SAMPLE CONTENT

TRIUMPH

MHT-CET MATHEMATICS

SOLUTIONS to MCQs

Target Publications® Pvt. Ltd.

Written in accordance with the latest MHT-CET Paper Pattern prescribed by
State Common Entrance Test Cell, Maharashtra State

TRIUMPH

MHT-CET
MATHEMATICS
SOLUTIONS to MCQs

Salient Features

- Detailed solutions provided for difficult MCQs as per the concepts emphasized in the syllabus
- **Smart Keys** (Caution, Shortcuts, Thinking Hatke) - Multiple Study Techniques to enhance understanding of concepts and problem solving skills
- Solutions to Evaluation Test for each chapter
- Solutions to Model Question Papers
- Solutions to MHT-CET 2023 Question Papers (9th May Shift - 1 & 10th May Shift - 1)

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PREFACE

Target's **Triumph MHT-CET Mathematics Solutions to MCQs** book provides students with holistic comprehension of principles of Mathematics through solutions to MCQs based on the concepts emphasized in the syllabus.

It includes **Smart Keys** (Caution, Shortcuts and Thinking Hatke), which offer supplemental explanations for the tricky questions and are intended to help students how to approach problems in novel ways in the shortest possible time with accuracy.

- **Caution** apprises students about mistakes often made while solving MCQs.
- **Shortcuts** comprise formulae based short cuts considering their usage in solving MCQ.
- **Thinking Hatke** reveals quick witted approach to crack the specific question.

Solutions to **Model Question Papers** and **MHT-CET 2023 Question Papers** (9^{th} May Shift - 1 & 10^{th} May Shift - 1) are also included in this book.

All the features of this book are designed keeping the following elements in mind:
Time management, easy memorization or revision, and non-conventional yet simple methods for MCQ solving.

We hope the book benefits the learner as we have envisioned.

Publisher
Edition: First

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.

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Chapter 8

Measures of Dispersion

Shortcuts

1. Standard deviation ≤ Range. i.e., Variance ≤ $(\text{Range})^2$

2. S.D. of first n natural numbers is $\sqrt{\frac{n^2-1}{12}}$.

Classical Thinking

8.1 Range, Variance and Standard Deviation

1. (A) 2. (D)

3. (C) Range = L – S = 100 – 50 = 50

4. (B) Least possible value of x
= Greatest Value – Range
= 35 – 23
= 12

5. (C) Upper limit of the highest class (L) = 50
Lower limit of the lowest class (S) = 10
∴ Range = L – S = 50 – 10 = 40

6. (A) 7. (B)

8. (B) $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{52}{10} = 5.2$

∴ Variance $= \frac{1}{N}\sum f_i(x_i - \bar{x})^2 = \frac{35.6}{10} = 3.56$

9. (B)

10. (C) Here, $\bar{x} = \frac{2+4+6+8+10}{5} = 6$

∴ variance $= \frac{1}{n}\Sigma(x_i - \bar{x})^2$

$= \frac{1}{5}\{(2-6)^2 + (4-6)^2 + (6-6)^2 + (8-6)^2 + (10-6)^2\}$

$= \frac{1}{5}\{16 + 4 + 0 + 4 + 16\}$

$= \frac{1}{5}\{40\} = 8$

11. (A) Variance of first n natural numbers

$= \frac{n^2-1}{12}$ …[Using ***Shortcut 2***]

for n = 20

Variance of first 20 natural numbers $= \frac{20^2-1}{12}$

$= \frac{133}{4}$

12. (C)

13. (B) S.D. of 1st n natural numbers $= \sqrt{\frac{n^2-1}{12}}$ …[Using ***Shortcut 2***]

For n = 7,

Required S.D. $= \sqrt{\frac{7^2-1}{12}} = \sqrt{4} = 2$

14. (A) S.D. $= \sqrt{\frac{1}{n}\left(\sum_{i=1}^{n} x_i^2\right) - (\bar{x})^2}$

$= \sqrt{\frac{1}{7}(619) - (9)^2}$

$= \sqrt{\frac{619-567}{7}}$

$= \sqrt{\frac{52}{7}}$

15. (C) S.D. is independent of change of origin.

