + 5)}$$
$$\therefore \quad \boldsymbol{T_{BS} = 196 + 100}$$
$$\boldsymbol{= 296 \text{ N}}$$

Therefore, the tension in the string when the elevator is moving up with an acceleration of 5 m/s^2 is 296 N. (*We observe* *that when the elevator is moving down with an acceleration of 5 m/s^2, then the tension in the string will be* 96 *N*). (Note:- *It is important to understand the real-life significance of this effect. The design engineer of the elevator has to consider not only the total weight of the people riding on the elevator but also the additional forces exerted on the floor as a result of the acceleration of the elevator*).

1.6.2 Normal force:

Let us analyze the forces exerted on a book resting on a table to understand the concept of a ‘normal force’.



[A] *[B]*

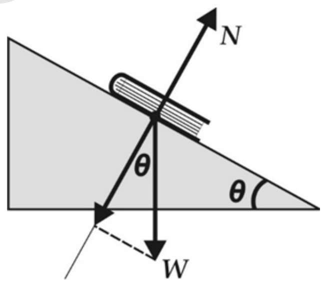
The Earth’s gravitational attraction exerts a **downward force** F_{BE} on the book, which is equal to the weight (W) of the book. However, the book is stationary on the table, and therefore it does not have any acceleration (*or any movement*) in a vertical direction. Hence, the net force acting on the book must be zero in a vertical direction. Therefore, there has to be **some other vertical force** acting on the book in the reverse direction, to nullify the downward force of gravity. That force is the **normal force** N = F_{BT} exerted on the book by the table (*Fig. A and B*). In this case, **‘normal’** means **‘perpendicular’**. The normal force exerted by a surface is always perpendicular (*or normal to*) the surface.

The normal force N acting in an upward direction on the book is always equal and opposite to the force due to the weight W of the book acting in a downward direction. Therefore, the normal force N always balances the weight W of the book and the book does not have any acceleration in a vertical direction.

$$\therefore \quad N = W$$

If someone places his hand on the book and pushes it downward with force F, even then the book will continue to remain at rest on the table. For the vertical acceleration of the book to be still zero, the net force on the book must again be zero. Hence the upward normal force must now equal the total downward force ($W + F$). The normal force N must therefore increase as F increases, since $N = W + F$. Eventually, $W + F$ could become so large that it would exceed the ability of the table to provide enough upward normal force N, and the book would break through the tabletop.

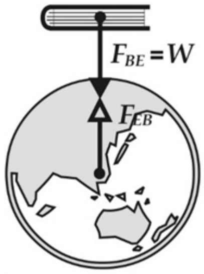
As stated earlier, the normal force exerted by a surface is always perpendicular (*or normal*) to the surface.



Therefore, if a book is kept on an inclined surface, then the direction of the normal force exerted by the surface on the book will be at an angle that will be perpendicular to the inclined surface.

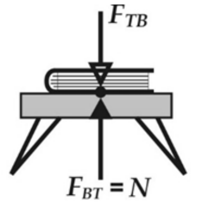
(Note:- *The normal force* N *shown in the free-body diagram of the book is equal and opposite to the downward force* W *(fig. A and B). But* N *is not the* ***reaction force*** *to weight* W.

This can be explained by referring to the next figure. The reaction force to weight W *is the force of attraction* F_{EB} *exerted on the earth by the book, as shown in fig. C. And reaction force to normal force* N *is the downward force* F_{TB} *exerted on the table by the book, as can be seen from fig. D.*



Free-body diagram of the Earth (Action-reaction pair)

[C]



Free-body diagram of the Table (Action-reaction pair)

[D]

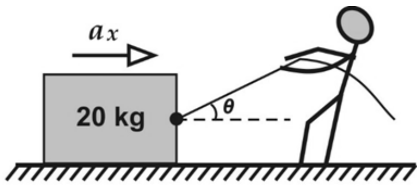
It may be observed that action-reaction pairs of Newton's third law never act on the same object*. Therefore, the forces N and W acting on the book are not an action-reaction pair*).

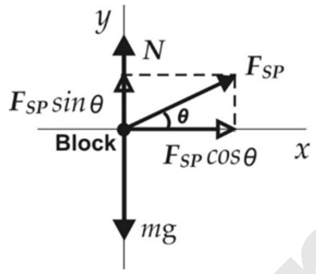
1.6.3 Origin of contact forces:

Tension and normal forces are examples of contact forces in which one object exerts a force on another because of the contact between them. These forces originate with atoms of each object. Each atom exerts a force on its neighboring atom (*which may be an atom from the same or another object*). A contact force can be maintained between neighboring atoms only if it does not exceed the inter-atomic bond within either of the objects. Otherwise, when the binding force between the atoms is overcome, the string or the surface breaks.

