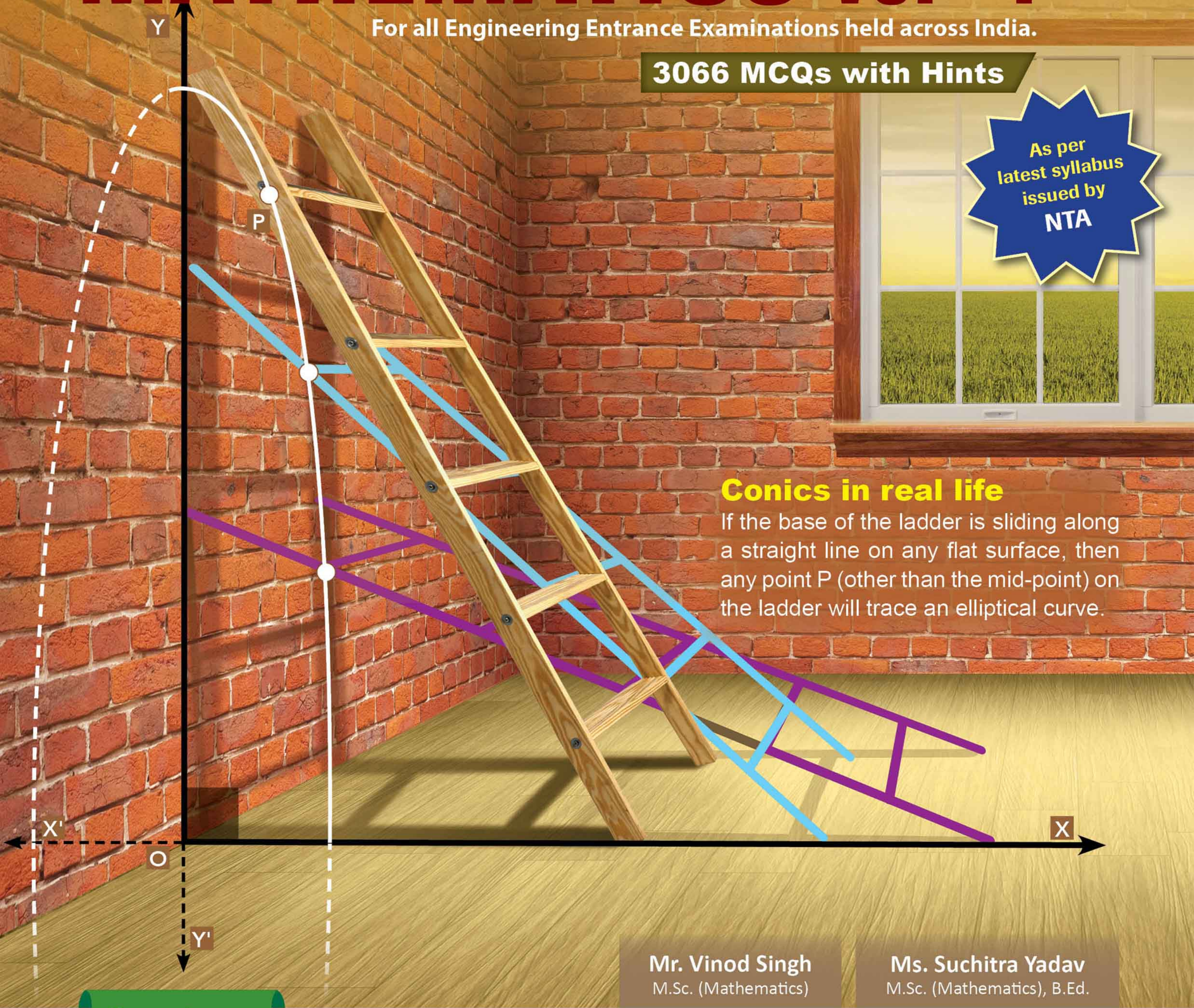
SAMPLE CONTENT

JEE (Main)
MATHEMATICS Vol - I

For all Engineering Entrance Examinations held across India.

3066 MCQs with Hints

As per latest syllabus issued by NTA



Conics in real life

If the base of the ladder is sliding along a straight line on any flat surface, then any point P (other than the mid-point) on the ladder will trace an elliptical curve.

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For all Engineering Entrance Examinations held across India.

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Absolute JEE (Main) Mathematics Vol. I

Updated as per latest syllabus for

JEE (Main) 2024 issued by **NTA on 01st November, 2023**

Salient Features

- Precise theory for every topic
- Subtopic-wise segregation of MCQs for efficient practice
- Exhaustive coverage of questions including questions from previous years' JEE (Main) and other competitive examinations **till year 2023**:
  - **3066** MCQs
  - **103** Numerical Value Type (NVT) questions
  - Solutions to the questions are provided for better understanding
  - **Shortcuts** for quick problem solving
- Includes relevant Solved Questions from:
  - JEE (Main) 2023 24th Jan (Shift - II)
- Topic Test with Answer keys provided in each chapter for self-assessment
- Q.R. codes provide:
  - Answers & Solutions to Topic Tests
  - Answers & Solutions to exam paper of JEE (Main) 2024 31st January (Shift – I)

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PREFACE

Target's **"Absolute Mathematics Vol. I"** has been compiled according to the notified syllabus for JEE (Main), which in turn has been framed after reviewing various national syllabi.

All the questions included in a chapter have been specially created and compiled to enable students solve complex problems which require strenuous effort with promptness.

Features in each chapter:

- Coverage of **'Theoretical Concepts'** that form a vital part of any competitive examination.
- **'Multiple Choice Questions'** are segregated topic-wise to enable easy assimilation of questions based on the specific concept.
- **'Important Note'** highlights the unique points about the topic.
- **'Formulae'** covers all the key formulae in the chapter, making it useful for students to glance at while solving problems and revising at the last minute.
- **'Shortcuts'** to help students save time while dealing with lengthy questions.
- **'Topic Test'** at the end of each chapter to assess the level of preparation of the student on a competitive level.

The level of difficulty of the questions is at par with that of various competitive examinations like JEE (Main), AIEEE, EAMCET, BCECE & the likes. Also to keep students updated, questions from most examinations such as MHT CET, Karnataka CET, EAMCET, WB JEE, BCECE, JEE (Main), etc. are covered.

Question Paper and Answer Keys of JEE (Main) 2024 31st Jan (Shift - I) have been provided to offer students glimpse of the complexity of questions asked in entrance examination. Solutions are also provided through a separate Q.R. code. The papers have been split unit-wise to let the students know which of the units were more relevant in the latest examinations.

This edition of **"Absolute Mathematics Vol. I"** has been conceptualized with absolute focus on the assistance students would require answering tricky questions and would give them an edge over the competition.

We hope the book benefits the learner as we have envisioned.

A book affects eternity; one can never tell where its influence stops.

Publisher

Edition: Fourth

KEY FEATURES

Important Notes

Important Notes

Highlights the unique points about the topic

Shortcuts

Shortcuts

'**Shortcuts**' help students to save time while dealing with lengthy questions.

Formulae

Formulae

'**Formulae**' cover all of the key formulae in the chapter useful for students while solving problems and revising at the last minute, at a glance.

Questions from competitive exams

Questions from competitive exams

To ensure students are well prepared, questions from various competitive exams are covered exclusively.

Topic Test

Topic Test

'**Topic Test**' covers questions from the chapter for self-evaluation purpose.

This is our attempt to offer students with revision and help them assess their knowledge of each chapter.

Q.R. Codes

QR codes

'**QR codes**' provide:

i. Answers & Solutions to the Topic Tests of each chapter
ii. Answers & Solutions to JEE (Main) 2024 31st January (Shift - I) paper.

Frequently Asked Questions

➢ **Why Absolute Series?**

Gradually, every year the nature of competitive entrance exams is inching towards conceptual understanding of topics. Moreover, it is time to bid adieu to the stereotypical approach of solving a problem using a single conventional method.

To be able to successfully crack the JEE (Main) examination, it is imperative to develop skills such as data interpretation, appropriate time management, knowing various methods to solve a problem, etc. With *Target's Absolute Series*, we are sure, you'd develop all the aforementioned skills and take a more holistic approach towards problem solving. The way you'd tackle advanced level MCQs with the help of hints, tips, shortcuts and necessary practice would be a game changer in your preparation for the competitive entrance examinations.

➢ **What is the intention behind the launch of Absolute Series?**

The sole objective behind the introduction of Absolute Series is to severely test the students' preparedness to take competitive entrance examinations. With a healthy mix of MCQs, we intend to develop a student's MCQ solving skills within a stipulated time period.

➢ **What do I gain out of Absolute Series?**

After using Absolute Series, students would be able to:

a. Assimilate the given data and apply relevant concepts with utmost ease.

b. Tackle MCQs of different pattern such as match the columns, diagram based questions and multiple concepts efficiently.

c. Garner the much needed confidence to appear for various competitive exams.

d. Easy and time saving methods to tackle tricky questions will help ensure that time consuming questions do not occupy more time than you can allot per question.

➢ **How to derive the best advantage of the book?**

To get the maximum benefit of the book, we recommend :

a. Go through the detailed theory and Examples solved alongwith at the beginning of a chapter for concept clarity.

b. Know all the Formulae compiled at the end of theory by-heart.

c. Using subtopic wise segregation as a leverage, complete MCQs in each subtopic at your own pace. Questions from exams such as JEE (Main) are tagged and placed along the flow of the subtopic. Mark these questions specially to gauge the trends of questions in various exams.

d. Be more open to Shortcuts and Alternate Method. Assimilate them into your thinking.