16. (B) Here, $\sigma_x = 10$
Let $y = 5x + 50$
∴ $\sigma_y = 5\sigma_x$
$= 5(10)$
$= 50$

17. (C) $\sigma = \sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2}$

∴ $4 = \sqrt{\frac{\sum x_i^2}{100} - (50)^2}$

∴ $16 = \frac{\sum x_i^2}{100} - 2500 \Rightarrow \sum x_i^2 = 251600$

18. (D) If X and Y are two variables such that
$Y = \frac{X}{a}$ $(a \neq 0)$, then $\sigma_y = \frac{1}{|a|}\sigma_x$
Here, $\sigma_x = 8$
∴ S.D. of the new observations $= \frac{8}{|-2|} = 4$

19. (B) $\bar{x} = \frac{2+3+a+11}{4} = \frac{16+a}{4}$

Now,

$\sigma^2 = \frac{1}{N}\Sigma(x-\bar{x})^2$

$\therefore (3.5)^2 = \frac{(4+9+a^2+121)}{4} - \left(\frac{16+a}{4}\right)^2$

$\Rightarrow \frac{49}{4} = \frac{134+a^2}{4} - \frac{256+32a+a^2}{16}$

$\Rightarrow 3a^2 - 32a + 84 = 0$

20. (D) We know that $\sigma(ax + b) = |a| (\sigma(x))$
So $\sigma(1 - 4x) = |-4| \sigma(x) = 4 \times 2.6 = 10.4$

21. (B) As S.D. is independent of change of origin.
S.D. of $y_1 - 3, y_2 - 3, \ldots, y_n - 3$ is also 6.
So, their variance is 36.

8.2 Standard Deviation for Combined data, Coefficient of variation

1. (B) Here, $\sigma_1^2 = 4$, $\sigma_2^2 = 5$, $\overline{X}_1 = 2$, $\overline{X}_2 = 4$
and $n_1 = n_2 = 5$

$\therefore \overline{X} = \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2} = 3$

$d_1 = \overline{X}_1 - \overline{X} = 2 - 3 = -1$,

$d_2 = \overline{X}_2 - \overline{X} = 4 - 3 = 1$

Let σ^2 be the combined variance. Then,

$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$

$= \frac{(4+1)+(5+1)}{2} = \frac{11}{2}$

2. (D) C.V. $= \frac{\text{S.D.}}{|\text{Mean}|} \times 100 = \frac{19.76}{35.16} \times 100$

3. (A) We have, C.V. = 50 and S.D. = 20

$\therefore$ C.V. $= \frac{\text{S.D.}}{|\text{Mean}|} \times 100$

$\therefore 50 = \frac{20}{|\text{Mean}|} \times 100 \times$

$\therefore |\text{Mean}| = \frac{20}{50} \times 100$

$\therefore$ Mean = 40

4. (C) Coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

$\therefore 60 = \frac{21}{\bar{x}} \times 100$

$\Rightarrow \bar{x} = 35$

5. (B) Coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

$\therefore 7.2 = \frac{\sqrt{3.24}}{\bar{x}} \times 100$

$\therefore \bar{x} = \frac{\sqrt{3.24}}{7.2} \times 100 = 25$

6. (D) $\frac{\sigma}{|\bar{x}|} \times 100 = 16$ and $\bar{x} = 25$

$\Rightarrow \sigma = 4$

$\Rightarrow \sigma^2 = 16$

7. (C) $\bar{x} = \frac{530}{10} = 53$, $\sum x_i = 530$, $\sum(x_i - \bar{x})^2 = 70$

$\therefore$ S.D. $= \sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}} = \sqrt{\frac{70}{10}} = \sqrt{7} = 2.64$

$\therefore$ C.V. $= \frac{\sigma}{|\bar{x}|} \times 100 = \frac{2.64}{53} \times 100 = 4.98$

8. (C) Coefficient of variation $= \frac{\text{S.D.}}{|\text{Mean}|} \times 100$

$\Rightarrow 45 = \frac{\sigma}{12} \times 100$

$\Rightarrow \sigma = \frac{45 \times 12}{100} = \frac{540}{100} = 5.4$

9. (C) S.D. $(\sigma) = \sqrt{\frac{250}{10}} = \sqrt{25} = 5$

Hence, coefficient of variation $= \frac{\sigma}{\text{mean}} \times 100$

$= \frac{5}{50} \times 100 = 10$

Critical Thinking

8.1 Range, Variance and Standard Deviation

1. (D) Variance $= \frac{\sum x_i^2}{n} - (\bar{x})^2$

$= \frac{(2^2 + 4^2 + \ldots. + 100^2)}{50} - \left(\frac{2+4+\ldots.+100}{50}\right)^2$

$= \frac{4(1^2 + 2^2 + \ldots. + 50^2)}{50} - (51)^2$

$= 4\left(\frac{50 \times 51 \times 101}{50 \times 6}\right) - (51)^2$

$= 3434 - 2601 = 833$

2. (C) Here,
$N = \sum f_i = 12$, $\Sigma f_i x_i = 132$, $\sum f_i x_i^2 = 1692$

$\therefore V(X) = \frac{1692}{12} - \left(\frac{132}{12}\right)^2 = 141 - 121 = 20$

3. (D) Since, root mean square ≥ A.M.

