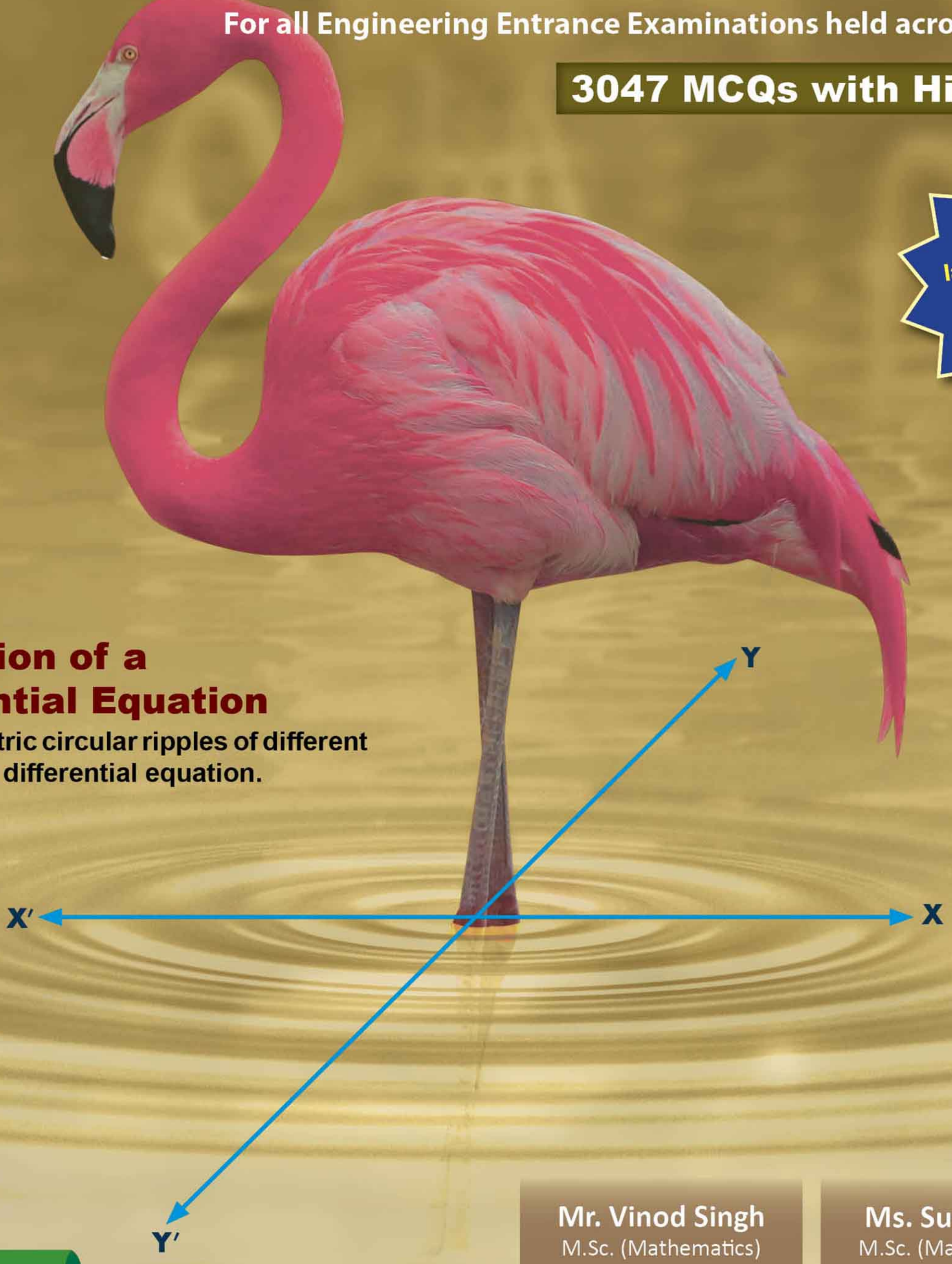
SAMPLE CONTENT

JEE (Main)
MATHEMATICS Vol - II

For all Engineering Entrance Examinations held across India.

3047 MCQs with Hints

As per latest syllabus issued by NTA



Formation of a Differential Equation

The concentric circular ripples of different radii form a differential equation.

Mr. Vinod Singh
M.Sc. (Mathematics)

Ms. Suchitra Yadav
M.Sc. (Mathematics), B.Ed.

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For all Engineering Entrance Examinations held across India.

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Absolute JEE (Main) Mathematics Vol. II

Updated as per latest syllabus for
JEE (Main) 2024 issued by **NTA on 01st November, 2023**

Salient Features

- Precise theory for every topic
- Subtopic-wise segregation of MCQs for efficient practice
- Exhaustive coverage of questions including questions from previous years' JEE (Main) and other competitive examinations **till year 2023**:
  - **3047** MCQs
  - **100** Numerical Value Type (NVT) questions
  - Solutions to the questions are provided for better understanding
  - **Shortcuts** for quick problem solving
- Includes relevant Solved Questions from:
  - JEE (Main) 2023 24th Jan (Shift - II)
- Topic Test with Answer keys provided in each chapter for self-assessment
- Q.R. codes provide:
  - Answers & Solutions to Topic Tests
  - Answers & Solutions to exam paper of JEE (Main) 2024 31st January (Shift – I)

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PREFACE

Target's **"Absolute Mathematics Vol. II"** has been compiled according to the notified syllabus for JEE (Main), which in turn has been framed after reviewing various national syllabi.

All the questions included in a chapter have been specially created and compiled to enable students solve complex problems which require strenuous effort with promptness.

Features in each chapter:

- Coverage of **'Theoretical Concepts'** that form a vital part of any competitive examination.
- **'Multiple Choice Questions'** are segregated topic-wise to enable easy assimilation of questions based on the specific concept.
- **'Important Note'** highlights the unique points about the topic.
- **'Formulae'** covers all the key formulae in the chapter, making it useful for students to glance at while solving problems and revising at the last minute.
- **'Shortcuts'** to help students save time while dealing with lengthy questions.
- **'Topic Test'** at the end of each chapter to assess the level of preparation of the student on a competitive level.

The level of difficulty of the questions is at par with that of various competitive examinations like JEE (Main), AIEEE, EAMCET, BCECE & the likes. Also to keep students updated, questions from most examinations such as MHT CET, Karnataka CET, EAMCET, WB JEE, BCECE, JEE (Main), etc. are covered.

Question Paper and Answer Keys of JEE (Main) 2024 31st Jan (Shift - I) have been provided to offer students glimpse of the complexity of questions asked in entrance examination. Solutions are also provided through a separate Q.R. code. The papers have been split unit-wise to let the students know which of the units were more relevant in the latest examinations.

This edition of **"Absolute Mathematics Vol. II"** has been conceptualized with absolute focus on the assistance students would require answering tricky questions and would give them an edge over the competition.

We hope the book benefits the learner as we have envisioned.

A book affects eternity; one can never tell where its influence stops.

Publisher

Edition: Fourth

KEY FEATURES

Important Notes

Important Notes

Highlights the unique points about the topic

Shortcuts

Shortcuts

'**Shortcuts**' help students to save time while dealing with lengthy questions.

Formulae

Formulae

'**Formulae**' cover all of the key formulae in the chapter useful for students while solving problems and revising at the last minute, at a glance.

Questions from competitive exams

Questions from competitive exams

To ensure students are well prepared, questions from various competitive exams are covered exclusively.

Topic Test

Topic Test

'**Topic Test**' covers questions from the chapter for self-evaluation purpose.

This is our attempt to offer students with revision and help them assess their knowledge of each chapter.

Q.R. Codes

QR codes

'**QR codes**' provide:

i. Answers & Solutions to the Topic Tests of each chapter
ii. Answers & Solutions to JEE (Main) 2024 31st January (Shift - I) paper.

Frequently Asked Questions

- **Why Absolute Series?**

  Gradually, every year the nature of competitive entrance exams is inching towards conceptual understanding of topics. Moreover, it is time to bid adieu to the stereotypical approach of solving a problem using a single conventional method.

  To be able to successfully crack the JEE (Main) examination, it is imperative to develop skills such as data interpretation, appropriate time management, knowing various methods to solve a problem, etc. With *Target's Absolute Series*, we are sure, you'd develop all the aforementioned skills and take a more holistic approach towards problem solving. The way you'd tackle advanced level MCQs with the help of hints, tips, shortcuts and necessary practice would be a game changer in your preparation for the competitive entrance examinations.

- **What is the intention behind the launch of Absolute Series?**

  The sole objective behind the introduction of Absolute Series is to severely test the students' preparedness to take competitive entrance examinations. With a healthy mix of MCQs, we intend to develop a student's MCQ solving skills within a stipulated time period.