Best of luck to all the aspirants!

CONTENTS

4 Binomial Theorem and its Simple Applications

- Binomial theorem for a positive integral index, General term, Middle term and simple applications

➢ **Introduction:**

Any algebraic expression which contains two dissimilar terms is called **binomial expression.**

e.g.

$x + y$, $2 - x$, $2a - 5b$, $x^2y + \frac{1}{xy^2}$, $\frac{2a^2}{5} + \frac{5b^3}{3}$; etc.

We know the following formulae:

$(x + y)^0 = 1$

$(x + y)^1 = x + y$

$(x + y)^2 = x^2 + 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

We observe that the coefficients in the above expansions follow a particular pattern as given below:

Index	Coefficients
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1

The above pattern is known as **Pascal's Triangle.**

Here,

i. each row starts with 1 and ends with 1.

ii. The coefficients of an expansion are obtained by sum of two coefficients, one just before it and other just after it, in the previous row.

➢ **Binomial theorem for positive integral index:**

If x and a are real numbers, then $\forall$ n $\in$ N,

$(x + a)^n = {}^nC_0\, x^n a^0 + {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 + \ldots + {}^nC_r\, x^{n-r} a^r + \ldots + {}^nC_{n-1}\, x^1 a^{n-1} + {}^nC_n\, x^0 a^n$

i.e., $\mathbf{(x + a)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} a^r}$

The above expansion is called **Binomial Theorem** or **Binomial Expansion.**

Important Notes

❖ The binomial expansion is also valid when x and a are complex numbers.

❖ The coefficients ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, …, ${}^nC_n$ or simply $C_0, C_1, C_2, \ldots, C_n$ are called binomial coefficients and they can be evaluated with the help of **Pascal's triangle.**

➢ **Properties of Binomial Expansion $(x + a)^n$:**

i. There are (n + 1) terms in the expansion.

ii. The coefficients of the terms equidistant from the beginning and the end in a binomial expansion, are equal.

i.e., ${}^nC_0 = {}^nC_n = 1$

${}^nC_1 = {}^nC_{n-1} = n$

${}^nC_2 = {}^nC_{n-2} = \frac{n(n-1)}{2!}$ and so on…

iii. In each term, sum of the indices of x and a is equal to n.

iv. If we put a = – a in $(x + a)^n$, then

$(x - a)^n = {}^nC_0\, x^n a^0 - {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 - {}^nC_3\, x^{n-3} a^3 + \ldots + (-1)^r\, {}^nC_r\, x^{n-r} a^r + \ldots + (-1)^n\, {}^nC_n\, a^n$

i.e., $\mathbf{(x - a)^n = \sum_{r=0}^{n} (-1)^r\, {}^nC_r\, x^{n-r} a^r}$

v. Putting $x = 1$ and a = x, we get

a. $(1 + x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \ldots + {}^nC_r\, x^r + \ldots + {}^nC_n\, x^n$

i.e., $\mathbf{(1 + x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r}$

This is the expansion of $(1 + x)^n$ in **ascending powers of x.**

b. $(1 + x)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1} + {}^nC_2\, x^{n-2} + \ldots + {}^nC_r\, x^{n-r} + \ldots + {}^nC_n$

i.e., $\mathbf{(1 + x)^n} = \sum_{r=0}^{n} {}^nC_r\, x^{n-r}$

This is the expansion of $(1 + x)^n$ in **descending powers** of x.

vi. Putting $x = 1$ and a = – x, we get

$(1 - x)^n = {}^nC_0 - {}^nC_1\, x + {}^nC_2\, x^2 - {}^nC_3\, x^3 + \ldots + (-1)^r\, {}^nC_r\, x^r + \ldots + (-1)^n\, {}^nC_n\, x^n$

i.e., $\mathbf{(1 - x)^n = \sum_{r=0}^{n} (-1)^r\, {}^nC_r\, x^r}$

vii. The coefficient of $(r + 1)^{th}$ term in the expansion of $(1 + x)^n$ is ${}^nC_r$.

viii. The coefficient of x^r in the expansion of $(1 + x)^n$ is ${}^nC_r$.

ix. a. $(x + a)^n + (x - a)^n = 2[{}^nC_0 x^n a^0 + {}^nC_2 x^{n-2} a^2 + {}^nC_4 x^{n-4} a^4 + \ldots]$

b. $(x + a)^n - (x - a)^n = 2[{}^nC_1 x^{n-1} a^1 + {}^nC_3 x^{n-3} a^3 + \ldots]$

c. $(1 + x)^n + (1 - x)^n = 2[{}^nC_0 + {}^nC_2 x^2 + {}^nC_4 x^4 + \ldots]$

d. $(1 + x)^n - (1 - x)^n = 2[{}^nC_1 x + {}^nC_3 x^3 + {}^nC_5 x^5 + \ldots]$

Important Notes

❖ If **n is odd**, then $\{(x + a)^n + (x - a)^n\}$ and $\{(x + a)^n - (x - a)^n\}$ both have the same number of terms = $\left(\frac{n+1}{2}\right)$ **terms.**

❖ If **n is even**, then $\{(x + a)^n + (x - a)^n\}$ has $\left(\frac{n}{2}+1\right)$ **terms** and $\{(x + a)^n - (x - a)^n\}$ has $\left(\frac{n}{2}\right)$ **terms.**

➢ **General term:**

The general term of the expansion is $(r + 1)^{th}$ term, usually denoted by T_{r+1} or t_{r+1}.

i. General term of the expansion $(x + a)^n$ is $T_{r+1} = {}^nC_r x^{n-r} a^r$

ii. General term of the expansion $(x - a)^n$ is $T_{r+1} = (-1)^r\, {}^nC_r x^{n-r} a^r$

iii. General term of the expansion $(1 + x)^n$ is $T_{r+1} = {}^nC_r x^r$
$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r$$

iv. General term of the expansion $(1 - x)^n$ is $T_{r+1} = (-1)^r\, {}^nC_r x^r$
$$= \frac{(-1)^r n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r$$

Important Note

❖ In the binomial expansion of $(x + a)^n$; the r^{th} term from the end = $(n - r + 2)^{th}$ term from the beginning
i.e., ${}^nC_{n-r+1} x^{r-1} a^{n-r+1}$

➢ **Independent term or constant term:**

Independent term or constant term of a binomial expansion is the term in which exponent of the variable is zero.

Condition:

In the expansion of $(x + a)^n$

$(n - r)$ [power of x] + r[power of a] = 0

e.g.

Find the constant term in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^{15}$

Solution:

Given $\left(x^3 - \frac{1}{x^2}\right)^{15}$

Here, power of x = 3, power of a = – 2 and n = 15

Let $(r + 1)^{th}$ term be the constant term in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^{15}$

$\therefore$ $(15 - r)\,3 + r(-2) = 0$

$\therefore$ $45 - 3r - 2r = 0$

$\therefore$ $45 - 5r = 0$

$\therefore$ $r = 9$

$\therefore$ $(9 + 1)^{th}$ term is independent of x.

➢ **Middle term (s):**

In the expansion of $(x + a)^n$, the middle term depends upon the value of n.

i. If **n is even**, there is only one middle term, which is $\left(\frac{n+2}{2}\right)^{th}$ term.

i.e., $T_{\left[\frac{n}{2}+1\right]} = {}^nC_{\frac{n}{2}}\, x^{\frac{n}{2}} a^{\frac{n}{2}}$

e.g.

Find the middle term in the expansion of $\left(\frac{2}{3}x^2 - \frac{3}{2x}\right)^{20}$

Solution:

Here n = 20, which is an even number.

$\therefore$ $\left(\frac{20+2}{2}\right)^{th}$ term i.e., 11^{th} term is the middle term.

ii. If **n is odd,** there are two middle terms, which are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms

i.e., $T_{\left(\frac{n+1}{2}\right)} = {}^nC_{\frac{n-1}{2}}\, x^{\frac{n+1}{2}} a^{\frac{n-1}{2}}$

and $T_{\left(\frac{n+3}{2}\right)} = {}^nC_{\frac{n+1}{2}}\, x^{\frac{n-1}{2}} a^{\frac{n+1}{2}}$

e.g.

Find the middle terms in the expansion of $\left(3x - \frac{x^3}{6}\right)^7$.

Solution:

Here n = 7, which is an odd number.

$\therefore$ $\left(\frac{7+1}{2}\right)^{th}$ and $\left(\frac{7+3}{2}\right)^{th}$ i.e., 4^{th} and 5^{th} terms are two middle terms.

Important Notes

- When there are two middle terms in the expansion then their binomial coefficients are equal.
- Binomial coefficient of middle term is the greatest binomial coefficient.

➢ **Greatest coefficient in the expansion of $(x + a)^n$:**

i. If **n is even,** then greatest coefficient is $^nC_{\frac{n}{2}}$.

ii. If **n is odd**, then greatest coefficient is $^nC_{\frac{n-1}{2}}$ or $^nC_{\frac{n+1}{2}}$.

➢

i. Greatest term in the expansion of $(1 + x)^n$:

If T_r and T_{r+1} be the r^{th} and $(r + 1)^{th}$ terms in the expansion of $(1 + x)^n$, then

$$\frac{T_{r+1}}{T_r} = \frac{^nC_r x^r}{^nC_{r-1}x^{r-1}} = \frac{n-r+1}{r}|x|$$

Let numerically T_{r+1} be the greatest term in the above expansion. Then $T_{r+1} \geq T_r$

or $\frac{T_{r+1}}{T_r} \geq 1$

$\therefore$ $\frac{n-r+1}{r}|x| \geq 1$

or $r \leq \frac{(n+1)}{(1+|x|)}|x|$ ...(i)

Now substituting values of n and x in (i), we get

Case I : $r \leq m + f$ or

Case II : $r \leq m$

where m is a positive integer and f is a fraction such that $0 < f < 1$.