$\therefore \sqrt{\dfrac{\sum_{i=1}^{n} x_i^2}{n}} \geq \dfrac{\sum_{i=1}^{n} x_i}{n}$

$\Rightarrow \sqrt{\dfrac{400}{n}} \geq 5$

$\Rightarrow \dfrac{400}{n} \geq 25 \Rightarrow n \leq 16$

4. (A) It is given that each of the two populations has 100 observations which are 100 consecutive integers. So, sum of the squares of deviations from their respective means are same.

$\therefore V_A = V_B \Rightarrow \dfrac{V_A}{V_B} = 1$

5. (C) Let the unknown numbers be x and y.

Mean = 8

$\Rightarrow \dfrac{2+4+10+12+14+x+y}{7} = 8$

$\Rightarrow x + y = 14$ …(i)

Variance = 16

$\Rightarrow \dfrac{2^2+4^2+10^2+12^2+14^2+x^2+y^2}{7} - (\text{mean})^2 = 16$

$\Rightarrow 460 + x^2 + y^2 = 7[16 + (8)^2]$

$\Rightarrow 460 + x^2 + y^2 = 560$

$\Rightarrow x^2 + y^2 = 100$ …(ii)

Solving (i) and (ii), we get

$x = 6, y = 8$ or $x = 8, y = 6$

$\therefore$ Product = 48

6. (C) Since, mean = 6

$\therefore \dfrac{a+b+8+5+10}{5} = 6$

$\Rightarrow a + b = 7$

$\Rightarrow (a - 6) = (1 - b)$ ….(i)

$6.80 = \dfrac{\sum(x_i - \bar{x})^2}{n}$

$\Rightarrow 6.80 = \dfrac{(a-6)^2+(b-6)^2+4+1+16}{5}$

$\Rightarrow 34 = (a - 6)^2 + (b - 6)^2 + 21$

$\Rightarrow (a - 6)^2 + (b - 6)^2 = 13$

$\Rightarrow (1 - b)^2 + (b - 6)^2 = 13$ ….[From (i)]

$\Rightarrow b^2 - 2b + 1 + b^2 - 12b + 36 = 13$

$\Rightarrow 2b^2 - 14b + 24 = 0$

$\Rightarrow b^2 - 7b + 12 = 0$

$\Rightarrow b = 3, 4$

$\therefore b = 3 \Rightarrow a = 4$ and

$b = 4 \Rightarrow a = 3$

7. (D) Using ***Shortcut 1***, we get

$\text{Var}(X) \leq (\text{Range})^2$

i.e., $\text{Var}(x) \leq (b - a)^2$

8. (B) Corrected $\sum x^2 = 2830 - 20^2 + 30^2 = 3330$

Corrected $\sum x = 170 - 20 + 30 = 180$

$\therefore$ Corrected Variance $= \dfrac{3330}{15} - \left(\dfrac{180}{15}\right)^2$

$= 78$

9. (C) Mean $\bar{x} = \dfrac{31+32+33+....+47}{17}$

$= \left[\dfrac{\frac{17}{2}(31+47)}{17}\right]$ ....$\left[\because S_n = \dfrac{n}{2}(a + t_n)\right]$

$\Rightarrow \bar{x} = 39$

Now,

$\sigma^2 = \dfrac{1}{N}\Sigma(x - \bar{x})^2$

$= \dfrac{1}{17}\left[(31-39)^2 + (32-39)^2 + ... + (47-39)^2\right]$

$= \dfrac{1}{17}\left[8^2 + 7^2 + 6^2 + ... + 1^2 + 0 + 1^2 + 2^2 + ... + 8^2\right]$

$= \dfrac{2}{17}\left[1^2 + 2^2 + 3^2 + ... + 8^2\right]$

$= \dfrac{2}{17}\left[\dfrac{1}{6}(8)(8+1)(2\times8+1)\right]$

....$\left[\because \sum_{x=1}^{n} x^2 = \dfrac{1}{6}[n(n+1)(2n+1)]\right]$

$= 24$

$\therefore$ S.D. $= \sigma = \sqrt{24} = 2\sqrt{6}$

10. (B) Corrected $\sum x = 40 \times 200 - 50 + 40 = 7990$

$\therefore$ Corrected $= \bar{x} = \dfrac{7990}{200} = 39.95$

Incorrect $\sum x^2 = n\,[\sigma^2 + \bar{x}^2]$

$= 200\,[15^2 + 40^2]$

$= 365000$

Corrected $\sum x^2 = 365000 - 2500 + 1600$

$= 364100$

$\therefore$ Corrected $\sigma = \sqrt{\dfrac{364100}{200} - (39.95)^2} = 14.98$

11. (B) Let $x_1, x_2, ..., x_{30}$ be actual weights of 30 fishes and $y_1, y_2, ..., y_{30}$ be the weights of fishes taken from misaligned increasing scale. Then,