- **What do I gain out of Absolute Series?**

  After using Absolute Series, students would be able to:

  a. Assimilate the given data and apply relevant concepts with utmost ease.

  b. Tackle MCQs of different pattern such as match the columns, diagram based questions and multiple concepts efficiently.

  c. Garner the much needed confidence to appear for various competitive exams.

  d. Easy and time saving methods to tackle tricky questions will help ensure that time consuming questions do not occupy more time than you can allot per question.

- **How to derive the best advantage of the book?**

  To get the maximum benefit of the book, we recommend :

  a. Go through the detailed theory and Examples solved alongwith at the beginning of a chapter for concept clarity.

  b. Know all the Formulae compiled at the end of theory by-heart.

  c. Using subtopic wise segregation as a leverage, complete MCQs in each subtopic at your own pace. Questions from exams such as JEE (Main) are tagged and placed along the flow of the subtopic. Mark these questions specially to gauge the trends of questions in various exams.

  d. Be more open to Shortcuts and Alternate Method. Assimilate them into your thinking.

Best of luck to all the aspirants!

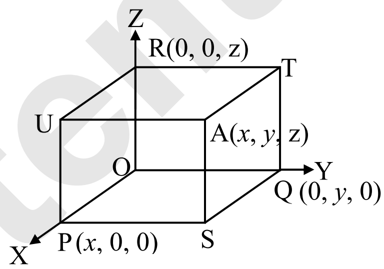
CONTENTS

6 Three Dimensional Geometry

- Co-ordinates of a point in space, Distance between two points, Section formula, Direction ratios and cosines
- Equations of a line, Angle between two intersecting lines, Skew lines (the shortest distance between them and its equation)

➢ **Co-ordinates of a point in space:**

In the adjoining figure, O is the origin. OX, OY and OZ are three mutually perpendicular lines which are also known as X-axis, Y-axis and Z-axis respectively. Let A be any point in space. Through A we draw three planes which are parallel to the co-ordinate planes to meet the axes at P, Q and R. The planes **XOY**, **YOZ** and **ZOX** are known as **XY-plane, YZ-plane, XZ-plane** respectively.



Also, **OP = x, OQ = y, OR = z**

These three real numbers taken in this order determined by the point A are called the **cartesian co-ordinates of the point A** written as (x, y, z). x, y, z are positive or negative accordingly as they are measured along positive or negative directions of the co-ordinate axes.

The three co-ordinate planes (XOY, YOZ and ZOX) divide space into eight parts, each part is called an **octant**.

Signs of co-ordinates of a point A(x, y, z) in different octants.

Octant (x, y, z)	O-XYZ (+, +, +)	O-X′YZ (–, +, +)	O-XY′Z (+, –, +)	O-X′Y′Z (–, –, +)
Octant (x, y, z)	O-XYZ′ (+, +, –)	O-X′YZ′ (–, +, –)	O-XY′Z′ (+, –, –)	O-X′Y′Z′ (–, –, –)

Important Note

❖ The co-ordinates of any point in XY plane, YZ plane and ZX plane are given by (x, y, 0), (0, y, z), (x, 0, z) respectively.

➢ **Distance between two points in space:**

i. Distance formula:

The distance between two points A(x_1, y_1, z_1) and B(x_2, y_2, z_2) in space is given by

AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

ii. Distance from origin:

Let O ≡ (0, 0, 0) be the origin and P(x, y, z) be any point. Then OP = $\sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$

∴ **OP** = $\sqrt{x^2 + y^2 + z^2}$

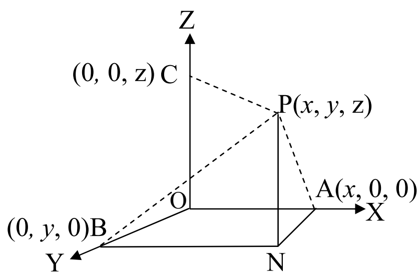
iii. Distance of a point from co-ordinate axes:

Let P(x, y, z) be any point in the space. Let PA, PB and PC be the perpendiculars drawn from P to the axes OX, OY and OZ respectively.

The distance of a point P(x, y, z) from X-axis is PA = $\sqrt{y^2 + z^2}$.

The distance of a point P(x, y, z) from Y-axis is PB = $\sqrt{x^2 + z^2}$.

The distance of a point P(x, y, z) from Z-axis is PC = $\sqrt{x^2 + y^2}$.



- **Section formulae:**

  Internal Division: The co-ordinates of a point P $\equiv (x, y, z)$ which divides the join of (x_1, y_1, z_1) and (x_2, y_2, z_2) **internally** in the ratio m : n are

  $$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right), \text{ where } m + n \neq 0$$

  eg.

  Find the co-ordinates of a point P which divides the line joining the points (2, 4, 5) and (3, 5, –4) internally in the ratio –2 : 3.

Solution:

$$P \equiv \left(\frac{-2(3)+3(2)}{-2+3}, \frac{-2(5)+3(4)}{-2+3}, \frac{-2(-4)+3(5)}{-2+3}\right)$$

$\therefore$ P $\equiv$ (0, 2, 23)

$\therefore$ the co-ordinates of point P are (0, 2, 23).

External Division:

The co-ordinates of a point P $\equiv (x, y, z)$ which divides the join of (x_1, y_1, z_1) and (x_2, y_2, z_2) **externally** in the ratio m : n are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right), \text{ where } m - n \neq 0$$

eg.

Find the co-ordinates of the point M which divides the line joining the points (–2, 4, 7) and (3, –5, 8) externally in the ratio 2 : 3.

Solution:

$$M \equiv \left(\frac{2(3)-3(-2)}{2-3}, \frac{2(-5)-3(4)}{2-3}, \frac{2(8)-3(7)}{2-3}\right)$$

$\therefore$ M $\equiv$ (–12, 22, 5)

$\therefore$ the co-ordinates of point M are (–12, 22, 5).

- **Centroid of a Triangle:**

  If G is the centroid of the triangle with vertices $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$, then

  $$\mathbf{G} \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

  eg.

  Find the co-ordinates of centroid of the triangle whose vertices are (2, –4, 3) , (3, –1, –2) and (–2, 5, 8).

Solution:

If G is the centroid of the triangle,

then $G \equiv \left(\frac{2+3-2}{3}, \frac{-4-1+5}{3}, \frac{3-2+8}{3}\right)$

$\therefore$ G $\equiv$ (1, 0, 3)

$\therefore$ the co-ordinates of centroid of the triangle is (1, 0, 3).

- **Centroid of a Tetrahedron:**

  If T is the centroid of the tetrahedron with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$, then

  $$\mathbf{T} \equiv \left(\frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{y_1 + y_2 + y_3 + y_4}{4}, \frac{z_1 + z_2 + z_3 + z_4}{4}\right)$$

  eg.

  Find the co-ordinates of centroid of the tetrahedron whose vertices are (2, –1, 3), (–1, 3, 1), (3, 4, –2), (4, 6, 2).

Solution:

If T is the centroid of the tetrahedron, then

$$T \equiv \left(\frac{2-1+3+4}{4}, \frac{-1+3+4+6}{4}, \frac{3+1-2+2}{4}\right)$$

$\therefore$ T $\equiv$ (2, 3, 1)

$\therefore$ the co-ordinates of centroid of the tetrahedron are (2, 3, 1).

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To see complete chapter buy **Target Notes** or **Target E-Notes**

- **Skew lines:**

  Two straight lines in space which are neither parallel nor intersecting are called **skew lines**. These are the lines that do not lie in same plane, hence we say that these lines are non-coplanar.

  Shortest distance between two skew lines (Vector form):

  The shortest distance between the lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$ is given by

$$\mathbf{d} = \left| \frac{\left(\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2\right) \cdot \left(\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1\right)}{\left|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2\right|} \right|$$

  Cartesian form:

  The shortest distance between the lines

  $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ is

$$\mathbf{d} = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1 n_2 - m_2 n_1)^2 + (l_2 n_1 - l_1 n_2)^2 + (l_1 m_2 - l_2 m_1)^2}}$$

  eg.

  Find the shortest distance between the lines l_1 and l_2 whose vector equations are

  $\vec{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$ and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$.

Solution:

Here, get $\vec{a}_1 = \hat{i} + \hat{j}$, $\vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\therefore$ $\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$ and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$

$\therefore$ $\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{9 + 1 + 49} = \sqrt{59}$

$\therefore$ the shortest distance between the given lines is $d = \left| \frac{\left(\vec{b}_1 \times \vec{b}_2\right) \cdot \left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|} \right| = \frac{|3 - 0 + 7|}{\sqrt{59}} = \frac{10}{\sqrt{59}}$ units

Formulae

1. The distance between two points A(x_1, y_1, z_1) and B(x_2, y_2, z_2) in space is given by

AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

2.
i. The distance of a point P(x, y, z) from the origin is $\sqrt{x^2 + y^2 + z^2}$.
ii. The distance of a point (x, y, z) from X-axis is $\sqrt{y^2 + z^2}$.
iii. The distance of a point (x, y, z) from Y-axis is $\sqrt{x^2 + z^2}$.
iv. The distance of a point (x, y, z) from Z-axis is $\sqrt{x^2 + y^2}$.