In the first case T_{m+1} is the greatest term, while in the second case T_m and T_{m+1} are the greatest terms and both are equal.

OR

When n is even, T_{m+1} is the greatest term, when n is odd, T_m and T_{m+1} are the greatest terms and both are equal.

e.g.

Find the greatest term in the expansion of $(1 + x)^{10}$, when $x = \frac{2}{3}$.

Solution:

In the expansion of $(1 + x)^{10}$, we have

$$\frac{T_{r+1}}{T_r} = \frac{10-r+1}{r}|x|$$

$$= \frac{11-r}{r}\left(\frac{2}{3}\right) \quad \ldots\left[\because x = \frac{2}{3}\right]$$

Now, $\frac{T_{r+1}}{T_r} > 1$

$\Rightarrow \left(\frac{11-r}{r}\right)\left(\frac{2}{3}\right) > 1$

$\Rightarrow 22 > 5r$

$\Rightarrow r < 4\frac{2}{5}$

$\therefore$ $(4 + 1)^{th}$ i.e., 5^{th} term is the greatest term.

ii. Greatest term in the expansion of $(x + a)^n$:

Greatest term for $(x + a)^n$ can be found as follows:

$$(x + a)^n = x^n\left(1+\frac{a}{x}\right)^n$$

Find the greatest term of $\left(1+\frac{a}{x}\right)^n$. Then multiply it by x^n, we get the result.

e.g.

Find the greatest term in the expansion of $(2 + 3x)^9$, when $x = \frac{3}{2}$.

Solution:

Since, $(2 + 3x)^9 = 2^9\left(1+\frac{3x}{2}\right)^9$

Now, in the expansion of $\left(1+\frac{3x}{2}\right)^9$, we have

$$\frac{T_{r+1}}{T_r} = \frac{9-r+1}{r}\left|\frac{3}{2}x\right|$$

$$= \frac{(10-r)}{r}\left|\frac{3}{2}\times\frac{3}{2}\right| \quad \ldots\left[\because x = \frac{3}{2}\right]$$

$$= \frac{90-9r}{4r}$$

$\therefore$ $\frac{T_{r+1}}{T_r} \geq 1 \Rightarrow \frac{90-9r}{4r} \geq 1$

$\Rightarrow 90 \geq 13r$

$\Rightarrow r \leq \frac{90}{13} = 6\frac{12}{13}$

$\therefore$ $r \leq 6\frac{12}{13}$

$\therefore$ $(6 + 1)^{th}$ i.e., 7^{th} term is the greatest term.

Important Notes

- The greatest term in $(1 + x)^{2n}$ has the greatest coefficient if $\frac{n}{n+1} < x < \frac{n+1}{n}$.
- $\sum_{r=0}^{n} {^nC_r} = 2^n$, $\sum_{r=1}^{n} {^{n-1}C_{r-1}} = 2^{n-1}$, $\sum_{r=0}^{n}(-1)^r\, {^nC_r} = 0$

➢ **Multinomial theorem (for a positive integral index):**

If n is a positive integer and $a_1, a_2, a_3, \ldots, a_m \in C$, then

$(a_1 + a_2 + a_3 + \ldots + a_m)^n$

$= \sum \dfrac{n!}{n_1!\, n_2!\, n_3! \ldots n_m!} a_1^{n_1} a_2^{n_2} a_3^{n_3} \ldots a_m^{n_m}$

where $n_1, n_2, n_3, \ldots, n_m$ are all non-negative integers subject to the condition

$n_1 + n_2 + n_3 + \ldots + n_m = n$

i. The coefficient of $a_1^{n_1} a_2^{n_2} a_3^{n_3} \ldots a_m^{n_m}$ in the expansion of $(a_1 + a_2 + a_3 + \ldots + a_m)^n$ is $\dfrac{n!}{n_1!\, n_2!\, n_3! \ldots n_m!}$.

e.g.

Find the coefficient of $a^8 b^6 c^4$ in the expansion of $(a + b + c)^{\mathbf{18}}$**.**

Solution:

The coefficient of $a^8 b^6 c^4$ in the expansion of $(a + b + c)^{18}$ is $\dfrac{18!}{8!\,6!\,4!}$

…[$\because$ n = 18, $n_1 = 8$, $n_2 = 6$, $n_3 = 4$]

ii. The **greatest coefficient** in the expansion of $(a_1 + a_2 + a_3 + \ldots + a_m)^n$ is $\dfrac{n!}{(q!)^{m-r}[(q+1)!]^r}$, where q is the quotient and r is the remainder when n is divided by m.

e.g.

Find the greatest coefficient in the expansion of $(x + y + z + u)^{10}$.

Solution:

Here, n = 10 and m = 4

…[$\because$ x, y, z, u are four terms]

Now,

$$\begin{array}{r} 2 \\ 4\overline{)\,10} \\ \underline{-8} \\ 2 \end{array}$$

$\therefore$ q = 2 and r = 2

Hence, greatest coefficient $= \dfrac{10!}{(2!)^{4-2}\left[(2+1)!\right]^2}$

$= \dfrac{10!}{(2!)^2 (3!)^2}$

iii. The number of distinct terms in the expansion of $(a_1 + a_2 + a_3 + \ldots + a_m)^n$ is ${}^{n+m-1}C_{m-1}$.

e.g.

Find the total number of terms in the expansion of $(2x + y + z)^{12}$.

Solution:

Here n = 12, m = 3

$\therefore$ total number of terms is ${}^{12+3-1}C_{3-1} = 91$.

Important Notes

❖ Number of terms in the expansion of $(a + b + c)^n$

$= {}^{n+2}C_2 = \dfrac{(n+1)(n+2)}{2}$

❖ Number of terms in the expansion of $(a + b + c + d)^n$

$= {}^{n+3}C_3 = \dfrac{(n+1)(n+2)(n+3)}{6}$

➢ **Binomial theorem for any index:**

Let n be a rational number and x be a real number such that $|x| < 1$, then

$(1 + x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3$

$+ \ldots + \dfrac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \ldots\infty$

This is called **Binomial theorem for any index.**

General term in the expansion of $(1 + x)^n$, $|x| < 1$, $n \in Q$

$T_{r+1} = \dfrac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r$

i.e., $T_{r+1} = {}^nC_r\, x^r$

Some important expansions:

For $|x| < 1$, we have

i. $(1 + x)^{-1} = 1 - x + x^2 - x^3 + \ldots$ to ∞

ii. $(1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots$ to ∞

iii. $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots$ to ∞

iv. $(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots$ to ∞

v. $(1 + x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \ldots$ to ∞

vi. $(1 - x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \ldots$ to ∞

Particular Expansions	General Term (T_{r+1})
$(1 + x)^{-1}$	$(-1)^r x^r$
$(1 + x)^{-2}$	$(-1)^r (r + 1) x^r$
$(1 + x)^{-3}$	$(-1)^r \dfrac{(r+1)(r+2)}{2!}x^r$
$(1 + x)^{-4}$	$(-1)^r \dfrac{(r+1)(r+2)(r+3)}{3!}x^r$
$(1 - x)^{-1}$	x^r
$(1 - x)^{-2}$	$(r + 1) x^r$
$(1 - x)^{-3}$	$\dfrac{(r+1)(r+2)}{2!}x^r$
$(1 - x)^{-4}$	$\dfrac{(r+1)(r+2)(r+3)}{3!}x^r$

Important Notes

❖ If x is so small that x^2, x^3 are all neglected, then $(1 + x)^n = 1 + nx$.

❖ $(1 + x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2$, if x is small so that $x^3, x^4, \ldots$ are all neglected.

Formulae

1. Binomial theorem for positive integral index:
If x and a are real numbers, then $\forall$ n $\in$ N,
$(x + a)^n$
$= {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + \ldots + {}^nC_r x^{n-r} a^r + \ldots + {}^nC_{n-1} x^1 a^{n-1} + {}^nC_n x^0 a^n$
i.e., $(x + a)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} a^r$

2. Number of terms in the expansion of $(x + a)^n$ is $n + 1$.

3. The coefficients of the terms equidistant from the beginning and the end in a binomial expansion $(x + a)^n$ are equal.

4. $(r + 1)^{th}$ term from the end in the expansion $(x + a)^n$ is equal to $(r + 1)^{th}$ term from the beginning in the expansion of $(a + x)^n$.