Example 4:- A person is pulling a block of 20 kg with a rope by applying a constant force of 100 N (*it is assumed that the block is moving on a horizontal frictionless surface, and the rope is massless*). The rope is making an angle θ = 18^0 to the horizontal as shown in the following figure. Our objective is to calculate the horizontal acceleration of the block and the normal force acting on the block.

Solution:- Free-body diagram of the block with pulling force F_{SP} applied at 18^0 to the horizontal is given in the following figure.





The net force $\sum F_x$ acting on the block in X-axis is as follows. $\sum F_x = F_{SP}\cos\theta$

Since the block gets accelerated, this same net force $\sum F_x$ can also be obtained by using Newton's second law as $\sum F_x = ma_x$. Therefore, a force equation for the block for X-axis can be written as follows.

$$\therefore \quad F_{SP}\cos\theta = ma_x$$

We can now use this force equation for x component of the block to find the horizontal acceleration of the block as follows.

$$F_{SP}\cos\theta = ma_x$$
$$\therefore \quad 100 \times \cos 18 = 20a_x$$
$$\therefore \quad 100 \times 0.95 = 20a_x$$
$$\therefore \quad a_x = 4.76 \text{ m/s}^2$$

The components of the net force $\sum F_y$ acting on the block along the Y-axis are as follows.

$$\sum F_y = N + F_{SP}\sin\theta - mg$$

This same net force $\sum F_y$ can also be obtained by using Newton's second law as $\sum F_y = ma_y$. Therefore, a force equation for the block for Y-axis can be written as follows.

$$\therefore \quad N + F_{SP}\sin\theta - mg = ma_y$$

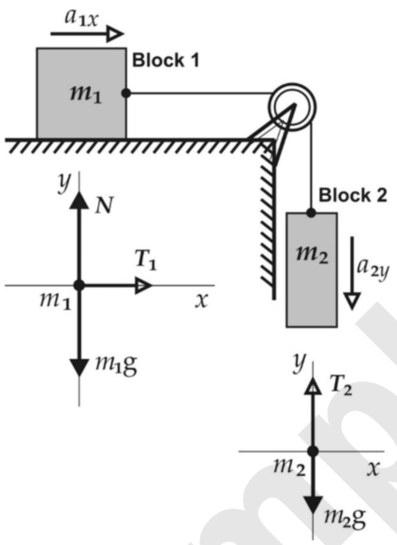
We can assume that the block stays on the surface while it is getting pulled by the person along the X-axis. Hence vertical acceleration of the block is zero. $\therefore\ a_y = 0$

$$\because \quad N + F_{SP}\sin\theta - mg = ma_y$$
$$\therefore \quad N + F_{SP}\sin\theta - mg = m \times 0$$

$\therefore \quad N = mg - F_{SP}\sin\theta$
$= 20 \times 9.8 - 100 \times \sin 18$
$\therefore \quad N = 196 - 100 \times 0.31$
$= 165$ newton

Example 5:- Following figure shows a system of two blocks connected by a string going over a frictionless pulley. Block 1 of mass m_1 is connected to block 2 of mass m_2 by a string passing over a pulley. Block 1 is moving on a frictionless horizontal surface. Our objective is to find the tension in the string and acceleration of each block.

Solution:- Let us refer to the free-body diagrams of block 1 and block 2. We will assume that the accelerations of block 1 and block 2 along the X and Y-axis are a_{1x} and a_{2y} respectively.



Block 1 is acted on by tension force T_1 in the horizontal direction. The net force $\sum F_x$ acting on block 1 in the X-axis is as follows.

$$\sum \boldsymbol{F_x} = \boldsymbol{T_1}$$

This same net force $\sum F_x$ acting on block 1 can also be obtained by using Newton's second law as $\sum F_x = m_1 a_{1x}$. Therefore, a force equation for block 1 for X-axis can be written as follows.

$$\therefore \quad T_1 = m_1 a_{1x} \ldots\ldots\ldots (6)$$

Block 1 is acted on by force N and m_1g in a vertical direction. The components of the net force $\sum F_y$ acting on block 1 in the Y-axis are as follows.

$$\sum F_y = N - m_1 g$$

This same net force $\sum F_y$ acting on block 1 can also be obtained by using Newton's second law as $\sum F_y = m_1 a_{1y}$. Therefore, a force equation for block 1 for Y-axis can be written as follows.

$$\therefore \quad N - m_1 g = m_1 a_{1y}$$

We may observe that block 1 does not move in a vertical direction. Hence its vertical acceleration $a_{1y} = 0$. Therefore, we can simplify 'y component' of the force equation of block 1 as follows.

$$N - m_1 g = 0 \ \ldots\ldots\ldots\ldots (7)$$

Block 2 is acted upon by tension force T_2 in the string and its weight m_2g in the vertical direction. (*There is no horizontal force acting on block 2*).

