SAMPLE CONTENT

Challenger

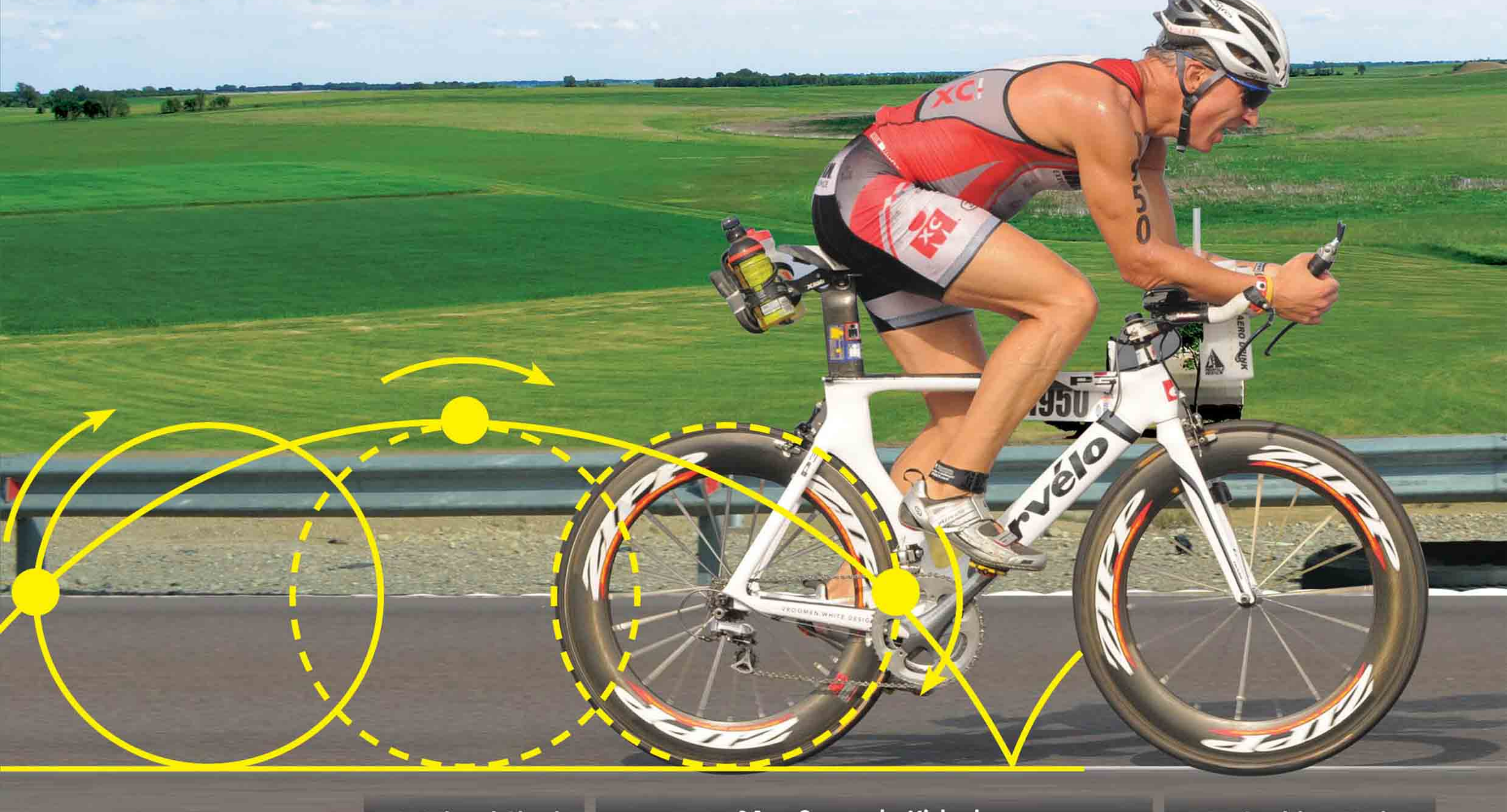
JEE (Main) MATHEMATICS Vol - II

As per latest syllabus issued by NTA

1506 MCQs with Hints

For all Engineering Entrance Examinations held across India.