5. $(1 + x)^n$
$= {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_r x^r + \ldots + {}^nC_n x^n$
i.e., $(1 + x)^n = \sum_{r=0}^{n} {}^nC_r x^r$
This is the expansion of $(1 + x)^n$ in **ascending powers of x.**

6. $(1 + x)^n$
$= {}^nC_0 x^n + {}^nC_1 x^{n-1} + {}^nC_2 x^{n-2} + \ldots + {}^nC_r x^{n-r} + \ldots + {}^nC_n$
i.e., $(1 + x)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r}$
This is the expansion of $(1 + x)^n$ in **descending powers of x.**

7. $(x + a)^n + (x - a)^n$
$= 2[{}^nC_0 x^n a^0 + {}^nC_2 x^{n-2} a^2 + {}^nC_4 x^{n-4} a^4 + \ldots]$

8. $(x + a)^n - (x - a)^n$
$= 2[{}^nC_1 x^{n-1} a^1 + {}^nC_3 x^{n-3} a^3 + \ldots]$

9. $(1 + x)^n + (1 - x)^n$
$= 2[{}^nC_0 + {}^nC_2 x^2 + {}^nC_4 x^4 + \ldots]$

10. $(1 + x)^n - (1 - x)^n = 2[{}^nC_1 x + {}^nC_3 x^3 + \ldots]$

11. **General term** of the expansion $(x + a)^n$ is
$T_{r+1} = {}^nC_r x^{n-r} a^r$
General term of the expansion $(x - a)^n$ is
$T_{r+1} = (-1)^r\, {}^nC_r x^{n-r} a^r$
General term of the expansion $(1 + x)^n$ is
$T_{r+1} = {}^nC_r x^r$
$= \dfrac{n(n-1)(n-2)\ldots(n-r+1)}{r!} x^r$
General term of the expansion $(1 - x)^n$ is
$T_{r+1} = (-1)^r\, {}^nC_r x^r$
$= \dfrac{(-1)^r n(n-1)(n-2)\ldots(n-r+1)}{r!} x^r$

12. Middle term in the expansion of $(x + a)^n$:
i. If **n is even**, then the middle term
$= \left(\dfrac{n}{2}+1\right)^{th}$ term.
ii. If **n is odd**, then the middle term
$= \left(\dfrac{n+1}{2}\right)^{th}$ and $\left(\dfrac{n+3}{2}\right)^{th}$ terms.

13. Greatest coefficient of $(x + a)^n$:
i. If **n is even,** then greatest coefficient $= {}^nC_{\frac{n}{2}}$
ii. If **n is odd,** then greatest coefficient
$= {}^nC_{\frac{n-1}{2}}$ or ${}^nC_{\frac{n+1}{2}}$

14. i. If $x > 0$, then the greatest term in the expansion of $(1 + x)^n$, n $\in$ N is T_{m+1}, where m is largest positive integer 'r' satisfying
$\dfrac{T_{r+1}}{T_r} \ge 1$ i.e., $\dfrac{n-r+1}{r}\, x \ge 1$.
If for r = m, the equality holds, then both T_m and T_{m+1} are greatest and equal.
ii. In the expansion of $(x + a)^n$, x, a > 0, greatest term is T_{m+1}, where m is the largest positive integer 'r' satisfying
$\dfrac{T_{r+1}}{T_r} \ge 1$ i.e., $\dfrac{n-r+1}{r} \cdot \dfrac{a}{x} \ge 1$.
In case equality holds for r = m, then both T_m and T_{m+1} are greatest and equal.

15. Total number of terms in the expansion $(a_1 + a_2 + a_3 + \ldots + a_m)^n$ is ${}^{n+m-1}C_{m-1}$.

16. Let n be a rational number and x be a real number such that $|x| < 1$, then
$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \ldots\infty$$
This is called **Binomial theorem for any index.**

17. If $(1 + x)^n$
$= C_0 + C_1x + C_2x^2 + \ldots + C_n x^n$; n $\in$ N, then
i. $C_0a + C_1\dfrac{a^2}{2} + C_2\dfrac{a^3}{3} + \ldots + C_n\dfrac{a^{n+1}}{n+1} = \dfrac{(1+a)^{n+1}-1}{n+1}$
ii. $C_0a + \dfrac{C_2a^3}{3} + \dfrac{C_4a^5}{5} + \ldots = \dfrac{(1+a)^{n+1}-(1-a)^{n+1}}{2(n+1)}$
iii. $C_1 + 2C_2a + 3C_3a^2 + \ldots + nC_na^{n-1} = n(1 + a)^{n-1}$
iv. $C_1 + 3C_3a^2 + 5C_5a^4 + \ldots$
$= \dfrac{n}{a}[(1 + a)^{n-1} + (1 - a)^{n-1}]$

Shortcuts

1. Coefficient of x^m in the expansion of $\left(ax^p + \dfrac{b}{x^q}\right)^n$ = coefficient of T_{r+1},

where $r = \dfrac{np - m}{p+q}$.

e.g.

Coefficient of x^4 in $\left(\dfrac{x}{3} - \dfrac{2}{x^2}\right)^{10}$ = coefficient of T_{r+1}

where $r = \dfrac{10.1 - 4}{1+2} = 2$

$\therefore$ required coefficient $=$ coefficient of T_3

$= {}^{10}C_2 \left(\dfrac{1}{3}\right)^8 (-2)^2$

2. i. If the coefficients of p^{th}, q^{th} terms in the expansion of $(1 + x)^n$ are equal, then $p + q = n + 2$.

ii. If coefficients of x^r, x^{r+1} in the expansion of $\left(a + \dfrac{x}{b}\right)^n$ are equal, then

$n = (r + 1)(ab + 1) - 1$.

3. The term independent of x in the expansion of $\left(ax^p + \dfrac{b}{x^q}\right)^n = T_{r+1}$, where $r = \dfrac{np}{p+q}$.

e.g.

Term independent of x in

$\left(x^2 - \dfrac{1}{3x}\right)^9 = T_{r+1}$, where $r = \dfrac{9(2)}{2+1} = 6$

$\therefore$ required term $= T_{6+1} = T_7 = {}^9C_6\left(-\dfrac{1}{3}\right)^6$

$= 28$

4. i. The coefficient of x^{n-1} in the expansion of $(x - 1)(x - 2) \dots (x - n) = \dfrac{-n(n+1)}{2}$

ii. The coefficient of x^{n-1} in the expansion of $(x + 1)(x + 2) \dots (x + n) = \dfrac{n(n+1)}{2}$

5. i. If coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the expansion of $(1 + x)^n$ are in A.P., then $n^2 - n(4r + 1) + 4r^2 - 2 = 0$.

ii. If coefficients of x^{r-1}, x^r, x^{r+1} in the expansion of $(1 + x)^n$ are in A.P., then $n^2 - (4r + 1) + 4r^2 - 2 = 0$.

6. Sum of the coefficients of $(ax + by)^n = (a + b)^n$.

7. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the expansion of $(1 + x)^n$ are in H.P., then $n + (n - 2r)^2 = 0$.

8. To find the greatest term in the expansion of $(1 + x)^n$

i. Calculate, $r = \left|\dfrac{x(n+1)}{x+1}\right|$

ii. If r is integer, then T_r and T_{r+1} are equal and both are greatest terms.

iii. If r is not an integer, then $T_{[r]+1}$ is the greatest term, where [.] denotes the greatest integer $n \le x$.

e.g.

To find the greatest term of $(2 + 3x)^9$ at $x = \dfrac{3}{2}$.

Solution:

$(2 + 3x)^9 = 2^9\left(1 + \dfrac{3x}{2}\right)^9$

$= 2^9\left(1 + \dfrac{9}{4}\right)^9 \quad \dots\left[\because x = \dfrac{3}{2}\right]$

Now, $r = \left|\dfrac{x(n+1)}{x+1}\right| = \left|\dfrac{\frac{9}{4}(9+1)}{\frac{9}{4}+1}\right|$

$= 6\dfrac{12}{13}$

$\neq$ an integer

$\therefore$ $[r] = 6$

$\therefore$ the greatest term $= T_{6+1} = T_7$

9. If a_r is coefficient of x^r in the expansion of $(1 + x)^n$, $n \in N$, then

$\dfrac{a_{r+1}}{a_r} = \dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{n-r}{r+1}$

10. To find the sum of coefficients in the expansion of

i. $(x + y)^n$, put $x = y = 1$

ii $(x + y + z)^n$, put $x = y = z = 1$

iii. $(ax + by)^n$, put $x = y = 1$

iv. $(ax + by + cz)^n$, put $x = y = z = 1$

Multiple Choice Questions

Binomial theorem for a positive integral index, General term, Middle term and simple applications

1. If n is a positive integer, then the number of terms in the expansion of $(x + a)^n$ is
   (A) n (B) n + 1
   (C) n – 1 (D) none of these

2. $\left(\sqrt{3}+1\right)^4 + \left(\sqrt{3}-1\right)^4$ is equal to
   (A) a rational number
   (B) an irrational number
   (C) a negative integer
   (D) none of these

3. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n+1}+\left(\sqrt{3}-1\right)^{2n+1}$ is
(A) an odd positive integer
(B) an even positive integer
(C) an irrational number
(D) a rational number

4. When n = 8, $\left(\sqrt{3}+i\right)^{n}+\left(\sqrt{3}-i\right)^{n}=$ **[TS EAMCET 2018]**
(A) –256 (B) –128
(C) 256i (D) 128i

5. $(\sqrt{2}+1)^6-(\sqrt{2}-1)^6=$
(A) $101\sqrt{2}$ (B) $70\sqrt{2}$
(C) $140\sqrt{2}$ (D) $120\sqrt{2}$

6. $\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4=$ **[KEAM 2018]**
(A) $20\sqrt{6}$ (B) $30\sqrt{6}$
(C) $5\sqrt{10}$ (D) $40\sqrt{6}$

7. The value of $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$ will be
(A) – 198 (B) 198
(C) 99 (D) – 99

8. The number of non-zero terms in the expansion of $(1+3\sqrt{2}x)^9+(1-3\sqrt{2}x)^9$ is
(A) 9 (B) 0 (C) 5 (D) 10

9. The total number of terms in the expansion of $(x+a)^{47}-(x-a)^{47}$ after simplification is **[Karnataka CET 2017]**
(A) 24 (B) 47 (C) 48 (D) 96