$y_i = x_i + 2;\ i = 1,2,....,30$

$\Rightarrow \bar{Y} = \bar{X} + 2$ and $\sigma_Y = \sigma_X$

...[$\because$ Standard deviation is independent of change of origin]

$\Rightarrow 30 = \bar{X} + 2$ and $\sigma_Y = 2$

$\Rightarrow \bar{X} = 28$ and $\sigma_Y = 2$

12. (C)

Class	f_i	x_i	$d_i = x_i - A$, A = 25	f_id_i	$f_id_i^2$
0-10	1	5	– 20	– 20	400
10-20	3	15	– 10	– 30	300
20-30	4	25	0	0	0
30-40	2	35	10	20	200
Total	**10**			**– 30**	**900**

$\sigma^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2 = \frac{900}{10} - \left(\frac{-30}{10}\right)^2$

$\sigma^2 = 90 - 9 = 81$

$\Rightarrow \sigma = 9$

13. (C) Let a, a, ...n times and – a, – a, – a, – a, ... n times i.e., mean = 0 and

$\text{S.D.} = \sqrt{\frac{n(a-0)^2 + n(-a-0)^2}{2n}}$

$\therefore \quad 2 = \sqrt{\frac{na^2 + na^2}{2n}} = \sqrt{a^2} = \pm a$

Hence, $|a| = 2$.

8.2 Standard Deviation for Combined data, Coefficient of variation

1. (C) Here, $\bar{x} = 13$, $\bar{y} = 17$, $\sigma_x = 3$, $\sigma_y = 2$, $n_x = 20$, $n_y = 30$

Combined mean $(\bar{x}_c) = \frac{n_x\bar{x} + n_y\bar{y}}{n_x + n_y}$

$= \frac{20(13) + 30(17)}{20+30}$

$= 15.4$

Now, $d_x = \bar{x} - \bar{x}_c = 13 - 15.4 = -2.4$

$d_y = \bar{y} - \bar{x}_c = 17 - 15.4 = 1.6$

$\therefore$ Combined standard deviation (σ_c)

$= \sqrt{\frac{n_x(\sigma_x^2 + d_x^2) + n_y(\sigma_y^2 + d_y^2)}{n_x + n_y}}$

$= \sqrt{\frac{20\left[3^2 + (-2.4)^2\right] + 30(2^2 + 1.6^2)}{20 + 30}}$

$= 3.14$

2. (A) Let $n_1 = 60$, $n_2 = 120$, $\bar{x}_1 = 35.4$, $\bar{x}_2 = 30.9$, $\sigma_1 = 4$, $\sigma_2 = 5$

Combined mean $(\bar{x}_c) = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$= \frac{60 \times 35.4 + 120 \times 30.9}{60 + 120}$

$= \frac{2124 + 3708}{180}$

$= \frac{5832}{180} = 32.4$

Now, $d_1 = \bar{x}_1 - \bar{x}_c = 35.4 - 32.4 = 3$

$d_2 = \bar{x}_2 - \bar{x}_c = 30.9 - 32.4 = -1.5$

$\therefore$ Combined standard deviation (σ_c)

$= \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$

$= \sqrt{\frac{60(4^2 + 3^2) + 120\left[5^2 + (-1.5)^2\right]}{60 + 120}}$

$= \sqrt{\frac{4770}{180}}$

$= \sqrt{26.5} = 5.15$

3. (B) Here,

$\sum_{i=1}^{n} x_i = 30$, $\sum_{i=1}^{n} y_i = 40$, $\sum_{i=1}^{n} x_i^2 = 220$, $\sum_{i=1}^{n} y_i^2 = 340$

$\bar{x} = \frac{\sum x_i}{n} = \frac{30}{5} = 6$,

$\bar{y} = \frac{\sum y_i}{n} = \frac{40}{5} = 8$

Combined mean $(\bar{x}_c) = \frac{n_x\bar{x} + n_y\bar{y}}{n_x + n_y}$

$= \frac{5(6) + 5(8)}{5+5} = 7$

Now, $d_x = \bar{x} - \bar{x}_c = 6 - 7 = -1$

$d_y = \bar{y} - \bar{x}_c = 8 - 7 = 1$

$\sigma_x^2 = \frac{1}{n}\sum x_i^2 - (\bar{x})^2 = \frac{1}{5}(220) - (6)^2$

$= 44 - 36 = 8$

$\sigma_y^2 = \frac{1}{n}\sum y_i^2 - (\bar{y})^2 = \frac{1}{5}(340) - (8)^2 = 68 - 64$

$= 4$

$\therefore$ Combined standard deviation (σ_c)

$= \sqrt{\frac{n_x(\sigma_x^2 + d_x^2) + n_y(\sigma_y^2 + d_y^2)}{n_x + n_y}}$

$= \sqrt{\frac{5\left[8 + (-1)^2\right] + 5\left[4 + (1)^2\right]}{5 + 5}}$

$= \sqrt{\frac{70}{10}}$

$= \sqrt{7} = 2.65$

4. (A) Let n_1 and n_2 be the number of boys and girls respectively.