The net force $\sum F_y$ acting on block 2 in the vertical direction is $\sum F_y = T_2 - m_2 g$.

This same net force $\sum F_y$ acting on block 2 can also be obtained by using Newton's second law as $\sum F_y = m_2 a_{2y}$. Therefore, a force equation for block 2 for Y-axis can be written as follows.

$$\therefore \quad T_2 - m_2 g = m_2 a_{2y} \ldots\ldots (8)$$

The string is assumed to be mass-less, and the pulley is assumed to be mass-less and frictionless. Therefore, magnitudes of both the tensions T_1 and T_2 in the string are equal. Hence, we will represent common tension in string as equal to T. Similarly, the accelerations of both blocks will also be equal since the string does not stretch. Hence, the acceleration of both the blocks is the same and is represented as equal to a.

The direction of acceleration of a_{2y} of block 2 is in the downward direction.

$$\therefore \quad a_{2y} = -a$$

The direction of acceleration a_{1x} of block 1 is considered towards the right-hand side on X-axis.

$$\therefore \ a_{1x} = +a$$

We will simplify equations (6) and (8) as follows by substituting these values.

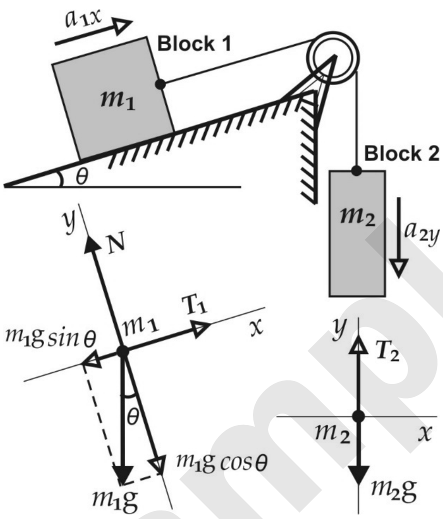
$T = m_1 a$ and $T - m_2 g = m_2(-a)$

We can solve both equations simultaneously to get the following expressions for values of T and a.

$$T = \frac{m_1 m_2}{m_1 + m_2} g \quad \text{and} \quad a = \frac{m_2}{m_1 + m_2} g$$

Example 6:- Following figure shows the system of two blocks similar to the last example. However, now block 1 is sliding over a plane inclined at an angle $\theta = 30^0$. The masses of the two blocks are $m_1 = 20$ kg and $m_2 = 5$ kg. The system is released from rest. Our objective is to find the acceleration of the blocks and tension in the string.

Solution:- The free-body diagrams of block 1 and block 2 are given in the figure.



(*We have already seen that the magnitude of the tension in the string on both sides of a pulley is the same, and the magnitude of the acceleration of both the blocks is also the same.* $\therefore$ $T_1 = T_2 = T$, and $a_{1x} = a_{2y} = a$).

We will intentionally choose XY coordinate system for block 1 in such a way that the X-axis is parallel to the acceleration of block 1. Therefore, we will choose X-axis for block 1 at an angle of 30^0 to the horizontal.

Let us assume that block 1 moves in the positive direction on X-axis as shown in the figure, moving up on the inclined plane.

The components of the net force $\sum F_x$ acting on block 1 in the X-axis are as follows.

$$\sum F_x = T - m_1 g \sin\theta$$

This same net force $\sum F_x$ acting on block 1 can also be obtained by using Newton's second law as $\sum F_x = m_1 a_{1x}$. We observe that the acceleration of block 1 is assumed in a positive direction on X-axis.

$$\therefore \sum F_x = m_1 a_{1x} = m_1 a$$

Therefore, a force equation for block 1 for X-axis can be written as follows.

$$\therefore \quad T - m_1 g \sin\theta = m_1 a$$

$$\therefore \quad T = m_1 g \sin\theta + m_1 a$$

$$= 20 \times 9.8 \times \sin 30 + 20a$$

$$= 196 \times 0.5 + 20a$$

$$\therefore \quad T = 98 + 20a \quad \ldots\ldots\ldots \quad (9)$$

The components of net force $\sum F_y$ acting on block 1 in the Y-axis are as follows.

$$\sum F_y = N - m_1 g \cos\theta$$

This same net force $\sum F_y$ acting on block 1 can also be obtained by using Newton's second law as $\sum F_y = m_1 a_{1y}$. Therefore, a force equation for block 1 for Y-axis can be written as follows.