10. The expression $\left(x+\sqrt{x^2-1}\right)^5+\left(x-\sqrt{x^2-1}\right)^5$ is a polynomial of degree
(A) 5 (B) 6 (C) 10 (D) 20

11. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is **[JEE (Main) 2018]**
(A) 0 (B) 1 (C) 2 (D) –1

12. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6$, $(x > 1)$ is equal to **[JEE (Main) April 2019]**
(A) 29 (B) 32 (C) 26 (D) 24

13. In the expansion of $(x + a)^n$, the sum of odd terms is P and sum of even terms is Q, then the value of $(P^2 - Q^2)$ will be
(A) $(x^2 + a^2)^n$ (B) $(x^2 - a^2)^n$
(C) $(x - a)^{2n}$ (D) $(x + a)^{2n}$

14. Sum of odd terms is A and sum of even terms is B in the expansion $(x + a)^n$, then
(A) $AB = \frac{1}{4}(x - a)^{2n} - (x + a)^{2n}$
(B) $2AB = (x + a)^{2n} - (x - a)^{2n}$
(C) $4AB = (x + a)^{2n} - (x - a)^{2n}$
(D) none of these

15. The last digit in 7^{300} is
(A) 7 (B) 9 (C) 1 (D) 3

16. The number $(101)^{100} - 1$ is divisible by **[WB JEE 2018]**
(A) 10^4 (B) 10^6 (C) 10^8 (D) 10^{12}

17. The remainder when 2^{2000} is divided by 17 is **[KEAM 2018]**
(A) 1 (B) 2 (C) 8 (D) 12

18. The coefficient of x^5 in the expansion of $(x + 3)^8$ is **[KEAM 2018]**
(A) 1542 (B) 1512
(C) 2512 (D) 12

19. The coefficient of x^5 in the expansion of $(1 + x^2)^5 (1 + x)^4$ is **[KEAM 2017]**
(A) 30 (B) 60
(C) 40 (D) 10

20. The coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$ is **[KEAM 2018]**
(A) 17 (B) 19
(C) –17 (D) –19

21. The coefficient of x^4 in the expansion of $(1 - x + x^2 - x^3)^4$ is **[EAMCET 2016]**
(A) 31 (B) 30
(C) 25 (D) –14

22. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^n$ is
(A) ${}^nC_4$
(B) ${}^nC_4 + {}^nC_2$
(C) ${}^nC_4 + {}^nC_2 + {}^nC_4 . {}^nC_2$
(D) ${}^nC_4 + {}^nC_2 + {}^nC_1 . {}^nC_2$

23. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is
(A) 144 (B) – 132
(C) – 144 (D) 132

24. The coefficient of x^5 in the expansion of $(x^2 - x - 2)^5$ is
(A) – 82 (B) – 81 (C) – 83 (D) 0

25. In the expansion of $(1 + 3x + 2x^2)^6$, the coefficient of x^{11} is
(A) 144 (B) 288 (C) 216 (D) 576

26. If $(1 + 2x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $a_r =$
(A) $({}^nC_r)^2$ (B) ${}^nC_r\,{}^nC_{r+1}$
(C) ${}^{2n}C_r$ (D) ${}^{2n}C_{r+1}$

Page no. **107** to **110** are purposely left blank.

To see complete chapter buy **Target Notes** or **Target E-Notes**

99. The coefficient of middle term in the expansion of $(1+x)^{10}$ is
(A) $\frac{10!}{5!6!}$ (B) $\frac{10!}{(5!)^2}$
(C) $\frac{10!}{5!7!}$ (D) $\frac{10!}{5!8!}$

100. In the expansion of $(3x + 2)^4$, the coefficient of middle term is
(A) 81 (B) 54 (C) 216 (D) 36

101. The middle term in the expansion of $\left(\frac{10}{x}+\frac{x}{10}\right)^{10}$ is **[Karnataka CET 2015]**
(A) ${}^9C_5$ (B) ${}^7C_5$ (C) ${}^{10}C_5$ (D) ${}^8C_5$

102. The middle term in the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$ is
(A) $C(12, 7)\, x^{-3} y^3$ (B) $C(12, 6)\, x^3 y^{-3}$
(C) $C(12, 7)\, x^3 y^{-3}$ (D) $C(12, 6)\, x^{-3} y^3$

103. The coefficient of the middle term in the binomial expansion, in powers of x, of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is same, if α equals
(A) $\frac{3}{5}$ (B) $\frac{10}{3}$ (C) $-\frac{3}{10}$ (D) $-\frac{5}{3}$

104. The greatest coefficient in the expansion of $(1+x)^{12}$ is
(A) ${}^{12}C_6$ (B) ${}^{12}C_4$ (C) ${}^{12}C_5$ (D) ${}^{12}C_8$

105. The greatest coefficient in the expansion of $(1+x)^{2n+2}$ is
(A) $\frac{(2n)!}{(n!)^2}$ (B) $\frac{(2n+2)!}{\{(n+1)!\}^2}$
(C) $\frac{(2n+2)!}{n!(n+1)!}$ (D) $\frac{(2n)!}{n!(n+1)!}$

106. The greatest term in the expansion of $(1 + 3x)^{54}$, when $x = \frac{1}{3}$, is
(A) 28^{th} (B) 25^{th} (C) 26^{th} (D) 24^{th}

107. The largest term in the expansion of $(3 + 2x)^{50}$, when $x = \frac{1}{5}$, is
(A) 15^{th} (B) 51^{st} (C) 7^{th} (D) 8^{th}

108. The numerically greatest term in the binomial expansion of $(2a - 3b)^{19}$ when $a = \frac{1}{4}$ and $b = \frac{2}{3}$ is **[AP EAMCET 2018]**
(A) ${}^{19}C_5.2^{11}$ (B) ${}^{19}C_3.\frac{1}{2^{11}}$
(C) ${}^{19}C_4.\frac{1}{2^{13}}$ (D) ${}^{19}C_3.2^{13}$

109. The greatest term in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$ is
(A) $\frac{25840}{9}$ (B) $\frac{24840}{9}$
(C) $\frac{26840}{9}$ (D) $\frac{23840}{9}$

110. If the middle term in the expansion of $(1 + x)^{2n}$ is the greatest term, then x lies in the interval **[EAMCET 2016]**
(A) $\left(\frac{n}{n+1},\frac{n+1}{n}\right)$ (B) $\left(\frac{n+1}{n},\frac{n}{n+1}\right)$
(C) $(n - 2, n)$ (D) $(n - 1, n)$

111. If n is even positive integer, then the condition that the greatest term in the expansion of $(1 + x)^n$ may also have the greatest coefficient is **[WB JEE 2018]**
(A) $\frac{n}{n+2} < x < \frac{n+2}{n}$
(B) $\frac{n}{n+1} < x < \frac{n+1}{n}$
(C) $\frac{n+1}{n+2} < x < \frac{n+2}{n+1}$
(D) $\frac{n+2}{n+3} < x < \frac{n+3}{n+2}$

112. If the 4^{th} term in the expansion of $\left(2+\frac{3}{8}x\right)^{10}$ has the maximum numerical value, then the range of value of x for which this will be true is given by
(A) $-\frac{64}{21} < x < -2$ (B) $-\frac{64}{21} < x < 2$
(C) $\frac{64}{21} < x < 4$ (D) none of these

Miscellaneous

1. The fourth term in the expansion of $(1-2x)^{\frac{3}{2}}$ will be
(A) $-\frac{3}{4}x^4$ (B) $\frac{x^3}{2}$
(C) $-\frac{x^3}{2}$ (D) $\frac{3}{4}x^4$

2. The first four terms in the expansion of $(1-x)^{\frac{3}{2}}$ are
(A) $1-\frac{3}{2}x+\frac{3}{8}x^2-\frac{x^3}{16}$ (B) $1-\frac{3}{2}x-\frac{3}{8}x^2-\frac{x^3}{16}$
(C) $1-\frac{3}{2}x+\frac{3}{8}x^2+\frac{x^3}{16}$ (D) $1+\frac{3}{2}x+\frac{3}{8}x^2-\frac{x^3}{16}$

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Numerical Value Type Questions

1. If the 4^{th} term in the expansion of $\left(\sqrt{x^{\frac{1}{\log_{10}x+1}}}+x^{\frac{1}{12}}\right)^6$ is 200, $x > 1$, find the value of x.

2. The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(1+x)^n$ are in the ratio 1 : 3 : 5. Find the average of the coefficients of $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms.

3. If the middle term in the binomial expansion of $\left(\frac{1}{x}+x\sin x\right)^{10}$ is $\frac{63}{8}$, then find the value of $6\sin^2 x+\sin x-2$.