Let n = 200, $\bar{x}_c = 65$, $\bar{x}_1 = 70$, $\bar{x}_2 = 62$, $\sigma_1 = 8$, $\sigma_2 = 10$

Here, $n_1 + n_2 = n$

$\therefore \quad n_1 + n_2 = 200$ …(i)

Combined mean $(\bar{x}_c) = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$\therefore \quad 65 = \frac{n_1(70) + n_2(62)}{200}$ …[From (i)]

$\therefore$ $70n_1 + 62n_2 = 13000$

$\therefore$ $35n_1 + 31n_2 = 6500$ …(ii)

Solving (i) and (ii), we get

$n_1 = 75$, $n_2 = 125$

$\therefore$ Number of boys = 75

$d_1 = \bar{x}_1 - \bar{x}_c = 70 - 65 = 5$

$d_2 = \bar{x}_2 - \bar{x}_c = 62 - 65 = -3$

Combined S.D. (σ_c)

$$= \sqrt{\frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}}$$

$$= \sqrt{\frac{75(64+25)+125(100+9)}{200}}$$

$$= \sqrt{\frac{6675+13625}{200}}$$

$$= \sqrt{\frac{20300}{200}} = \sqrt{101.5} = 10.07$$

5. (B) Here, $\bar{x} = \frac{75+78+80+86+91+88+83}{7}$

$= \frac{581}{7} = 83$

$\sum x_i = 581$, $\sum (x_i - \bar{x})^2 = 196$

$\therefore$ S.D. $= \sigma = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2} = \sqrt{\frac{1}{7}(196)} = 5.29$

$\therefore$ C.V. $= \frac{\sigma}{|\bar{x}|} \times 100 = \frac{5.29}{83} \times 100 = 6.37$

6. (C) Here, $\sum f_i = 50$, $\sum f_i x_i = 3130$,

$\sum f_i (x_i - \bar{x})^2 = 106$

$\therefore$ $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3130}{50} = 62.6$

$\therefore$ $\sigma = \sqrt{\frac{1}{N}\sum f_i (x_i - \bar{x})^2} = \sqrt{\frac{106}{50}} = 1.46$

Now, Variance $= \sigma^2 = 2.12$

$\therefore$ C.V. $= \frac{\sigma}{|\bar{x}|} \times 100 = 2.33$

7. (B) C.V. $= \frac{\sigma}{\bar{x}} \times 100$

When C.V. = 55, $\sigma = 22$

$55 = \frac{22}{\bar{x}_1} \times 100$

$\Rightarrow \bar{x}_1 = \frac{22}{55} \times 100 = 40$

For C.V.= 65, $\sigma = 39$

$\bar{x}_2 = \frac{39}{65} \times 100 = 60$

$\therefore$ Means are 40, 60.

8. (C) C.V. of A $= \frac{\sigma_A}{\bar{x}} \times 100$

$\therefore$ $4 = \frac{\sigma_A}{\bar{x}} \times 100$

$\Rightarrow \sigma_A = \frac{4\bar{x}}{100}$ …(i)

and C.V. of B $= \frac{\sigma_B}{\bar{x}} \times 100$

$\therefore$ $2 = \frac{\sigma_B}{\bar{x}} \times 100$

$\Rightarrow \sigma_B = \frac{2\bar{x}}{100}$ …(ii)

From (i) and (ii),

$\sigma_A = 2\sigma_B$

MHT-CET Previous Years' Questions

1. (A) Note that: Required variance = Variance of first 10 natural numbers

…[$\because$ Variance is independent of change of origin]

$= \frac{10^2 - 1}{12}$ …[Using ***Shortcut 2***]

$= 8.25$

2. (A) Mean $= \frac{2+3+11+x}{4} = \frac{16+x}{4}$

Variance $= \frac{1}{n}\sum\left(x_i - \bar{x}\right)^2$

$$\Rightarrow \frac{49}{4} = \frac{1}{4}\left[\left(2-\left(\frac{16+x}{4}\right)\right)^2 + \left(3-\left(\frac{16+x}{4}\right)\right)^2 + \left(11-\left(\frac{16+x}{4}\right)\right)^2 + \left(x-\left(\frac{16+x}{4}\right)\right)^2\right]$$

$$\Rightarrow \frac{49}{4} = \frac{1}{4}\left[\frac{(x+8)^2}{16} + \frac{(x+4)^2}{16} + \frac{(28-x)^2}{16} + \frac{(3x-16)^2}{16}\right]$$