3. The co-ordinates of a point P ≡ (x, y, z) which divides the join of (x_1, y_1, z_1) and (x_2, y_2, z_2) internally in the ratio m : n are $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n}\right)$, where m + n $\neq$ 0

4. The co-ordinates of a point $P \equiv (x, y, z)$ which divides the join of (x_1, y_1, z_1) and (x_2, y_2, z_2) externally in the ratio m : n are $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$, where $m - n \neq 0$

5. If G is the centroid of the triangle with vertices $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$, then $G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

6. If T is the centroid of the tetrahedron with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$, then $T \equiv \left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$

7. The line segment joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is divided by

i. YZ-plane in the ratio $-x_1 : x_2$

ii. ZX-plane in the ratio $-y_1 : y_2$

iii. XY-plane in the ratio $-z_1 : z_2$

8. The direction ratios of a line joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is $x_2 - x_1$, $y_2 - y_1$ and $z_2 - z_1$.

9. If l, m, n are direction cosines of a vector $\vec{r}$, then

i. $\vec{r} = |\vec{r}|(l\hat{i}+m\hat{j}+n\hat{k}) \Rightarrow \hat{r} = l\hat{i}+m\hat{j}+n\hat{k}$

ii. $l^2 + m^2 + n^2 = 1$

iii. Projections of $\vec{r}$ on the co-ordinate axes are $l|\vec{r}|, m|\vec{r}|, n|\vec{r}|$.

10. Projection of a line obtained by joining two given points to another line whose direction cosines are l, m, n is given by $|l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$.

11. The equation of a line passing through the point $A\left(\vec{a}\right)$ and parallel to the vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a scalar.

12. The equation of a line passing through the point (x_1, y_1, z_1) and having direction ratios a, b, c is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$.

13. The equation of a line passing through the point $A\left(\vec{a}\right)$ and $B\left(\vec{b}\right)$ is $\vec{r} = (1-\lambda)\vec{a} + \lambda\vec{b}$, where λ is a scalar.

14. The equation of a line passing through the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

15. The length of perpendicular of the point $P\left(\vec{\alpha}\right)$ from the line $\vec{r} = \vec{a} + \lambda\vec{b}$ is given by $\sqrt{\left|\vec{\alpha}-\vec{a}\right|^2 - \left[\frac{\left(\vec{\alpha}-\vec{a}\right).\vec{b}}{\left|\vec{b}\right|}\right]^2}$.

16. The length of perpendicular from the point P(a, b, c) to the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is given by

$\sqrt{\left[(a-x_1)^2 + (b-y_1)^2 + (c-z_1)^2\right] - \left[(a-x_1)l + (b-y_1)m + (c-z_1)n\right]^2}$, where l, m, n are the direction cosines of the line.

17. The angle θ between the two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r}_2 = \vec{a}_2 + \mu\vec{b}_2$ is given by $\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}$.

18. The angle θ between the lines whose direction ratios are a_1, b_1, c_1 and a_2, b_2, c_2 is given by

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

19. The shortest distance between the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ is given by

$$d = \left|\frac{\left(\vec{b}_1\times\vec{b}_2\right).\left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$$

20. The shortest distance between the lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ is

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2}}$$

Shortcuts

1. An angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.
2. Any vector equally inclined to all the three axes have direction cosines as $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$.
3. The number of lines which are equally inclined to the co-ordinate axes is 4.
4. If l, m, n are the direction cosines of a line, then the maximum value of $l$mn = $\frac{1}{3\sqrt{3}}$.
5. The angle between a diagonal of a cube and the diagonal of a faces of the cube is $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$.
6. If a straight line makes angles α, β, γ, δ with the diagonals of a cube, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

Multiple Choice Questions

Co-ordinates of a point in space, Distance between two points, Section formula, Direction ratios and cosines

1. For every point (x, y, z) on the X-axis
(A) $x = 0$
(B) $y = 0$
(C) $x = 0, y = 0$
(D) $y = 0$, z = 0

2. The shortest distance of the point (a, b, c) from the X-axis is
(A) $\sqrt{(a^2 + b^2)}$ (B) $\sqrt{(b^2 + c^2)}$
(C) $\sqrt{(c^2 + a^2)}$ (D) $\sqrt{(a^2 + b^2 + c^2)}$

3. The distance of the point (4, 3, 5) from the Y-axis is
(A) $\sqrt{34}$ (B) 5
(C) $\sqrt{41}$ (D) $\sqrt{15}$

4. Distance of the point (1, 2, 3) from the co-ordinate axes are
(A) 13, 10, 5
(B) $\sqrt{13}, \sqrt{10}, \sqrt{5}$
(C) $\sqrt{3}, \sqrt{13}, \sqrt{10}$
(D) $\frac{1}{\sqrt{13}}, \frac{1}{\sqrt{10}}, \frac{1}{\sqrt{5}}$

5. From which of the following the distance of the point (1, 2, 3) is $\sqrt{10}$?
(A) Origin (B) X-axis
(C) Y-axis (D) Z-axis

6. If A(1, 2, 3), B(–1, –1, –1) be the points, then the distance AB is
(A) $\sqrt{5}$ (B) $\sqrt{21}$
(C) $\sqrt{29}$ (D) None of these

7. Distance between the points (1, 3, 2) and (2, 1, 3) is
(A) 12 (B) $\sqrt{12}$
(C) $\sqrt{6}$ (D) 6

8. If the extremities of the diagonal of a square are (1, –2, 3) and (2, –3, 5), then the length of the side is
(A) $\sqrt{6}$ (B) $\sqrt{3}$
(C) $\sqrt{5}$ (D) $\sqrt{7}$

9. If the sum of the squares of the distance of a point from the three co-ordinate axes be 36, then its distance from the origin is
(A) 6 (B) $3\sqrt{2}$
(C) $2\sqrt{3}$ (D) none of these

10. Points (1, 1, 1), (–2, 4, 1), (–1, 5, 5) and (2, 2, 5) are the vertices of a
(A) rectangle (B) square
(C) parallelogram (D) trapezium

11. The points A(5, –1, 1), B(7, –4, 7), C(1, –6, 10) and D(–1, –3, 4) are vertices of a
(A) square
(B) rhombus
(C) rectangle
(D) none of these

12. If the points (0, 1, 2), (2, –1, 3) and (1, –3, 1) are the vertices of a triangle, then the triangle is
(A) right angled
(B) isosceles right angled
(C) equilateral
(D) none of these

13. The triangle formed by the points (0, 7, 10), (–1, 6, 6), (– 4, 9, 6) is
(A) equilateral
(B) isosceles
(C) right angled
(D) right angled isosceles

14. The point equidistant from the points O(0, 0, 0), A(a, 0, 0), B(0, b, 0) and C(0, 0, c) has the co-ordinates
(A) (a, b, c) (B) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$
(C) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ (D) $\left(\frac{a}{4}, \frac{b}{4}, \frac{c}{4}\right)$

15. For every point P(x, y, z) on the XY-plane,
(A) $x = 0$ (B) $y = 0$
(C) z = 0 (D) none of these

16. The point dividing the line joining the points (1, 2, 3) and (3, –5, 6) in the ratio 3 : –5 is
(A) $\left(2, \frac{-25}{2}, \frac{3}{2}\right)$ (B) $\left(-2, \frac{25}{2}, \frac{-3}{2}\right)$
(C) $\left(2, \frac{25}{2}, \frac{3}{2}\right)$ (D) none of these

17. If A(1, 2, –1) and B(–1, 0, 1) are given, then the co-ordinates of P which divides AB externally in the ratio 1 : 2 are
(A) (1, 4, –1) (B) (3, 4, –3)
(C) (3, 4, 1) (D) none of these

18. The co-ordinates of the point which divides the join of the points (2, –1, 3) and (4, 3, 1) in the ratio 3 : 4 internally are given by
(A) $\frac{2}{7}, \frac{20}{7}, \frac{10}{7}$ (B) $\frac{15}{7}, \frac{20}{7}, \frac{3}{7}$
(C) $\frac{10}{7}, \frac{15}{7}, \frac{2}{7}$ (D) $\frac{20}{7}, \frac{5}{7}, \frac{15}{7}$

19. XY-plane divides the line joining the points (2, 4, 5) and (–4, 3, –2) in the ratio
(A) 3 : 5 (B) 5 : 2
(C) 1 : 3 (D) 3 : 4

20. XOZ plane divides the join of (2, 3, 1) and (6, 7, 1) in the ratio
(A) 3 : 7 (B) 2 : 7
(C) –3 : 7 (D) –2 : 7

21. The ratio in which the line joining the points (1, 2, 3) and (–3, 4, –5) is divided by the XY-plane, is
(A) 3 : 5 internally (B) 5 : 3 externally
(C) 3 : 5 externally (D) 5 : 3 internally