4. If $C_r = {}^{25}C_r$ and $C_0+5C_1+9C_2+\ldots+(101)C_{25}=2^{25}\cdot k$, then k is equal to _______.

5. If $p=\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots\infty$, find p^2+2p-4.

6. If the sum of the coefficients of all even powers of x in the product $(1+x+x^2+\ldots+x^{2n})(1-x+x^2-x^3+\ldots+x^{2n})$ is 61, then n is equal to_______.
[JEE (Main) Jan 2020]

7. If the maximum value of the term independent of t in the expansion of $\left(t^2x^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geq 0$, is K, then 8K is equal to ________.
[JEE (Main) July 2022]

8. Let the sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n$, $x \neq 0$, $n \in N$, be 376. Then the coefficient of x^4 is _______.
[JEE (Main) Jan 2023]

Topic Test

1. For $n \in N$, in the expansion of $\left(\sqrt[3]{x^{-2}}+p\sqrt[3]{x^4}\right)^n$, the sum of all binomial coefficients lies between 50 and 100 and the term independent of x is 240. Then, the value of p^2 is
(A) 4 (B) 12
(C) 16 (D) 25

2. If the binomial coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $\left(\sqrt{2^{\log 10\left(10-3^x\right)}}+\sqrt[5]{2^{(x-2)\log 10\,3}}\right)^n$ are in A.P. and the 6^{th} term is 21, then the values of x are
(A) 0, 2 (B) 0, 3
(C) 1, 2 (D) 1, 3

3. If the last term in the expansion of $\left(2^{\frac{1}{3}}+2^{-\frac{1}{2}}\right)$ is $\left[\frac{1}{3(9)^{\frac{1}{3}}}\right]^{\log_3 8}$, then the 5^{th} term is equal to
(A) 90 (B) 150
(C) 210 (D) 270

4. 7^{103}, when divided by 25 leaves the remainder
(A) 15 (B) 16 (C) 18 (D) 20

5. If $S_n = {}^nC_0\cdot{}^nC_1+{}^nC_1\cdot{}^nC_2+\ldots+{}^nC_{n-1}\,{}^nC_n$ and $\frac{S_{n+1}}{S_n}=\frac{15}{4}$, then n equals
(A) 4 (B) 5 (C) 6 (D) 8

6. If the coefficients of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$ are equal, then the value of ab is
(A) $\frac{1}{7}$ (B) $\frac{1}{8}$ (C) 1 (D) 2

7. The value of r for which ${}^{40}C_r\,{}^{40}C_0+{}^{40}C_{r-1}\,{}^{40}C_1+{}^{40}C_{r-2}\,{}^{40}C_2+\ldots+{}^{40}C_0\,{}^{40}C_r$ is maximum, is
(A) 10 (B) 20
(C) 30 (D) 40

8. In the expansion $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, if the 7^{th} term from the beginning and end are equal, then the value of n is
(A) 10 (B) 12
(C) 14 (D) 16

9. The sum of all real values of x for which the middle term in the expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equal 5670 is
(A) 0 (B) 2 (C) 4 (D) 6

10. The sum of the coefficients of integral powers of x in the expansion of $\left(1+2\sqrt{x}\right)^{40}$, is
(A) $3^{40}+1$ (B) $3^{40}-1$
(C) $\frac{1}{2}\left(3^{40}-1\right)$ (D) $\frac{1}{2}\left(3^{40}+1\right)$

11. The coefficient of x^{11} in the expansion of $(1-x)(1-4x)(1-4^2x)...(1-4^{11}x)$ is

(A) $\frac{1}{3}(4^{25}-4^{37})$ (B) $\frac{1}{3}(4^{37}-4^{25})$

(C) $\frac{1}{3}(4^{55}-4^{67})$ (D) $\frac{1}{3}(4^{67}-4^{55})$

12. If { } represents fractional part of x, then $\left\{\frac{3^{2009}}{26}\right\}$ is

(A) $\frac{1}{26}$ (B) $\frac{3}{26}$

(C) $\frac{9}{26}$ (D) $\frac{11}{26}$

13. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is

(A) 23 (B) 35

(C) 38 (D) 43

14. If ${}^{40}C_1 + 2^2\ {}^{40}C_2 + 3^2\ {}^{40}C_3 + \ldots + 40^2\ {}^{40}C_{40} = p \times 2^q$, then the value of p + q is

(A) 1528 (B) 1678

(C) 1728 (D) 1838

15. If the 4th term in the binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$, $x > 0$, is 20×8^7, then the value of x is

(A) 8 (B) 8^2

(C) 8^3 (D) 8^4

16. If the coefficient of r^{th} and $(r+1)^{th}$ terms in the expansion of $(1+x)^{24}$ are in the ratio 12:13, then r is the root of the quadratic equation.

(A) $x^2 - 7x + 12 = 0$

(B) $x^2 - 14x + 13 = 0$

(C) $x^2 - 14x + 24 = 0$

(D) $x^2 - 7x + 1 = 0$

17. If x is positive, the first negative term in the expansion of $(1+x)^{\frac{21}{5}}$ is

(A) 4^{th} term (B) 5^{th} term

(C) 6^{th} term (D) 7^{th} term

18. The number of irrational terms in the binomial expansion of $\left(\sqrt[8]{5}+\sqrt[6]{2}\right)^{100}$ is

(A) 4 (B) 11

(C) 95 (D) 97

19. $2\,{}^{20}C_0 + 5\,{}^{20}C_1 + 8\,{}^{20}C_2 + \ldots + 62\,{}^{20}C_{20} =$

(A) 2^{23} (B) 2^{24}

(C) 2^{25} (D) 2^{26}

20. If k is an integral multiple of 4 lying in between the coefficients of x and x^4 in the expansion of $\left(x^2+\frac{1}{x}\right)^8$, then the number of values of k is

(A) 2 (B) 3

(C) 4 (D) 5

21. Find the digit at the unit's place in the number $17^{1995} + 11^{1995} - 7^{1995}$.

22. The coefficients of $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14. Find the average of the coefficients of $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms.

23. For $x \in R$, $x \neq -1$, if

$$(1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + \ldots + x^{2016} = \sum_{i=0}^{2016} a_i x^i$$

and $a_{24} = \frac{p!}{24!\,q!}$, then find the value of p – q.

24. If the sum of the coefficients of 1st, 2nd and 3rd terms in the expansion of $\left(x^2-\frac{2}{x}\right)^n$, $x \neq 0$, $n \in N$ is 391, then find the value of n.

25. If the sum of the coefficients in the expansion of $(x+y)^n$ is 2048, then find the value of the greatest coefficient in $(x+y)^n$.

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4. Binomial Theorem and its Simple Applications

Binomial theorem for a positive integral index, General term, Middle term and simple applications

1. **(B)**
Number of terms = one more than the power of the binomial

2. **(A)**
Since, $(x + a)^n + (x - a)^n$
$= 2[{}^nC_0\, x^n\, a^0 + {}^nC_2\, x^{n-2}\, a^2 + {}^nC_4\, x^{n-4}\, a^4 + \ldots]$
$\therefore$ $\left(\sqrt{3}+1\right)^4 + \left(\sqrt{3}-1\right)^4$
$= 2\left[{}^4C_0\left(\sqrt{3}\right)^4 + {}^4C_2\left(\sqrt{3}\right)^2(1)^2 + {}^4C_4\left(\sqrt{3}\right)^0(1)^4\right]$
$= 2\,[9 + 6(3) + 1]$
$= 56$
= a rational no.

3. **(C)**
$\left(\sqrt{3}+1\right)^{2n+1} + \left(\sqrt{3}-1\right)^{2n+1}$
$= 2\left\{{}^{2n+1}C_0\left(\sqrt{3}\right)^{2n+1} + {}^{2n+1}C_2\left(\sqrt{3}\right)^{2n-1} + \ldots + {}^{2n+1}C_{2n}\sqrt{3}\right\}$
$= 2\sqrt{3}\left\{{}^{2n+1}C_0\left(\sqrt{3}\right)^{2n} + {}^{2n+1}C_2\left(\sqrt{3}\right)^{2n-2} + \ldots + {}^{2n+1}C_{2n}\right\}$
$= 2\sqrt{3}$ {a positive integer}
= an irrational number.

4. **(A)**
$\left(\sqrt{3}+i\right)^8 + \left(\sqrt{3}-i\right)^8$
$= 2\left[{}^8C_0\left(\sqrt{3}\right)^8 + {}^8C_2\left(\sqrt{3}\right)^6 i^2 + {}^8C_4\left(\sqrt{3}\right)^4 i^4 + {}^8C_6\left(\sqrt{3}\right)^2 i^6 + {}^8C_8\, i^8\right]$
$= 2(81 - 756 + 630 - 84 + 1)$
$= 2(-128)$
$= -256$

5. **(C)**
Since, $(x + a)^n - (x - a)^n$
$= 2[{}^nC_1\, x^{n-1}\, a + {}^nC_3\, x^{n-3}\, a^3 + {}^nC_5\, x^{n-5}\, a^5 + \ldots]$
$\therefore$ $(\sqrt{2}+1)^6 - (\sqrt{2}-1)^6$
$= 2[{}^6C_1\,(\sqrt{2})^5\,(1)^1 + {}^6C_3\,(\sqrt{2})^3\,(1)^3 + {}^6C_5\,(\sqrt{2})^1\,(1)^5]$
$= 2\,[6\times 4\sqrt{2} + 20\times 2\sqrt{2} + 6\sqrt{2}]$
$= 2\,[24\sqrt{2} + 40\sqrt{2} + 6\sqrt{2}]$
$= 140\sqrt{2}$

6. **(D)**
$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$
$= 2\left[{}^4C_1\left(\sqrt{3}\right)^3\left(\sqrt{2}\right) + {}^4C_3\left(\sqrt{3}\right)\left(\sqrt{2}\right)^3\right]$
$= 2\left(12\sqrt{6} + 8\sqrt{6}\right)$
$= 40\sqrt{6}$

7. **(B)**
$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$
$= 2\left[\left(\sqrt{2}\right)^6 + {}^6C_2\left(\sqrt{2}\right)^4(1)^2 + {}^6C_4\left(\sqrt{2}\right)^2(1)^4 + {}^6C_6\right]$
$= 2(8 + 15 \times 4 + 15 \times 2 + 1)$
$= 198$

8. **(C)**
Given expression
$\left(1+3\sqrt{2}x\right)^9 + \left(1-3\sqrt{2}x\right)^9$
$= 2\left[1 + {}^9C_2(3\sqrt{2}x)^2 + {}^9C_4(3\sqrt{2}x)^4 + {}^9C_6(3\sqrt{2}x)^6 + {}^9C_8(3\sqrt{2}x)^8\right]$
$\therefore$ the number of non-zero terms is 5.