$784 = (x^2 + 16x + 64) + (x^2 + 8x + 16) + (784 - 56x + x^2) + (9x^2 - 96x + 256)$

$\Rightarrow 12x^2 - 128x + 336 = 0$

$\Rightarrow 3x^2 - 32x + 84 = 0$

$\Rightarrow x = 6$ or $x = \frac{14}{3}$

3. (C) Co-efficient of variation $= \frac{\text{S.D.}}{\text{Mean}} \times 100$

C.V. of Physics $= \frac{3}{20} \times 100 = 15$

C.V. of Chemistry $= \frac{2}{25} \times 100 = 8$

C.V. of Mathematics = $\frac{4}{23} \times 100 = 17.39$

C.V. of Biology = $\frac{5}{27} \times 100 = 18.52$

$\therefore$ Biology shows the highest variability in marks.

4. (D) $\overline{X} = \frac{\sum x_i}{N} = \frac{528}{16} = 33$

$\sum(x_i - \overline{x})^2 = 9158$

Variance = $\frac{1}{N}\sum(x_i - \overline{x})^2$

$= \frac{9158}{16} = 572.375$

5. (D) C.V. = $\frac{\text{S.D}}{|\text{Mean}|} \times 100$

$= \frac{12}{72} \times 100 = 16.67\%$

6. (B) Co-efficient of variation = $\frac{\text{S.D.}}{\text{Mean}} \times 100$

C.V. of A = $\frac{12}{80} \times 100 = 15$

C.V. of B = $\frac{6}{75} \times 100 = 8$

C.V. of C = $\frac{8}{70} \times 100 = 11.43$

C.V. of D = $\frac{10}{72} \times 100 = 13.89$

C.V. is the least for division B.

7. (A) S.D. = $\sqrt{\frac{1}{n}\left(\sum_{i=1} x_i^2\right) - (\overline{x})^2}$

$= \sqrt{\frac{3050}{50} - 6^2} = 5$

8. (A) First 10 multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.

Variance = $\frac{1}{n}\left(\sum x_i^2\right) - (\overline{x})^2$

$= \frac{1}{10}\left[3^2(1^2 + 2^2 + \ldots + 10^2)\right]$

$- \left\{\frac{1}{10}\left[3(1 + 2 + \ldots + 10)\right]\right\}^2$

$= \frac{1}{10} \times 3465 - (16.5)^2$

$= 74.25$

9. (D) Using ***Shortcut 2***, we get

$2 = \sqrt{\frac{n^2 - 1}{12}}$

$\Rightarrow n = 7$

10. (B) Here, $\sum f_i = 20$, $\sum f_i x_i = 141$, $\sum f_i x_i^2 = 1051$

Var(X) $= \frac{\sum f_i x_i^2}{\sum f_i} - (\overline{x})^2$

$= \frac{1051}{20} - \left(\frac{141}{20}\right)^2$

$= 52.55 - 49.70$

$= 2.85$

11. (C) Here, $\sum f_i = (x + 1)^2 + 2x - 5 + x^2 - 3x + x$

$= 2x^2 + 2x - 4$

$\sum f_i x_i = 2(x + 1)^2 + 3(2x - 5) + 5(x^2 - 3x) + 7x$

$= 7x^2 + 2x - 13$

N = 20 …[Given]

$\Rightarrow \sum f_i = 20$

$\Rightarrow 2x^2 + 2x - 4 = 20$

$\Rightarrow x = -4, 3$

$\Rightarrow x = 3$ …[$\because x \in N$]