22. The plane XOZ divides the join of (1, –1, 5) and (2, 3, 4) in the ratio λ : 1, then λ is
(A) –3 (B) 3
(C) $-\frac{1}{3}$ (D) $\frac{1}{3}$

23. If the points A(9, 8, –10), B(3, 2, –4), and C(5, 4, –6) be collinear, then the point C divides the line AB in the ratio
(A) 2 : 1 (B) 3 : 1
(C) 1 : 2 (D) –1 : 2

24. If P(x, y, z) is a point on the line segment joining Q(2, 2, 4) and R(3, 5, 6) such that projections of $\vec{OP}$ on the axes are $\frac{13}{5}$, $\frac{19}{5}$, $\frac{26}{5}$ respectively, then P divides QR in the ratio
(A) 1 : 2 (B) 3 : 2
(C) 2 : 3 (D) 3 : 1

25. If x-coordinate of a point P of line joining the points Q(2, 2, 1) and R(5, 2, –2) is 4, then the z-coordinate of P is
(A) –2 (B) –1
(C) 1 (D) 2

26. Let A(2, –1, 4) and B(0, 2, –3) be two points and C be a point on AB produced such that 2AC = 3AB, then the co-ordinates of C are
(A) $\left(\frac{1}{2}, \frac{5}{4}, -\frac{5}{4}\right)$
(B) $\left(-\frac{1}{2}, \frac{7}{4}, -\frac{13}{4}\right)$
(C) (6, –7, 18)
(D) $\left(-1, \frac{7}{2}, \frac{-13}{2}\right)$

27. If the centroid of a triangle whose vertices are (a, 1, 3), (–2, b, –5) and (4, 7, c) be the origin, then the values of a, b, c are
(A) –2, –8, –2 (B) 2, 8, –2
(C) –2, –8, 2 (D) 7, –1, 0

28. If centroid of the tetrahedron OABC, where A, B, C are given by (a, 2, 3),(1, b, 2) and (2, 1, c) respectively be (1, 2, –1), then distance of P(a, b, c) from origin is equal to
(A) $\sqrt{107}$ (B) $\sqrt{14}$
(C) $\sqrt{\frac{107}{14}}$ (D) None of these

29. If the points (–1, 3, 2), (–4, 2, –2) and (5, 5, λ) are collinear, then λ =
(A) –10 (B) 5
(C) –5 (D) 10

30. Which of the following set of points are non- collinear?
(A) (1, –1, 1), (–1, 1, 1), (0, 0, 1)
(B) (1, 2, 3), (3, 2, 1), (2, 2, 2)
(C) (–2, 4, –3), (4, –3, –2), (–3, –2, 4)
(D) (2, 0, –1), (3, 2, –2), (5, 6, –4)

31. The direction ratios of the line joining the points (4, 3, –5) and (–2, 1, –8) are
(A) $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$ (B) 6, 2, 3
(C) 2, 4, –13 (D) none of these

32. If O is the origin and OP = 3 with direction ratios –1, 2, –2, then co-ordinates of P are
(A) (1, 2, 2) (B) (–1, 2, –2)
(C) (–3, 6, –9) (D) $\left(\frac{-1}{3}, \frac{2}{3}, \frac{-2}{3}\right)$

33. Direction ratios of the line which is perpendicular to the lines with direction ratios –1, 2, 2 and 0, 2, 1 are
[MHT CET 2016]
(A) 1, 1, 2 (B) 2, –1, 2
(C) – 2, 1, 2 (D) 2, 1, –2

34. If points P(4, 5, x), Q(3, y, 4) and R(5, 8, 0) are collinear, then the value of $x + y$ is
[MHT CET 2018]
(A) – 4 (B) 3
(C) 5 (D) 4

35. If the direction ratios of a line are 1, –3, 2, then the direction cosines of the line are
(A) $\frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$
(B) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$
(C) $\frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$
(D) $\frac{-1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}$

36. The direction cosines of the line passing through P(2, 3, –1) and the origin are
(A) $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$
(B) $-\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$
(C) $\frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$
(D) $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}$

37. The co-ordinates of a point P are (3, 12, 4) with respect to origin O, then the direction cosines of OP are
(A) 3, 12, 4 (B) $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$
(C) $\frac{3}{\sqrt{13}}, \frac{1}{\sqrt{13}}, \frac{2}{\sqrt{13}}$ (D) $\frac{3}{13}, \frac{12}{13}, \frac{4}{13}$

38. If $\frac{1}{2}, \frac{1}{3}$, n are direction cosines of a line, then the value of n is
(A) $\frac{\sqrt{23}}{6}$ (B) $\frac{23}{6}$
(C) $\frac{2}{3}$ (D) $\frac{1}{6}$

39. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then
(A) $0 < c < 1$ (B) $c > 2$
(C) $c > 0$ (D) $c = \pm\sqrt{3}$

40. Direction cosines of the line $\frac{x+2}{2}=\frac{2y-5}{3}$, z = –1 are _______ **[MHT CET 2016]**

(A) $\frac{4}{5},\frac{3}{5},0$ (B) $\frac{3}{5},\frac{4}{5},\frac{1}{5}$

(C) $-\frac{3}{5},\frac{4}{5},0$ (D) $\frac{4}{5},-\frac{2}{5},\frac{1}{5}$

41. The direction cosines of the line joining the points (4, 3, –5) and (–2, 1, –8) are

(A) $\left(\frac{6}{7},\frac{2}{7},\frac{3}{7}\right)$ (B) $\left(\frac{2}{7},\frac{3}{7},\frac{6}{7}\right)$

(C) $\left(\frac{6}{7},\frac{3}{7},\frac{2}{7}\right)$ (D) none of these

42. The direction cosines of a line segment AB are $\frac{-2}{\sqrt{17}}$, $\frac{3}{\sqrt{17}}$, $\frac{-2}{\sqrt{17}}$. If $AB=\sqrt{17}$ and the co-ordinates of A are (3, –6, 10), then the co-ordinates of B are

(A) (1, –2, 4) (B) (2, 5, 8)

(C) (–1, 3, –8) (D) (1, –3, 8)

43. If a line lies in the octant OXYZ and it makes equal angles with the axes, then

(A) $l=m=n=\frac{1}{\sqrt{3}}$

(B) $l=m=n=\pm\frac{1}{\sqrt{3}}$

(C) $l=m=n=-\frac{1}{\sqrt{3}}$

(D) $l=m=n=\pm\frac{1}{\sqrt{2}}$

44. If vector $\bar{r}$ with d.c.s. l, m, n is equally inclined to the co-ordinate axes, then the total number of such vectors is **[MHT CET 2017]**

(A) 4 (B) 6

(C) 8 (D) 2

45. ΔABC has vertices at A ≡ (2, 3, 5), B ≡ (–1, 3, 2) and C ≡ (λ, 5, μ). If the median through A is equally inclined to the axes, then the values of λ and μ respectively are **[MHT CET 2017]**

(A) 10, 7 (B) 9, 10

(C) 7, 9 (D) 7, 10

46. If α, β, γ be the direction angles of a vector and $\cos\alpha=\frac{14}{15}$, $\cos\beta=\frac{1}{3}$, then $\cos\gamma=$

(A) $\pm\frac{2}{15}$ (B) $\frac{1}{5}$

(C) $\pm\frac{1}{15}$ (D) none of these

47. A line which makes angle 60° with Y-axis and Z-axis, then the angle which it makes with X-axis is

(A) 45° (B) 60° (C) 75° (D) 30°

48. A line makes angles of 45° and 60° with the positive axes of X and Y respectively. The angle made by the same line with the positive axis of Z is

(A) 30° or 60° (B) 60° or 90°

(C) 90° or 120° (D) 60° or 120°

49. If a line makes angles 120° and 60° with the positive directions of X and Z axes respectively, then the angle made by the line with positive Y-axis is **[MHT CET 2018]**

(A) 150° (B) 60°

(C) 135° (D) 120°

50. A line makes angles α, β, γ with the co-ordinate axes. If α + β = 90°, then γ =

(A) 0° (B) 90°

(C) 180° (D) none of these

51. If a vector $\bar{x}$ makes angles with measure $\frac{\pi}{4}$ and $\frac{5\pi}{4}$ with positive directions of X-axis and Y-axis respectively, then $\bar{x}$ made angle of measure _______ with positive direction of Z-axis. **[Gujarat CET 2018]**

(A) $\frac{5\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

52. A line AB in three dimensional space marks angles 45° and 120° with the positive X-axis and the positive Y-axis respectively. If AB makes an acute angle θ with the positive Z-axis, then θ equals

(A) 60° (B) 75° (C) 30° (D) 45°

53. If a line make angles α, β and γ with the axes respectively, then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$

(A) 1 (B) 2

(C) 3 (D) none of these

54. If a line makes the angle α, β, γ with three dimensional co-ordinate axes respectively, then $\cos 2\alpha+\cos 2\beta+\cos 2\gamma=$ **[Karnataka CET 2016]**