9. **(A)**
If n is odd, then $(x + a)^n - (x - a)^n$ will have $\left(\dfrac{n+1}{2}\right)$ terms.
$\therefore$ $(x + a)^{47} - (x - a)^{47}$ has $\left(\dfrac{47+1}{2}\right) = 24$ terms

10. **(A)**
$\left(x+\sqrt{x^2-1}\right)^5 + \left(x-\sqrt{x^2-1}\right)^5$
$= 2\left[x^5 + {}^5C_2 x^3\left(\sqrt{x^2-1}\right)^2 + {}^5C_4 x^1\left(\sqrt{x^2-1}\right)^4\right]$
$= 2x^5 + 2.\ {}^5C_2 x^3(x^2 - 1) + 2.\ {}^5C_4\, x(x^2 - 1)^2$ which is a polynomial of degree 5.

11. **(C)**
$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$
$= 2\left[{}^5C_0\, x^5 + {}^5C_2\, x^3\left(x^3-1\right) + {}^5C_4\, x\left(x^3-1\right)^2\right]$
$= 2\left[x^5 + 10x^3\left(x^3-1\right) + 5x\left(x^6 - 2x^3 + 1\right)\right]$
$= 2\,(x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x)$
Sum of the coefficients of all the odd degree terms = 2 (1 – 10 + 5 + 5)
= 2

12. **(D)**

$\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6$

$= 2\,[{}^6C_0\,x^6 + {}^6C_2\,x^4\,(x^3-1) + {}^6C_4\,x^2\,(x^3-1)^2 + {}^6C_6\,(x^3-1)^3\,]$

$= 2\,[x^6 + 15x^4\,(x^3-1) + 15x^2\,(x^6-2x^3+1) + (x^9-3x^6+3x^3-1)]$

$= 2\,(x^6 + 15x^7 - 15x^4 + 15x^8 - 30x^5 + 15x^2 + x^9 - 3x^6 + 3x^3 - 1)$

Sum of the coefficients of all even degree terms

$= 2(1 - 15 + 15 + 15 - 3 - 1)$

$= 2(12)$

$= 24$

13. **(B)**

$(x+a)^n = x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + {}^nC_3x^{n-3}a^3 + \ldots$

$= (x^n + {}^nC_2\,x^{n-2}\,a^2 + \ldots) + ({}^nC_1\,x^{n-1}a + {}^nC_3\,x^{n-3}a^3 + \ldots)$

$= P + Q$

$\therefore$ $(x-a)^n = P - Q$, as the terms are alter. +ve and –ve

$\therefore$ $P^2 - Q^2 = (P+Q)(P-Q) = (x+a)^n(x-a)^n$

$\therefore$ $P^2 - Q^2 = (x^2-a^2)^n$

14. **(C)**

$(x+a)^n = {}^nC_0\,x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + {}^nC_3\;\;x^{n-3}\;\;a^3 + \ldots$

According to the given condition,

$A = {}^nC_0\,x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4 + \ldots$

and $B = {}^nC_1x^{n-1}a + {}^nC_3\,x^{n-3}a^3 + \ldots$

$\therefore$ $(x+a)^n = A + B$

and $(x-a)^n = A - B$

But, $4AB = (A+B)^2 - (A-B)^2$

$\therefore$ $4AB = (x+a)^{2n} - (x-a)^{2n}$

15. **(C)**

We have, $7^2 = 49 = 50 - 1$

Now, $7^{300} = (7^2)^{150} = (50-1)^{150}$

$= {}^{150}C_0(50)^{150}\,(-1)^0 + {}^{150}C_1(50)^{149}(-1)^1 + \ldots + {}^{150}C_{150}(50)^0(-1)^{150}$

Thus, the last digits of 7^{300} is ${}^{150}C_{150}.1.1$ i.e., 1.

16. **(A)**

$(101)^{100} - 1 = (1+100)^{100} - 1$

$= [{}^{100}C_0 + {}^{100}C_1(100) + {}^{100}C_2(100)^2 + \ldots + {}^{100}C_{100}(100)^{100}] - 1$

$= (100)^2\,[1 + {}^{100}C_2 + {}^{100}C_3(100) + \ldots + {}^{100}C_{100}(100)^{98}]$

$\therefore$ $(101)^{100} - 1$ is divisible by $(100)^2$ i.e. 10^4.

17. **(A)**

$2^{2000} = (16)^{500}$

$= (17-1)^{500}$

$= {}^{500}C_0(17)^{500} - {}^{500}C_1(17)^{499} + {}^{500}C_2(17)^{498} - \ldots - {}^{500}C_{499}(17) + 1$

$= 17[(17)^{499} - {}^{500}C_1(17)^{498} + {}^{500}C_2(17)^{497} - \ldots - {}^{500}C_{499}] + 1$

$\therefore$ Remainder = 1

18. **(B)**

$(x+3)^8 = {}^8C_0\,x^8 + {}^8C_1\,(x^7)(3) + {}^8C_2\,(x^6)(3^2) + {}^8C_3\,(x^5)(3^3) + \ldots$

Coefficient of $x^5 = {}^8C_3\,(3^3) = (56)(27) = 1512$

19. **(B)**

$(1+x^2)^5(1+x)^4 = ({}^5C_0 + {}^5C_1x^2 + {}^5C_2x^4 + \ldots) \times ({}^4C_0 + {}^4C_1\,x + {}^4C_2x^2 + {}^4C_3\,x^3 + {}^4C_4\,x^4)$

$\therefore$ Coefficient of $x^5 = {}^5C_1\,{}^4C_3 + {}^5C_2\,{}^4C_1$

$= (5)(4) + (10)(4) = 20 + 40 = 60$

20. **(D)**

$(1-3x+7x^2)(1-x)^{16}$

$= (1-3x+7x^2)({}^{16}C_0 - {}^{16}C_1\,x + \ldots)$

Coefficient of $x = -{}^{16}C_1 - 3\,{}^{16}C_0$

$= -16 - 3 = -19$

21. **(A)**

$(1-x+x^2-x^3)^4 = (1-x)^4\,(1+x^2)^4$

$= ({}^4C_0 - {}^4C_1x + {}^4C_2x^2 - {}^4C_3x^3 + {}^4C_4x^4) \times ({}^4C_0 + {}^4C_1x^2 + {}^4C_2x^4 + {}^4C_3x^6 + {}^4C_4x^8)$

$\therefore$ Coefficient of $x^4 = {}^4C_0\cdot{}^4C_2 + {}^4C_2\cdot{}^4C_1 + {}^4C_4\cdot{}^4C_0$

$= 31$

22. **(D)**

$(1+x+x^2+x^3)^n = \{(1+x) + x^2(1+x)\}^n$

$= \{(1+x)\,(1+x^2)\}^n$

$= (1+x)^n(1+x^2)^n$

$= (1 + {}^nC_1\,x + {}^nC_2\,x^2 + \ldots + {}^nC_n\,x^n) \times (1 + {}^nC_1\,x^2 + {}^nC_2\,x^4 + \ldots + {}^nC_n\,x^{2n})$

$\therefore$ coefficient of x^4

$= {}^nC_2 + {}^nC_2\,{}^nC_1 + {}^nC_4 = {}^nC_4 + {}^nC_2 + {}^nC_1\,{}^nC_2$

23. **(C)**

We have,

$(1-x-x^2+x^3)^6 = \{(1-x)(1-x^2)\}^6$

$= (1-x)^6(1-x^2)^6$

$= ({}^6C_0 - {}^6C_1\,x + {}^6C_2\,x^2 - {}^6C_3\,x^3 + \ldots + {}^6C_6\,x^6)$
$({}^6C_0 - {}^6C_1\,x^2 + {}^6C_2\,x^4 - {}^6C_3\,x^6 + \ldots + {}^6C_6\,x^{12})$

$\therefore$ coefficient of x^7

$= [-{}^6C_1 \times -{}^6C_3] + [-{}^6C_3 \times {}^6C_2] + [-{}^6C_5 \times -{}^6C_1]$

$= 120 - 300 + 36 = -144$

24. **(B)**

$(x^2-x-2)^5$

$= (x-2)^5\,(1+x)^5$

$= [{}^5C_0\,x^5 - {}^5C_1\,x^4 \times 2 + {}^5C_2\,x^3 \times (2)^2 - {}^5C_3\,x^2 \times (2)^3 + {}^5C_4\,x \times (2)^4 - {}^5C_5(2)^5 + \ldots]$
$\times [{}^5C_0 + {}^5C_1\,x + {}^5C_2\,x^2 + {}^5C_3\,x^3 + {}^5C_4\,x^4 + {}^5C_5\,x^5]$

$= -81$

$\therefore$ coefficient of x^5

${}^5C_0 \times {}^5C_0 - {}^5C_1\,{}^5C_1 \times 2 + {}^5C_2\,{}^5C_2 \times (2)^2 - {}^5C_3 \times {}^5C_3 \times (2)^3 + {}^5C_4\,{}^5C_4\,(2)^4 - {}^5C_5 \times {}^5C_5\,(2)^5$