We may observe that block 1 does not move in the direction of the Y-axis. Hence its vertical acceleration $a_{1y} = 0$.

$$\therefore \quad \sum F_y = m_1 a_{1y} = m_1 \times 0 = 0$$

$$\therefore \quad N - m_1 g \cos\theta = 0$$

Block 2 is acted upon by tension force T in string and weight $m_2 g$ of block 2 in the vertical direction. There is no horizontal force acting on block 2.

The net force $\sum F_y$ acting on block 2 in the vertical direction is $\sum F_y = T - m_2 g$.

This same net force $\sum F_y$ acting on block 2 can also be obtained by using Newton's second law as $\sum F_y = m_2 a_{2y}$. We observe that block 2 is assumed as moving with acceleration a_{2y} in the downward direction as shown in the figure.

$$\therefore \quad \sum F_y = m_2 a_{2y} = m_2(-a)$$

Therefore, a force equation for block 2 for Y-axis can be written as follows.

$\therefore \; T - m_2 g = m_2(-a)$

$\therefore \; T = m_2 g - m_2 a$

$\therefore \; T = (5 \times 9.8) - 5a$

$= 49 - 5a$ . . . . (9A)

Let us substitute the value of tension force T from equation (9) into equation (9A) to find the value of a as follows.

$\because \; T = 49 - 5a$

$\therefore \; 98 + 20a = 49 - 5a$

$\therefore \; 25a = -49$

$\therefore \; a = -1.96$ m/s^2

The negative value of acceleration means that our initial assumption of block 1 moving with acceleration towards a positive direction on the X-axis is not correct. Therefore, the block will slide down the inclined plane in a negative direction on X-axis with an acceleration of 1.96 m/s^2 when it is released from rest.

Let us calculate the tension in the string by substituting this value of acceleration in equation (9A).

$T = 49 - 5a$

$= 49 - (5 \times -1.96)$

$= 58.8$ newton

This value of tension is greater than the weight of block 2, (*5 × 9.8 = 49 newton*). Therefore, the tension T of the string will pull the weight of block 2 upwards, which is consistent with the acceleration of block 2 in a vertically upward direction.

(Note:- *We will need to make changes in the figure drawn earlier by reversing the arrows of acceleration a_{1x} of block 1 and a_{2y} of block 2, to correct the figure in line with the calculated direction of acceleration*).

1.6.4 Frictional forces:

We have so far assumed that objects are moving on frictionless surfaces, to simplify the study of Newton's laws of motion. But whenever the surface of one object slides over that of the other, each object exerts a frictional force over the other object. Friction brings every moving object to a halt if it is left to move on its own. Friction causes wear and tear in any moving part, and a lot of engineering work is directed towards reducing it. But we will not be able to walk without friction, and the movement of any vehicle will not be possible without friction. Therefore, 'friction' assumes a very important place in our daily life, and needs to be included in our study of motions.

The frictional force acting between two surfaces of objects, when the objects are at rest, is called a force of **static friction**. And frictional force acting between two surfaces of objects in relative motion are called forces of **kinetic friction**.

'Friction' is a very complicated phenomenon that is not easy to capture in simple and accurate laws. However, the maximum force of static and kinetic friction follows two empirical laws as follows. The maximum force of static or kinetic friction is **proportional to the normal force N acting on the object**, but it is **independent of the surface area** of the contact.

Coefficient of static friction μ_S is defined as the ratio of the magnitude of the maximum force of static friction f_S to the normal force N.

$$\therefore \; \mu_S = \frac{f_S}{N}, \text{ hence } f_S = \mu_S N$$

Coefficient of kinetic friction μ_K is defined as the ratio of the magnitude of the force of kinetic friction f_K to the normal force N.

$$\therefore \; \mu_K = \frac{f_K}{N}, \text{ hence } f_K = \mu_K N$$

Both μ_S and μ_K are dimensionless constants as they are both ratios of two forces. Usually for any given two surfaces, $\mu_S > \mu_K$. The values of both μ_S and μ_K, are less than 1.

Example 7:- The system of blocks is similar to example 6. The only difference is block 1 is **not moving** on a frictionless surface, but on a

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