(A) –2 (B) –1

(C) 1 (D) 2

55. If a line makes angles α, β, γ and δ with the four diagonals of a cube, then the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma+\sin^2\delta$ is **[EAMCET 2016]**

(A) $\frac{4}{3}$ (B) $\frac{8}{3}$ (C) $\frac{7}{3}$ (D) $\frac{5}{3}$

56. If the co-ordinates of A and B be (1, 2, 3) and (7, 8, 7), then the projections of the line segment AB on the co-ordinate axes are
(A) 6, 6, 4 (B) 4, 6, 4
(C) 3, 3, 2 (D) 2, 3, 2

57. The projection of the line segment joining the points A(–1, 0, 3) and B(2, 5, 1) on the line whose direction ratios are proportional to 6, 2, 3 is
(A) $\frac{10}{7}$ (B) $\frac{22}{7}$
(C) $\frac{18}{7}$ (D) none of these

58. The projection of any line on co-ordinate axes be respectively 3, 4, 5 then its length is
(A) 12 (B) 50
(C) $5\sqrt{2}$ (D) none of these

59. The projection of a line segment on the co-ordinate axes are 2, 3, 6. Then, the length of the line segment is
(A) 7 (B) 5
(C) 1 (D) 11

Equations of a line, Angle between two intersecting lines, Skew lines(the shortest distance between them and its equation)

1. The vector equation of the line
$3x - 2 = 2y + 1 = 3z - 3$ is:
(A) $\vec{r} = \frac{2}{3}\hat{i} - \frac{1}{2}\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} + 2\hat{k})$
(B) $\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$
(C) $\vec{r} = \lambda(2\hat{i} + 3\hat{j} + 2\hat{k})$
(D) $\vec{r} = \frac{2}{3}\hat{i} - \frac{1}{2}\hat{j} + \hat{k} + \lambda(2\hat{i} + \hat{j})$

2. The vector equation of the line passing through the points (1, – 2, 5) and (– 2, 1, 3) is
(A) $\bar{r} = -2\hat{i} + \hat{j} + 3\hat{k} + \lambda(3\hat{i} - 3\hat{j} + 2\hat{k})$
(B) $\bar{r} = -2\hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} + 3\hat{j} - 5\hat{k})$
(C) $\bar{r} = -\hat{i} - 2\hat{j} + 5\hat{k} + \lambda(-2\hat{i} - \hat{j} + 3\hat{k})$
(D) $\bar{r} = -2\hat{i} + \hat{j} + 3\hat{k} + \lambda(\hat{i} - 2\hat{j} + 5\hat{k})$

3. The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is
(A) parallel to X-axis
(B) parallel to Y-axis
(C) parallel to Z-axis
(D) perpendicular to Z-axis

4. The equation of line equally inclined to co-ordinate axes and passing through (–3, 2, –5) is **[MHT CET 2017]**
(A) $\frac{x+3}{1} = \frac{y-2}{1} = \frac{z+5}{1}$
(B) $\frac{x+3}{-1} = \frac{y-2}{1} = \frac{5+z}{-1}$
(C) $\frac{x+3}{-1} = \frac{y-2}{1} = \frac{z+5}{1}$
(D) $\frac{x+3}{-1} = \frac{2-y}{1} = \frac{z+5}{-1}$

5. The equation of a line passing through (1, –1, 0) and parallel to $\frac{x-2}{3} = \frac{2y+1}{2} = \frac{5-z}{-1}$ is
(A) $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-0}{-1}$
(B) $\frac{x-1}{3} = \frac{y+1}{1} = \frac{z-0}{-1}$
(C) $\frac{x-1}{3} = \frac{y+1}{1} = \frac{z-0}{1}$
(D) $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-0}{1}$

6. The equation of line passing through (3, –1, 2) and perpendicular to the lines
$\bar{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and
$\bar{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \mu(\hat{i} - 2\hat{j} + 2\hat{k})$ is
[MHT CET 2018]
(A) $\frac{x+3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$
(B) $\frac{x-3}{3} = \frac{y+1}{2} = \frac{z-2}{2}$
(C) $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$
(D) $\frac{x-3}{2} = \frac{y+1}{2} = \frac{z-2}{3}$

7. The equation of the straight line passing through the points (4, –5, –2) and (–1, 5, 3) is
(A) $\frac{x-4}{1} = \frac{y+5}{-2} = \frac{z+2}{-1}$
(B) $\frac{x+1}{1} = \frac{y-5}{2} = \frac{z-3}{-1}$
(C) $\frac{x}{-1} = \frac{y}{5} = \frac{z}{3}$
(D) $\frac{x}{4} = \frac{y}{-5} = \frac{z}{-2}$

8. The equation of the line passing through the points (3, 2, 4) and (4, 5, 2) is

(A) $\frac{x+3}{1}=\frac{y+2}{3}=\frac{z+4}{-2}$

(B) $\frac{x-3}{1}=\frac{y-2}{3}=\frac{z-4}{-2}$

(C) $\frac{x+3}{7}=\frac{y+2}{7}=\frac{z+4}{6}$

(D) $\frac{x-3}{7}=\frac{y-2}{7}=\frac{z-4}{6}$

9. The equation of the straight line passing through the points (a, b, c) and (a – b, b – c, c – a) is

(A) $\frac{x-a}{a-b}=\frac{y-b}{b-c}=\frac{z-c}{c-a}$

(B) $\frac{x-a}{b}=\frac{y-b}{c}=\frac{z-c}{a}$

(C) $\frac{x-a}{a}=\frac{y-b}{b}=\frac{z-c}{c}$

(D) $\frac{x-a}{2a-b}=\frac{y-b}{2b-c}=\frac{z-c}{2c-a}$

10. If $\frac{x-1}{l}=\frac{y-2}{m}=\frac{z+1}{n}$ is the equation of the line through (1, 2, –1) and (–1, 0, 1), then (l, m, n) is

(A) (–1, 0, 1) (B) (1, 1, –1)
(C) (1, 2, –1) (D) (0, 1, 0)

11. The line joining the points (–2, 1, –8) and (a, b, c) is parallel to the line whose direction ratios are 6, 2, 3. The values of a, b, c are

(A) 4, 3, –5 (B) 1, 2, $\frac{-13}{2}$
(C) 10, 5, –2 (D) none of these

12. The direction ratios of the line perpendicular to the lines $\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1}$ and $\frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$ are proportional to

(A) 4, 5, 7 (B) 4, –5, 7
(C) 4, –5, –7 (D) – 4, 5, 7

13. The point of intersection of the lines $\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}$, $\frac{x+3}{-36}=\frac{y-3}{2}=\frac{z-6}{4}$ is

(A) $\left(21,\frac{5}{3},\frac{10}{3}\right)$ (B) (2, 10, 4)
(C) (–3, 3, 6) (D) (5, 7, –2)

14. The lines $\frac{x-1}{2}=\frac{y+1}{2}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-6}{2}=\frac{z}{1}$ intersect each other at point

[MHT CET 2017]

(A) (–2, –4, 5) (B) (–2, –4, –5)
(C) (2, 4, –5) (D) (2, –4, –5)

15. Lines $\bar{r}=(3+t)\hat{i}+(1-t)\hat{j}+(-2-2t)\hat{k}$, t ∈ R and x = 4 + k, y = – k, z = – 4 – 2k, k ∈ R, then the relation between the lines is _______.

[Gujarat CET 2018]

(A) perpendicular (B) coincident
(C) skew (D) parallel

16. The distance of the point (–2, 4, –5) from the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is

[Karnataka CET 2017]

(A) $\frac{\sqrt{37}}{10}$ (B) $\sqrt{\frac{37}{10}}$
(C) $\frac{37}{\sqrt{10}}$ (D) $\frac{37}{10}$

17. The perpendicular distance of the point (2, 4, –1) from the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$ is

(A) 3 (B) 5 (C) 7 (D) 9

18. The length of the perpendicular drawn from the point (3, –1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is

(A) $\sqrt{33}$ (B) $\sqrt{53}$
(C) $\sqrt{66}$ (D) $\sqrt{29}$

19. The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is **[JEE (Main) April 2019]**

(A) greater than 3 but less than 4
(B) less than 2
(C) greater than 2 but less than 3
(D) greater than 4

20. The foot of the perpendicular drawn from the point (1, 8, 4) on the line joining the points (0, – 11, 4) and (2, – 3, 1) is **[WB JEE 2018]**

(A) (4, 5, 2) (B) (– 4, 5, 2)
(C) (4, – 5, 2) (D) (4, 5, – 2)

21. The shortest distance between A (1, 0, 2) and the line $\frac{x+1}{3}=\frac{y-2}{-2}=\frac{z+1}{-1}$ is given by line joining A and B, then B in the line is

[Assam CEE 2018]