25. **(D)**

$(1+3x+2x^2)^6 = (1+x)^6\,(1+2x)^6$

$= [{}^6C_0 + {}^6C_1x + \ldots + {}^6C_5x^5 + {}^6C_6x^6]$
$[{}^6C_0 + {}^6C_1x + \ldots + {}^6C_5(2x)^5 + {}^6C_6(2x)^6]$

$= {}^6C_5\,{}^6C_6\,2^6 + {}^6C_6\,{}^6C_5\,2^5$

$= 576$

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72. **(A)**

Let $S = (1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \ldots + 1000(1+x)x^{999} + 1001x^{1000}$

$\Rightarrow \frac{xS}{1+x} = x(1+x)^{999} + 2x^2(1+x)^{998} + \ldots + 1000x^{1000} + 1001\frac{x^{1001}}{1+x}$

On subtracting these two, we get

$S - \frac{xS}{1+x}$

$= \{(1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \ldots + x^{1000}\} - 1001\frac{x^{1001}}{1+x}$

$\Rightarrow \frac{S}{1+x}$

$= (1+x)^{1000}\left\{\frac{1-\left(\frac{x}{1+x}\right)^{1001}}{1-\frac{x}{1+x}}\right\} - 1001\frac{x^{1001}}{1+x}$

$\Rightarrow \frac{S}{1+x} = (1+x)^{1001} - x^{1001} - 1001\frac{x^{1001}}{1+x}$

$\Rightarrow S = (1+x)^{1002} - x^{1001}(1+x) - 1001x^{1001}$

$\Rightarrow S = (1+x)^{1002} - 1002x^{1001} - x^{1002}$

$\therefore$ coefficient of x^{50} in S

= coefficient of x^{50} in $(1+x)^{1002}$

$= {}^{1002}C_{50}$

Numerical Value Type Questions

1. **(10)**

$T_4 = {}^6C_3\left(\sqrt{x^{\frac{1}{\log_{10}x+1}}}\right)^3\left(x^{\frac{1}{12}}\right)^3$

$\Rightarrow 200 = 20\left(x^{\frac{1}{\log_{10}x+1}}\right)^{\frac{3}{2}}\left(x^{\frac{1}{4}}\right)$

$\Rightarrow 10 = x^{\frac{3}{2}\left(\frac{1}{\log_{10}x+1}\right)+\frac{1}{4}}$

$\Rightarrow \log_x 10 = \frac{3}{2}\left(\frac{1}{\log_{10}x+1}\right)+\frac{1}{4}$

$\Rightarrow \frac{1}{t} = \frac{3}{2}\left(\frac{1}{t+1}\right)+\frac{1}{4}$ …[Take $t = \log_{10}x$]

$\Rightarrow t^2 + 3t - 4 = 0$

$\Rightarrow (t-1)(t+4) = 0$

$\Rightarrow t = 1$ or $t = -4$

$\Rightarrow \log_{10}x = 1$ or $\log_{10}x = -4$

$\Rightarrow x = 10$ …[As $x > 1$]

2. **(21)**

${}^nC_{r-2} : {}^nC_{r-1} : {}^nC_r = 1 : 3 : 5$

$\Rightarrow \frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{3}$ and $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{5}$

$\Rightarrow \frac{r-1}{n-r+2} = \frac{1}{3}$ and $\frac{r}{n-r+1} = \frac{3}{5}$

$\Rightarrow 4r - n = 5$ …(i)

and $8r - 3n = 3$ …(ii)

Solving (i) and (ii), we get

$n = 7$ and $r = 3$

$\Rightarrow$ Average of coefficients $= \frac{{}^7C_1 + {}^7C_2 + {}^7C_3}{3}$

$= \frac{7+21+35}{3}$

$= 21$

3. **(0)**

Here, n = 10, which is even.

Middle term $= \left(\frac{10}{2}+1\right)^{\text{th}}$ term $= 6^{\text{th}}$ term

$T_6 = {}^{10}C_5\left(\frac{1}{x}\right)^5(x\sin x)^5$

$\Rightarrow \frac{63}{8} = 252(\sin x)^5$

$\Rightarrow (\sin x)^5 = \frac{1}{32}$

$\Rightarrow (\sin x)^5 = \left(\frac{1}{2}\right)^5$

$\Rightarrow \sin x = \frac{1}{2}$

$\Rightarrow 2\sin x - 1 = 0$

$\Rightarrow 6\sin^2 x + \sin x - 2 = (2\sin x - 1)(3\sin x + 2) = 0$

4. **(51)**

$C_0 + 5C_1 + 9C_2 + \ldots + (101)C_{25} = \sum_{r=0}^{25} {}^{25}C_r(4r+1)$

$= 4\sum_{r=0}^{25} r\,{}^{25}C_r + \sum_{r=0}^{25} {}^{25}C_r$

$= 4\sum_{r=1}^{25} r \times \frac{25}{r}\left({}^{24}C_{r-1}\right) + \sum_{r=0}^{25} {}^{25}C_r$

$= 100.2^{24} + 2^{25}$

$= 2^{25}(50+1)$

$= 2^{25}.51$

$\Rightarrow k = 51$

5. **(3)**

$p = \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \ldots\infty$

$\Rightarrow p + 1 = 1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \ldots\infty$ …(i)

$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots \infty$ …(ii)

$nx = \frac{3}{4}$ …(iii)[From (i) and (ii)]

and $\frac{n(n-1)x^2}{2} = \frac{3.5}{4.8}$ …(iv)

$\frac{\frac{n(n-1)x^2}{2}}{(nx)^2} = \frac{\frac{3.5}{4.8}}{\left(\frac{3}{4}\right)^2}$ …[From (iii) and (iv)]

$\Leftrightarrow \frac{n-1}{n} = \frac{5}{3}$

$\Leftrightarrow n = \frac{-3}{2}$

$nx = \frac{3}{4}$

$\Rightarrow \left(\frac{-3}{2}\right)x = \frac{3}{4}$

$\Rightarrow x = \frac{-1}{2}$

$p + 1 = \left(1 - \frac{1}{2}\right)^{\frac{-3}{2}}$

$\Rightarrow (p + 1) = 2\sqrt{2}$

$\Rightarrow p^2 + 2p + 1 = 8$

$\Rightarrow p^2 + 2p - 4 = 3$

6. **(30)**

Let

$(1 + x + x^2 + \ldots + x^{2n})(1 - x + x^2 - x^3 + \ldots + x^{2n})$
$= a_0 + a_1x + a_2x^2 + \ldots$

Substituting $x = 1$, we get

$(1 + 1 + 1 + \ldots + 1)(1 - 1 + 1 - 1 + \ldots + 1)$
$= a_0 + a_1 + a_2 + a_3 + \ldots$

$\Rightarrow a_0 + a_1 + a_2 + a_3 + \ldots = 2n + 1$ …(i)

Substituting $x = -1$, we get

$(1 - 1 + 1 - 1 + \ldots + 1)(1 + 1 + 1 + \ldots + 1)$
$= a_0 - a_1 + a_2 - a_3 + \ldots$

$\Rightarrow a_0 - a_1 + a_2 - a_3 + \ldots = 2n + 1$ …(ii)

Adding (i) and (ii), we get

$2(a_0 + a_2 + a_4 + \ldots) = 2(2n + 1)$

$\Rightarrow 61 = 2n + 1$

$\Rightarrow n = 30$

7. **(6006)**

General term,

$t_{r+1} = {}^{15}C_r \left(t^2 x^{\frac{1}{5}}\right)^{15-r} \left[\frac{(1-x)^{\frac{1}{10}}}{t}\right]^r$

t_{r+1} is independent of t,

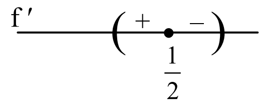
if $2(15 - r) - r = 0$

i.e. if $r = 10$

$\therefore$ $t_{11} = {}^{15}C_{10}\, x(1 - x)$

Let $f(x) = x(1 - x)$

$\therefore$ $f'(x) = 1 - 2x$



$f'(x) = 0$

$\Rightarrow x = \frac{1}{2}$

$\therefore$ $f(x)$ is maximum at $x = \frac{1}{2}$ and

maximum value $= f\left(\frac{1}{2}\right) = \frac{1}{4}$

$\therefore$ $K = {}^{15}C_{10} \times \frac{1}{4}$

$\Rightarrow K = \frac{3003}{4}$

$\Rightarrow 8K = 6006$

8. **(405)**

The sum of the coefficients of the first three terms is 376

$\Rightarrow {}^nC_0 - {}^nC_1(3) + {}^nC_2(3)^2 = 376$

$\Rightarrow 1 - 3n + \frac{9n(n-1)}{2} = 376$

$\Rightarrow 2 - 6n + 9n^2 - 9n = 752$

$\Rightarrow 9n^2 - 15n - 750 = 0$

$\Rightarrow 3n^2 - 5n - 250 = 0$

$\Rightarrow (n - 10)(3n + 25) = 0$

But, $n \neq \frac{-25}{3}$

$\therefore$ $n = 10$

The general term is given as

$T_{r+1} = {}^nC_r x^{n-r}\left(-\frac{3}{x^2}\right)^r$

$= {}^{10}C_r x^{10-r}(-3)^r \cdot x^{-2r}$

$= {}^{10}C_r (-3)^r x^{10-3r}$

For coefficient of x^4,

$10 - 3r = 4$

$\therefore$ $r = 2$

$\therefore$ Coefficient of $x^4 = {}^{10}C_2(-3)^2$

$= 45 \times 9$

$= 405$