Now mean $(\overline{x}) = \frac{\sum f_i x_i}{N}$

$= \frac{7(3)^2 + 2(3) - 13}{20}$

$= 2.8$

12. (A) Variance = $\frac{1}{N}\sum x^2 - \left(\frac{\sum x}{N}\right)^2$

$= \frac{18000}{60} - \left(\frac{960}{60}\right)^2$

$= 44$

13. (C) Var(X) $= \frac{1}{N}\sum x_i^2 - \left(\frac{\sum x_i}{N}\right)^2$

$= \frac{16.9}{10} - \left(\frac{12}{10}\right)^2 = 0.25$

S.D. = $\sqrt{0.25} = 0.5$

14. (B) Mean $(\overline{x}) = \frac{\sum x_i}{50}$

$\Rightarrow 16 = \frac{\sum x_i}{50} \Rightarrow \sum x_i = 800$

Standard Deviation = $\sqrt{\frac{\sum x_i^2}{50} - (\overline{x})^2}$

$\Rightarrow 16 = \sqrt{\frac{\sum x_i^2}{50} - (16)^2}$

$\Rightarrow \sum x_i^2 = (16^2 + 16^2)\, 50 \Rightarrow \sum x_i^2 = 25600$

Now, required mean = $\frac{\sum(x_i - 5)^2}{50}$

$= \frac{\sum x_i^2 + 25 \times 50 - 10\sum x_i}{50}$

$= \frac{25600 + 1250 - 8000}{50}$

$= 377$

15. (D) Variance remains the same i.e., 6

Mean $\left(\bar{x}\right) = \frac{\text{Sum of observations}}{15}$

$\Rightarrow 10 = \frac{\text{Sum of observations}}{15}$

$\Rightarrow$ Sum of observations = 150

Now, each observation is increased by 8.

New mean = $\frac{150+8(15)}{15} = 18$

16. (C) Mean = 5 …[Given]

$\therefore$ Mean = $\frac{\sum_{i=1}^{n} x_i}{n}$

$\Rightarrow 5 = \frac{3+5+7+a+b}{5}$

$\Rightarrow a + b = 10$ …(i)

S.D. = 2 …[Given]

$\therefore$ S.D. = $\sqrt{\frac{\sum x_i^2}{n} - \left(\bar{x}\right)^2}$

$\Rightarrow (2)^2 = \frac{3^2+5^2+7^2+a^2+b^2}{5} - (5)^2$

$\Rightarrow 4 = \frac{83+a^2+b^2}{5} - 25$

$\Rightarrow a^2 + b^2 = 62$ …(ii)

Now, (i) $\Rightarrow$ a + b = 10

Squaring both sides, we get

$(a + b)^2 = 100$

$a^2 + 2ab + b^2 = 100$

38 = 2ab …[From (ii)]

$\therefore$ ab = 19

Note that the required quadratic equation is expressed as

$x^2 - (a + b)x + ab = 0$

$\therefore$ $x^2 - 10x + 19 = 0$

17. (D) When each item of a data is multiplied by λ, variance is multiplied by λ^2.

$\therefore$ New variance = $3^2 \times 16$

$= 9 \times 16$

$= 144$

18. (C) Given that n = 50, $\bar{x} = 16$ and $\sigma_x^2 = 256$

$\therefore$ $\sigma_x^2 = \frac{1}{n}\left(\sum_{i=1}^{50} x_i^2\right) - \left(\bar{x}\right)^2$

$\therefore$ $256 = \frac{1}{50}\left(\sum_{i=1}^{50} x_i^2\right) - 256$

$\therefore$ $\frac{1}{50}\left(\sum_{i=1}^{50} x_i^2\right) = 512$

$\therefore$ $\sum_{i=1}^{50} x_i^2 = 25600$ …(i)

Now $\sum_{i=1}^{50}(x_i - 5)^2$

$= \sum_{i=1}^{50} x_i^2 + 25 \times 50 - 10\sum_{i=1}^{50} x_i$

$= 25600 + 1250 - 8000$ …[From (i) and (ii)]

$= 18850$

$\therefore$ Required Mean = $\frac{\sum_{i=1}^{50}(x_i - 5)^2}{50} = \frac{18850}{50} = 377$

19. (B) Variance = $\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2$

Here, n = 4 and variance = 5

$\therefore$ $5 = \frac{1}{4}[(-1)^2 + (0)^2 + (1)^2 + k^2] - \left(\frac{-1+0+1+k}{4}\right)^2$

$\therefore$ $5 = \frac{2+k^2}{4} - \frac{k^2}{16}$

$\therefore$ $80 = 8 + 4k^2 - k^2$

$\therefore$ $3k^2 = 72$

$\therefore$ $k^2 = 24$

$\therefore$ $k = 2\sqrt{6}$ …[$\because$ k > 0]

20. (A) Note that standard derivation is independent of change of origin.

$\therefore$ S.D. of x_i = S.D. of $(x_i - 2)$

$\therefore$ S.D. of $(x_i - 2)$

$= \sqrt{\frac{1}{n}\sum_{i=1}^{20}(x_i - 2)^2 - \left[\frac{\sum(x_i - 2)}{n}\right]^2}$

$= \sqrt{\frac{100}{20} - (1)^2}$

$= 2$

$\Rightarrow$ Requird S.D. = 2

21. (D) $\sigma^2 = \frac{1}{2n}[1^2 + 2^2 + 3^2 + \ldots + (2n)^2] - \left(\frac{1+2+3+\ldots+2n}{2n}\right)^2$

$= \frac{1}{2n}\left[\frac{2n(2n+1)(4n+1)}{6}\right] - \left[\frac{1}{2n} \times \frac{2n(2n+1)}{2}\right]^2$

$= \frac{(2n+1)(4n+1)}{6} - \left(\frac{2n+1}{2}\right)^2$

$= \frac{2n+1}{2}\left(\frac{4n+1}{3} - \frac{2n+1}{2}\right)$

$= \frac{2n+1}{2}\left(\frac{2n-1}{6}\right) = \frac{4n^2-1}{12}$

22. (D) When each term of a data is multiplied by λ, variance is multiplied by λ^2.