(A) $\left(\frac{1}{2},1,\frac{-3}{2}\right)$ (B) $\left(\frac{2}{3},1,-1\right)$
(C) $\left(\frac{2}{3},\frac{-1}{2},-2\right)$ (D) (1, – 2, – 1)

22. The image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ is **[Karnataka CET 2018]**

(A) (1, 0, 7) (B) (7, 0, 1)
(C) (2, 7, 0) (D) (–1, –6, –3)

23. The length and foot of the perpendicular from the point (2, –1, 5) to the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$ are

(A) $\sqrt{14}$, (1, 2, –3)
(B) $\sqrt{14}$, (1, –2, 3)
(C) $\sqrt{14}$, (1, 2, 3)
(D) none of these

24. The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is

(A) $\sqrt{30}$ (B) $2\sqrt{30}$
(C) $5\sqrt{30}$ (D) $3\sqrt{30}$

25. The shortest distance between the skew lines $\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1}$, $\frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2}$ is **[EAMCET 2015]**

(A) 6 (B) 7
(C) $3\sqrt{5}$ (D) $\sqrt{35}$

26. The shortest distance between lines

$L_1 : \frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$,

$L_2 : \frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$ is

(A) 0 (B) $\frac{17}{\sqrt{3}}$
(C) $\frac{41}{5\sqrt{3}}$ (D) $\frac{17}{5\sqrt{3}}$

27. A line from the origin meets the lines

$\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$

at P and Q respectively. If length PQ = d, then d^2 is equal to

(A) 3 (B) 4
(C) 5 (D) 6

28. If direction ratios of two lines are 5, –12, 13 and –3, 4, 5, then the angle between them is

(A) $\cos^{-1}\left(\frac{1}{65}\right)$ (B) $\cos^{-1}\left(\frac{2}{65}\right)$
(C) $\cos^{-1}\left(\frac{3}{65}\right)$ (D) $\frac{\pi}{2}$

29. The angle between the pair of lines with direction ratios (1, 1, 2) and $(\sqrt{3}-1, -\sqrt{3}-1, 4)$ is

(A) 30° (B) 45°
(C) 60° (D) 90°

30. If the co-ordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (–4, 3, –6) and (2, 9, 2) respectively, then the angle between the lines AB and CD is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) 0°

31. The angle between the pair of lines $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$ is **[KEAM 2017]**

(A) $\cos^{-1}\left(\frac{21}{9\sqrt{38}}\right)$ (B) $\cos^{-1}\left(\frac{23}{9\sqrt{38}}\right)$
(C) $\cos^{-1}\left(\frac{24}{9\sqrt{38}}\right)$ (D) $\cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$

32. **Statement I:** The straight lines whose d.c's are given by $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular then, $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$

Statement II: If l_1, m_1, n_1 and l_2, m_2, n_2 are d.c's of line and $l_1l_2 + m_1m_2 + n_1n_2 = 0$ then lines are perpendicular.

(A) Both Statement I and Statement II are false
(B) Both Statement I and Statement II are true
(C) Statement I is false but Statement II is true
(D) Statement I is true but Statement II is false

33. If A, B, C, D are the points (2, 3, –1), (3, 5, –3), (1, 2, 3), (3, 5, 7) respectively, then the angle between AB and CD is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

34. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is **[Karnataka CET 2018]**

(A) 0° (B) 30°
(C) 45° (D) 90°

35. If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$, $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ are at right angles, then k =

(A) –10 (B) $\frac{10}{7}$ (C) $\frac{-10}{7}$ (D) $\frac{-7}{10}$

36. The acute angle between the lines $x = -2 + 2t$, $y = 3 - 4t$, $z = -4 + t$ and $x = -2 - t$, $y = 3 + 2t$, $z = -4 + 3t$ is **[BCECE 2015]**

(A) $\sin^{-1}\frac{1}{\sqrt{3}}$ (B) $\cos^{-1}\frac{1}{\sqrt{6}}$
(C) $\cos^{-1}\frac{1}{\sqrt{5}}$ (D) $\cos^{-1}\frac{2}{3}$

37. The angle between a line with direction ratios proportional to 2, 2, 1 and a line joining (3, 1, 4) to (7, 2, 12) is
(A) $\cos^{-1}\left(\frac{2}{3}\right)$ (B) $\cos^{-1}\left(-\frac{2}{3}\right)$
(C) $\tan^{-1}\left(\frac{2}{3}\right)$ (D) none of these

38. The lines whose direction cosines are given by the relations $al + bm + cn = 0$ and $mn + nl + lm = 0$ are **[TS EAMCET 2018]**
(A) perpendicular if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 0$
(B) perpendicular if $\sqrt{a}+\sqrt{b}+\sqrt{c} = 0$
(C) parallel if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 0$
(D) parallel if $a + b + c = 0$

39. The acute angle between the lines whose direction ratios are given by $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$ is
(A) 0 (B) $\frac{\pi}{6}$
(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

MISCELLANEOUS

1. Points (–2, 4, 7), (3, –6, –8) and (1, –2, –2) are
(A) collinear
(B) vertices of an equilateral triangle
(C) vertices of an isosceles triangle
(D) none of these

2. Consider the lines:
$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$,
$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$.
The unit vector perpendicular to both L_1 and L_2 is
(A) $\frac{1}{\sqrt{99}}\left(-\hat{i}+7\hat{j}+7\hat{k}\right)$
(B) $\frac{1}{5\sqrt{3}}\left(-\hat{i}-7\hat{j}+5\hat{k}\right)$
(C) $\frac{1}{5\sqrt{3}}\left(-\hat{i}+7\hat{j}+5\hat{k}\right)$
(D) $\frac{1}{\sqrt{99}}\left(7\hat{i}-7\hat{j}-\hat{k}\right)$

3. A line makes angles $\frac{\alpha}{2}, \frac{\beta}{2}, \frac{\gamma}{2}$ with positive directions of co-ordinate axes, then $\cos\alpha + \cos\beta + \cos\gamma$ is equal to
(A) 1 (B) –1
(C) 2 (D) 3

4. The co-ordinates of the foot of perpendicular drawn from point P(1, 0, 3) to the join of points A(4, 7, 1) and B(3, 5, 3) is
(A) (5, 7, 1) (B) $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$
(C) $\left(\frac{2}{3}, \frac{5}{3}, \frac{7}{3}\right)$ (D) $\left(\frac{5}{3}, \frac{2}{3}, \frac{7}{3}\right)$

5. A line makes 45° angle with positive X-axis and makes equal angles with positive Y, Z-axes respectively. The sum of the three angles which the line makes with positive X, Y, Z-axes is
(A) 180° (B) 165°
(C) 150° (D) 135°

6. The angle between the two diagonals of a cube is
(A) 30° (B) 45°
(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $\cos^{-1}\left(\frac{1}{3}\right)$

7. If a line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, then the value of
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ is
(A) 1 (B) $\frac{4}{3}$
(C) variable (D) none of these

Numerical Value Type Questions

1. The line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersect the curve $xy = \lambda$, $z = 0$. Find the value of λ.

2. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is

3. If the shortest distance between the lines
$\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4}$ and
$\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5}$ is 6, then the square of sum of all possible values of λ is
[JEE (Main) Jan 2023]

Topic Test

1. The point in XY-plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1) has the co-ordinates

(A) (2, 0, 8) (B) (0, 3, 1)
(C) (3, 2, 0) (D) (3, 2, 1)

2. Let P(2, –1, 4) and Q(4, 3, 2) be two points, and a point R on PQ is such that 3PQ = 5QR, then the co-ordinates of R are

(A) $\left(\frac{14}{5}, \frac{3}{5}, \frac{16}{5}\right)$ (B) $\left(\frac{16}{5}, \frac{7}{5}, \frac{14}{5}\right)$
(C) $\left(\frac{11}{4}, \frac{1}{2}, \frac{13}{4}\right)$ (D) none of these

3. If a plane cuts off intercepts OA = a, OB = b, OC = c from the co-ordinate axes, then the area of the triangle ABC =

(A) $\frac{1}{2}\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$
(B) $\frac{1}{2}(bc + ca + ab)$
(C) $\frac{1}{2}abc$
(D) $\frac{1}{2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}$

4. The direction angles of the line $x = 4z + 3$, $y = 2 - 3z$ are α, β and γ, then $\cos\alpha + \cos\beta + \cos\gamma$ = ________.

(A) $\frac{2}{\sqrt{26}}$ (B) $\frac{8}{\sqrt{26}}$
(C) 1 (D) 2

5. A line makes the same angle θ with each of the X and Z-axes. If the angle β which it makes with Y-axis is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$
(C) $\frac{3}{5}$ (D) $\frac{2}{3}$

6. A line L_1 passes through the point, whose p. v. (position vector) $3\hat{i}$, is parallel to the vector $-\hat{i} + \hat{j} + \hat{k}$. Another line L_2 passes through the point having p.v. $\hat{i} + \hat{j}$ is parallel to vector $\hat{i} + \hat{k}$, then the point of intersection of lines L_1 and L_2 has p.v.