When a wheel rolls along a straight line, any point on the rim of the circular wheel traces a curve called a cycloid. Parametric form of the equation of a cycloid is more popular than the cartesian form.



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Challenger

JEE (Main)

Mathematics

Vol. II

Updated as per latest syllabus for

JEE (Main) 2024 issued by **NTA on 01st November, 2023**

Salient Features

- Concise theory for every topic
- Eclectic coverage of MCQs under each sub-topic
- Exhaustive coverage of questions including selective questions from previous JEE (Main) examinations updated upto year 2023:
  - **1506** Questions of the level of various competitive exams
  - Solutions to the questions are provided for better understanding
- Inclusion of 'Problems To Ponder' to enhance students' practical application of mathematical concepts.
- Study Techniques to Enhance Understanding and Problem Solving.
  - Shortcuts
  - Important Notes
- Includes Question Papers and Answer Keys (Solutions through Q.R. code) of:
  - JEE (Main) 2021 24th February, 16th March (Shift – I)
  - JEE (Main) 2022 25th July (Shift - I)
  - JEE (Main) 2023 24th Jan (Shift - II)
- Q.R. codes provide:
  - Solutions of previous years' exam papers of years 2021 to 2023
- Separate list of questions excluded from the JEE (Main) 2024 syllabus

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PREFACE

Target's **'Challenger Maths: Vol-II'** is a compact guidebook, extremely handy for preparation of various competitive exams like JEE (Main).

Features of each chapter:

- Coverage of **Theoretical Concepts** that form a vital part of any competitive examination.
- **Multiple Choice Questions** segregated into two sections. **Concept Building Problems** – Contains questions of various difficulty range and pattern. **Practice Problems** – Contains ample questions for thorough revision.
- **'Important Note'** highlights the unique points about the topic.
- **Shortcuts** to help students save time while dealing with lengthy questions.
- **Problems to Ponder:** Various types of questions of different pattern created with the primary objective of helping students to understand the application of various concepts of Maths.

MCQs have been created and compiled with the following objective in mind – to help students solve complex problems which require strenuous effort and understanding of multiple-concepts.

The level of difficulty of the questions is at par with that of various competitive examinations like JEE (Main), AIEEE, TS EAMCET (Engg.), BCECE, Assam CEE, AP EAMCET (Engg.) and the likes. Also to keep students updated, questions from the most recent examinations of JEE (Main), of years 2014, 2015, 2016, 2017, 2018, 2019 and 2020 are covered exclusively.

Question Papers and Answer Keys of JEE (Main) **2021** [24^{th} February, 16^{th} March (Shift - I)], JEE (Main) **2022** 25^{th} July (Shift - I) and JEE (Main) **2023** 24^{th} Jan (Shift - II) have been provided to offer students glimpse of the complexity of questions asked in entrance examination. Solutions are also provided through a separate Q.R. code. The papers have been split unit-wise to let the students know which of the units were more relevant in the latest examinations.

Considering the latest modifications in the syllabus of JEE (Main) examination, a list of questions based on the concepts excluded from the latest JEE (Main) 2024 syllabus is provided. The purpose of providing these questions is to display various question types and their level of difficulty that have been asked in previous examinations.

We hope the book benefits the learner as we have envisioned.

A book affects eternity; one can never tell where its influence stops.

- Publisher

Edition : Fifth

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.
Please write to us on : mail@targetpublications.org

KEY FEATURES

Subtopic wise segregation

Subtopic wise segregation

Every chapter is **segregated subtopic wise.**
This is our attempt to cater to each student's individualistic pace and preferences of studying a chapter and enable easy assimilation of questions based on the specific concept.

Miscellaneous

Miscellaneous

Miscellaneous MCQs covers concept of different sub-topics of same chapter or from different chapters.
This is our attempt to develop cognitive thinking in the students essential to solve questions involving fusion of multiple key concepts.

Questions from previous JEE exams:

Questions from previous JEE exams

To ensure students are well prepared, important questions from previous JEE exams are covered exclusively.