$\therefore$ New variance = $2^2 \times 5 = 20$

Evaluation Test

1. (D) When each item of a data is multiplied by λ, variance is multiplied by λ^2.
Hence, new variance = $5^2 \times 9 = 225$

2. (C) S. D. of first n natural numbers

$$= \sqrt{\frac{1}{n}\Sigma x^2 - \left(\frac{\Sigma x}{n}\right)^2} \quad \ldots\left[\because \bar{x} = \frac{\Sigma x}{n}\right]$$

$$= \sqrt{\frac{n(n+1)(2n+1)}{6n} - \left[\frac{n(n+1)}{2n}\right]^2}$$

$$= \sqrt{\frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2}$$

$$= \sqrt{\frac{n+1}{2}\left(\frac{2n+1}{3} - \frac{n+1}{2}\right)}$$

$$= \sqrt{\frac{n+1}{2}\left(\frac{4n+2-3n-3}{6}\right)}$$

$$= \sqrt{\frac{n^2-1}{12}}$$

3. (B) Let $y = \frac{ax+b}{c}$ i.e., $y = \frac{a}{c}x + \frac{b}{c}$

i.e., $y = Ax + B$, where $A = \frac{a}{c}$, $B = \frac{b}{c}$

$\therefore$ $\bar{y} = A\bar{x} + B$

$\therefore$ $y - \bar{y} = A(x - \bar{x})$

$\Rightarrow (y - \bar{y})^2 = A^2 (x - \bar{x})^2$

$\Rightarrow \Sigma(y - \bar{y})^2 = A^2 \Sigma(x - \bar{x})^2$

$\Rightarrow n.\sigma_y^2 = A^2.n\sigma_x^2 \Rightarrow \sigma_y^2 = A^2\sigma_x^2$

$\Rightarrow \sigma_y = |A|\sigma_x$

$\Rightarrow \sigma_y = \left|\frac{a}{c}\right|\sigma_x$

Thus, new S.D. $= \left|\frac{a}{c}\right|\sigma$

4. (D) $\sum_{j=1}^{18}(x_j - 8) = 9 \Rightarrow \sum_{j=1}^{18}x_j = 153$

and $\sum_{j=1}^{18}(x_j - 8)^2 = 45$

$\Rightarrow \sum_{j=1}^{18}(x_j^2 - 16x_j + 64) = 45$

$\Rightarrow \sum_{j=1}^{18}x_j^2 = 45 - 64 \times 18 + 16\sum_{j=1}^{18}x_j$

$= 45 - 1152 + 2448$
$= 1341$

$\therefore$ Standard deviation $= \sqrt{\frac{\Sigma x_j^2}{n} - \left(\frac{\Sigma x_j}{n}\right)^2}$

$= \sqrt{\frac{1341}{18} - \left(\frac{153}{18}\right)^2}$

$= \sqrt{74.5 - 72.25}$
$= 1.5$

5. (D)

6. (C) Let the two unknown items be x and y, then
Mean = 4

$\Rightarrow \frac{1+2+6+x+y}{5} = 4$

$\Rightarrow x + y = 11$ ...(i)

and variance = 5. 2

$\Rightarrow \frac{1^2+2^2+6^2+x^2+y^2}{5} - (\text{mean})^2 = 5.2$

$\Rightarrow 41 + x^2 + y^2 = 5\,[5.2 + (4)^2]$

$\Rightarrow 41 + x^2 + y^2 = 106$

$\Rightarrow x^2 + y^2 = 65$ ...(ii)

Solving (i) and (ii) for x and y, we get
$x = 4, y = 7$ or $x = 7, y = 4$

7. (B) Here $n_1 = 5$, $\bar{x}_1 = 8$, $\sigma_1^2 = 18$, $n_2 = 3$

$\bar{x}_2 = 8$, $\sigma_2^2 = 24$

$\bar{x}$ = combined mean $= \frac{5\times8+3\times8}{5+3} = \frac{64}{8} = 8$

Combined variance $= \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$,

where $d_1 = \bar{x}_1 - \bar{x}$, $d_2 = \bar{x}_2 - \bar{x}$

Now, $d_1 = 8 - 8$; $d_2 = 8 - 8 = 0$

Combined variance $= \frac{5(18)+3(24)}{5+3}$

$= \frac{90+72}{8}$

$= \frac{162}{8}$

$= 20.25$