(A) $2\hat{i} + 2\hat{j} + \hat{k}$ (B) $2\hat{i} + \hat{j} + \hat{k}$
(C) $2\hat{i} - \hat{j} - \hat{k}$ (D) $2\hat{i} - 2\hat{j} + \hat{k}$

7. Equation of the line of shortest distance between the lines $l_1 : \frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $l_2 : \frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ is $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$, where (a, b, c) is a point on line l_2, then the value of $c - b - a + l + m + n$ is

(A) 44 (B) 41 (C) 43 (D) 42

8. A line makes the same angle θ with each of the x and z-axis. If the line makes angle α with the y-axis and $\sin^2\alpha = 3\sin^2\theta$ then $\cos^2\theta$ equals

(A) $\frac{3}{5}$ (B) $\frac{2}{5}$ (C) 1 (D) 0

9. The projection of the line joining the points (3, 4, 5) and (4, 6, 3) on the line joining the points (–1, 2, 4) and (1, 0, 5), is

(A) $\frac{4}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

10. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection are given by

(A) (2a, a, 3a), (2a, a, a)
(B) (3a, 2a, 3a), (a, a, a)
(C) (3a, 2a, 3a), (a, a, 2a)
(D) (3a, 3a, 3a), (a, a, a)

11. ΔABC is formed by A (1, 8, 4), B (0, –11, 4) and C (2, –3, 1). If D is the foot of the perpendicular from A to BC, then the coordinates of D are

(A) (4, 5, –2) (B) (4, –5, 2)
(C) (–4, 5, 2) (D) (4, –5, –2)

12. If direction cosines of two lines are proportional to (2, 3, –6) and (3, –4, 5), then the acute angle between them is

(A) $\cos^{-1}\left(\frac{49}{36}\right)$ (B) $\cos^{-1}\left(\frac{18\sqrt{2}}{35}\right)$
(C) 96° (D) $\cos^{-1}\left(\frac{18}{35}\right)$

13. If the lines $x = y = z$ and $x = \frac{y}{2} = \frac{z}{3}$ and third line passing through (1, 1, 1) form a triangle of area $\sqrt{6}$ units then point of intersection of third line with second line is

(A) (1, 2, 3) (B) (2, 4, 6)
(C) $\left(\frac{4}{3}, \frac{8}{3}, \frac{12}{3}\right)$ (D) None of these

14. The angle between the lines whose direction ratios are given by $l + m + n = 0$ and $l^2 = m^2 + n^2$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

15. The lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other, if

(A) $aa' + cc' = 1$ (B) $aa' + cc' = -1$
(C) $ac + a'c' = 1$ (D) $ac + a'c' = -1$

16. Given that P (–1, 2, 3) and Q = (2, 3, 4). The point (s) which trisect the line segment PQ is (are)

(A) $\left(0, \frac{7}{3}, \frac{10}{3}\right)$ (B) $\left(1, \frac{8}{3}, \frac{11}{3}\right)$
(C) Both (A) and (B) (D) $\left(0, \frac{7}{3}, \frac{-1}{2}\right)$

17. If two straight lines whose direction cosines are given by the relation $m - n - l = 0$ and $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is.

(A) 6 (B) 4 (C) 3 (D) 2

18. The equation of line passing through (0, 1, 2) and parallel to $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is

(A) $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{4}$ (B) $\frac{x}{2} = \frac{y+1}{3} = \frac{z+2}{4}$
(C) $\frac{x}{2} = \frac{y-1}{-3} = \frac{z-2}{4}$ (D) $\frac{x}{2} = \frac{y+1}{-3} = \frac{z+2}{4}$

19. The equation of the line passing through the point (1, 2, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\bar{r} = \lambda\left(-3\hat{i} + 2\hat{j} + 5\hat{k}\right)$ is

(A) $\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} - 7\hat{j} + 4\hat{k}\right)$

(B) $\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 7\hat{j} - 4\hat{k}\right)$

(C) $\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} - 7\hat{j} - 4\hat{k}\right)$

(D) $\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 7\hat{j} + 4\hat{k}\right)$

20. The length of the perpendicular from the point (2,–1, 4) on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is

(A) greater than 3 but less than 4
(B) less than 2
(C) greater than 2 but less than 3
(D) greater than 4

21. If the reflection of the point P(1, 0, 0) in the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ is (a, b, c). Find $\left[\frac{bc}{a}\right]$, where [ ] represents the greatest integer function _______.

22. If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{\lambda}$ and $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then sum of possible values of λ is _______.

23. If the angle between the lines whose direction ratios are 2, –1 , 2 and a, 3, 5 be 45°, then a = _______

24. ABC is triangle in a plane with vertices A (2, 3, 5), B (–1, 3, 2) and C (λ, 5, μ). If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^2 + \mu^2 + 5)$ is _______

25. If the sum of squares of the distance of a point from the three co-ordinate axes be 24, then square distance from the origin is _______

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6. Three Dimensional Geometry

Co-ordinates of a point in space, Distance between two points, Section formula, Direction ratios and cosines

1. **(D)**
For every point (x, y, z) on the X-axis, $y = 0$, $z = 0$.

2. **(B)**
Distance from X-axis = $\sqrt{y^2 + z^2} = \sqrt{(b^2 + c^2)}$

3. **(C)**
Distance from Y-axis $= \sqrt{x^2 + z^2}$
$= \sqrt{16 + 25}$
$= \sqrt{41}$

4. **(B)**
Distance from X-axis $= \sqrt{4+9} = \sqrt{13}$
Distance from Y-axis $= \sqrt{1+9} = \sqrt{10}$
Distance from Z-axis $= \sqrt{1+4} = \sqrt{5}$

5. **(C)**
Distance from Y-axis $= \sqrt{1+9} = \sqrt{10}$

6. **(C)**
$AB = \sqrt{(-1-1)^2 + (-1-2)^2 + (-1-3)^2}$
$= \sqrt{4+9+16} = \sqrt{29}$

7. **(C)**
Required distance $= \sqrt{(2-1)^2 + (1-3)^2 + (3-2)^2}$
$= \sqrt{1+4+1} = \sqrt{6}$

8. **(B)**
Let P(1, –2, 3) and R(2, –3, 5) be the given points.
$\therefore$ $PR = \sqrt{(2-1)^2 + (-3+2)^2 + (5-3)^2} = \sqrt{6}$
$\therefore$ Length of the side $= \dfrac{\sqrt{6}}{\sqrt{2}}$
...[$\because$ diagonal $= \sqrt{2}$ (side)]
$= \sqrt{3}$

9. **(B)**
According to the given condition,
$\left[\sqrt{(x^2+y^2)}\right]^2 + \left[\sqrt{(y^2+z^2)}\right]^2 + \left[\sqrt{(z^2+x^2)}\right]^2 = 36$
$\Rightarrow x^2 + y^2 + z^2 = 18$
$\therefore$ Distance of the point $P(x, y, z)$ from the origin is
$\sqrt{x^2+y^2+z^2} = \sqrt{18} = 3\sqrt{2}$

10. **(B)**
Let A = (1, 1, 1), B = (–2, 4, 1), C = (–1, 5, 5) and D = (2, 2, 5).
$\therefore$ $AB = \sqrt{9+9+0} = 3\sqrt{2}$,
$BC = \sqrt{1+1+16} = 3\sqrt{2}$,
$CD = \sqrt{9+9+0} = 3\sqrt{2}$ and
$AD = \sqrt{1+1+16} = 3\sqrt{2}$
$\therefore$ It is a square.

11. **(B)**
Here, AB = BC = CD = DA = 7
Also, $\overrightarrow{AB}\cdot\overrightarrow{BC} \neq 0$
$\therefore$ ABCD is not a square.
But, $\overrightarrow{AC}\cdot\overrightarrow{BD} = 0$
$\therefore$ ABCD is a rhombus.

12. **(B)**
Let A = (0, 1, 2), B = (2, –1, 3) and C = (1, –3, 1)
$\therefore$ $AB = \sqrt{4+4+1} = \sqrt{9}$, $BC = \sqrt{1+4+4} = \sqrt{9}$ and
$AC = \sqrt{1+16+1} = \sqrt{18}$
$\therefore$ $AC^2 = AB^2 + BC^2$
$\therefore$ ΔABC is a right angled isosceles triangle.

13. **(D)**
Let A = (0, 7, 10), B = (–1, 6, 6) and C = (– 4, 9, 6)
$\therefore$ $AB = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$,
$BC = \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$,
$AC = \sqrt{16+4+16} = \sqrt{36} = 6$
$\therefore$ $AC^2 = AB^2 + BC^2$
$\therefore$ ΔABC is a right angled isosceles triangle.

14. **(B)**
Let $P(x, y, z)$ be the required point.
$\therefore$ OP = AP = BP = CP
Consider, OP = AP
$\Rightarrow OP^2 = AP^2$
$\Rightarrow x^2 + y^2 + z^2 = (x - a)^2 + y^2 + z^2$
$\Rightarrow -2ax + a^2 = 0$
$\Rightarrow x = \dfrac{a}{2}$
Similarly, OP = BP and OP = CP
$\Rightarrow y = \dfrac{b}{2}$ and $z = \dfrac{c}{2}$
$\therefore$ $P = \left(\dfrac{a}{2}, \dfrac{b}{2}, \dfrac{c}{2}\right)$

15. **(C)**
z-coordinate of every point on XY-plane is zero. The perpendicular distance of P(x, y, z) form XY-plane is zero.
$\therefore$ z = 0

16. **(B)**
$$x = \frac{mx_2 + nx_1}{m+n} = \frac{3(3)+(-5)(1)}{3-5} = -2$$
$$y = \frac{my_2 + ny_1}{m+n} = \frac{3(-5)+(-5)(2)}{3-5} = \frac{25}{2}$$
$$z = \frac{mz_2 + nz_1}{m+n} = \frac{3(6)+(-5)(3)}{3-5} = -\frac{3}{2}$$

17. **(B)**
By external division formula, we get
$$P = \left(\frac{1(-1)-2(1)}{1-2}, \frac{1(0)-2(2)}{1-2}, \frac{1(1)-2(-1)}{1-2}\right)$$
$$= (3, 4, -3)$$

18. **(D)**
By internal division formula, we get
$$P = \left(\frac{3(4)+4(2)}{3+4}, \frac{3(3)+4(-1)}{3+4}, \frac{3(1)+4(3)}{3+4}\right)$$
$$= \left(\frac{20}{7}, \frac{5}{7}, \frac{15}{7}\right)$$

19. **(B)**
The line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is divided by XY-plane in the ratio $-z_1 : z_2$
$\therefore$ Required ratio is 5 : 2.

20. **(C)**
XOZ plane divides the join of (2, 3, 1) and (6, 7, 1) in the ratio $-y_1 : y_2 = -3 : 7$

21. **(A)**
z-coordinate of every point on XY-plane is zero.
Now, $z = \frac{mz_2 + nz_1}{m+n}$
$\Rightarrow 0 = \frac{m(-5)+n(3)}{m+n}$
$\Rightarrow \frac{m}{n} = \frac{3}{5}$
$\therefore$ The line divides the XY-plane in the ratio 3 : 5 internally.

22. **(D)**
Since, XOZ plane i.e., $y = 0$ divides the join of (1, –1, 5) and (2, 3, 4) in the ratio λ : 1.
$\therefore$ $0 = \frac{3\lambda - 1}{\lambda + 1} \Rightarrow \lambda = \frac{1}{3}$

23. **(A)**
Let point C divides the line AB in the ratio λ : 1
$\therefore$ $5 = \frac{3\lambda + 9}{\lambda + 1}$
$\Rightarrow 5\lambda + 5 = 3\lambda + 9$
$\Rightarrow \lambda = 2$
$\therefore$ Required ratio is 2 : 1.

24. **(B)**
The coordinates of P are $\left(\frac{13}{5}, \frac{19}{5}, \frac{26}{5}\right)$.
Suppose, P divides QR in the ratio λ : 1.
$\therefore$ $\frac{13}{5} = \frac{3\lambda + 2}{\lambda + 1}$
$\Rightarrow \lambda = \frac{3}{2}$
$\therefore$ Required ratio is 3 : 2.

25. **(B)**
Suppose P divides QR in the ratio λ : 1.
Then, $P = \left(\frac{5\lambda+2}{\lambda+1}, \frac{2\lambda+2}{\lambda+1}, \frac{-2\lambda+1}{\lambda+1}\right)$.
But, x-coordinate of P is 4.
$\therefore$ $\frac{5\lambda+2}{\lambda+1} = 4 \Rightarrow \lambda = 2$
$\therefore$ z-coordinate of P = $\frac{-2\lambda+1}{\lambda+1} = \frac{-4+1}{2+1} = -1$

26. **(D)**
Given, 2AC = 3AB
$\Rightarrow$ 2AC = 3(AC – BC)
$\Rightarrow$ AC = 3BC
$\Rightarrow \frac{AC}{BC} = \frac{3}{1}$
$\Rightarrow$ C divides AB externally in the ratio 3 : 1.

$\therefore$ $C = \left(\frac{3(0)-1(2)}{3-1}, \frac{3(2)-1(-1)}{3-1}, \frac{3(-3)-1(4)}{3-1}\right)$
$\Rightarrow C = \left(-1, \frac{7}{2}, \frac{-13}{2}\right)$

27. **(C)**
According to the given condition,
$0 = \frac{a-2+4}{3} \Rightarrow a = -2, 0 = \frac{1+b+7}{3} \Rightarrow b = -8$
and $0 = \frac{3-5+c}{3} \Rightarrow c = 2$

28. **(A)**
Centroid $\equiv \left(\frac{\Sigma x}{4}, \frac{\Sigma y}{4}, \frac{\Sigma z}{4}\right) = (1, 2, -1)$
$\Rightarrow \left(\frac{a+3}{4}, \frac{3+b}{4}, \frac{5+c}{4}\right) = (1, 2, -1)$
$\Rightarrow$ a = 1, b = 5, c = –9
$\therefore$ $OP = \sqrt{a^2+b^2+c^2} = \sqrt{107}$

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$\therefore$ Angles between line and diagonals are

$$\cos\alpha = \frac{l+m+n}{\sqrt{3}}, \cos\beta = \frac{l+m-n}{\sqrt{3}},$$

$$\cos\gamma = \frac{-l+m+n}{\sqrt{3}}, \cos\delta = \frac{l-m+n}{\sqrt{3}}$$

$$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$$

$$= \frac{1}{3}\Big[(l+m+n)^2 + (l+m-n)^2 + (-l+m+n)^2 + (l-m+n)^2\Big]$$

$$= \frac{4}{3}$$

Numerical Value Type Questions

1. (5)

For the points where the line intersect the curve, we have z = 0

$$\Rightarrow \frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1} = 1$$

$$\Leftrightarrow \frac{x-2}{3} = 1 \quad \text{and} \quad \frac{y+1}{2} = 1$$

$$\Leftrightarrow x = 5 \quad \text{and} \quad y = 1$$

$$\Rightarrow \lambda = 5(1) = 5$$

2. (1)

$$x - \lambda = 2y - 1 = -2z$$

$$\Rightarrow \frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$$

$$\Rightarrow \frac{x-\lambda}{2} = \frac{y-\frac{1}{2}}{1} = \frac{z}{-1}$$

$$x = y + 2\lambda = z - \lambda$$

$$\Rightarrow \frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$$

$$\text{Distance (d)} = \frac{\begin{vmatrix} \lambda & 2\lambda+\frac{1}{2} & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}{\sqrt{(1+1)^2 + (-1-2)^2 + (2-1)^2}}$$

$$\Rightarrow \frac{\sqrt{7}}{2\sqrt{2}} = \frac{\left|-5\lambda - \frac{3}{2}\right|}{\sqrt{14}}$$

$$\Rightarrow |10\lambda + 3| = 7$$

$$\Rightarrow 10\lambda + 3 = 7 \text{ or } 10\lambda + 3 = -7$$

$$\Rightarrow \lambda = -1 \qquad \ldots[\lambda \text{ is an integer}]$$

$$\Rightarrow |\lambda| = 1$$

3. (384)

We know that,

the shortest distance between the lines

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \text{ and}$$

$$\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2} \text{ is}$$

$$d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2-m_2n_1)^2 + (l_1n_2-l_2n_1)^2 + (l_1m_2-l_2m_1)^2}}$$

$$\therefore \quad \frac{\begin{vmatrix} \lambda+\sqrt{6} & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}}{\sqrt{1+4+1}} = 6$$

$$\Rightarrow (\lambda+\sqrt{6})(-1) - \sqrt{6}(-2) - 3\sqrt{6}(-1) = \pm 6\sqrt{6}$$

$$\Rightarrow -\lambda - \sqrt{6} + 2\sqrt{6} + 3\sqrt{6} = \pm 6\sqrt{6}$$

$$\Rightarrow -\lambda = \pm 6\sqrt{6} - 4\sqrt{6}$$

$$\Rightarrow \lambda = \pm 6\sqrt{6} + 4\sqrt{6}$$

$$\therefore \quad \lambda = 6\sqrt{6} + 4\sqrt{6} \quad \text{or} \quad \lambda = -6\sqrt{6} + 4\sqrt{6}$$

$$\therefore \quad \lambda = 10\sqrt{6} \quad \text{or} \quad \lambda = -2\sqrt{6}$$

$\therefore$ The square of sum of all possible values of λ

$$= \left(10\sqrt{6} - 2\sqrt{6}\right)^2$$

$$= \left(8\sqrt{6}\right)^2$$

$$= 384$$