Important Note

Important Note

Highlights the unique points about the topic.

Shortcuts

Shortcuts

Shortcuts comprises important formula based short tricks considering their utility in solving MCQ.
This is our attempt to highlight content that would come handy while solving questions.

Problems to Ponder

Problems to Ponder

Problems to Ponder contains MCQs of different pattern created with the primary objective of helping students to understand the application of various concepts of Maths.

Q.R. Codes

Q.R. Code

'**QR code**' provides:
Solutions to JEE (Main) 2021 [24th February, 16th March (Shift - I)], JEE (Main) 2022 25th July (Shift - I) and JEE (Main) 2023 24th Jan (Shift - II) papers.

Frequently Asked Questions

- **Why Challenger Series?**

  Gradually, every year the nature of competitive entrance exams is inching towards conceptual understanding of topics. Moreover, it is time to bid adieu to the stereotypical approach of solving a problem using a single conventional method.

  To be able to successfully crack the JEE (Main) examination, it is imperative to develop skills such as data interpretation, appropriate time management, knowing various methods to solve a problem, etc. With Challenger Series, we are sure, you'd develop all the aforementioned skills and take a more holistic approach towards problem solving. The way you'd tackle advanced level MCQs with the help of hints, tips, shortcuts and necessary practice would be a game changer in your preparation for the competitive entrance examinations.

- **What is the intention behind the launch of Challenger Series?**

  The sole objective behind the introduction of Challenger Series is to severely test the student's preparedness to take competitive entrance examinations. With an eclectic range of critical and advanced level MCQs, we intend to test a student's MCQ solving skills within a stipulated time period.

- **What do I gain out of Challenger Series?**

  After using Challenger Series, students would be able to:

  a. assimilate the given data and apply relevant concepts with utmost ease.

  b. tackle MCQs of different pattern such as match the columns, diagram based questions, multiple concepts and assertion-reason efficiently.

  c. garner the much needed confidence to appear for various competitive exams.

- **Can the Questions presented in Problems to Ponder section be a part of the JEE (Main) Examination?**

  No, the questions would not appear as it is in the JEE (Main) Examination. However, there are fair chances that these questions could be covered in parts or with a novel question construction.

- **Why is then Problems to Ponder a part of this book?**

  The whole idea behind introducing Problems to Ponder was to cover an entire concept in one question. With this approach, students would get **more variety and less repetition** in the book.

Best of luck to all the aspirants!

CONTENTS

Note: ⊠ – *Complete chapter excluded from the JEE (Main) 2024 syllabus (in index)*
◑ – *Part of the chapter excluded from the JEE (Main) 2024 syllabus (in index)*

Questions based on the concepts excluded from the JEE (Main) 2024 Syllabus

Chapter and Excluded Subtopic		Questions excluded from 2024 Syllabus	Page No.
4. Applications of Derivatives 4.3 Tangents and Normals	Theory	-	97 to 99
	Concept Building Problems	All Questions	103 to 105
4.6 Mean value theorems	Theory	-	101
	Concept Building Problems	All Questions	109 and 110
Practice Problems	-	34, 35, 36, 38 to 56, 105 to 113	112, 113 and 117
Problems to Ponder	-	1, 2	117
6. Definite Integrals 6.3 Integral as a limit of a sum	Theory	-	208 and 209
	Concept Building Problems	All Questions	212
Practice Problems	-	64	217
8. Differential Equations 8.2 Formation of a differential equation	Theory	-	265 and 266
	Concept Building Problems	All Questions	270
Practice Problems	-	3, 6 to 10, 26, 39	274 to 277
9. Determinants and Matrices 9.1 Minors and cofactors of a determinant, *Evaluation of a determinant using properties*	Theory	-	302 to 305, 324
	Concept Building Problems	3, 5, 6, 7, 9, 12, 13, 15, 19	325 and 326
9.2 Evaluation of special determinants, multiplication of determinants, Factor theorem *(Properties of determinants)*	Theory	-	302 to 305, 324
	Concept Building Problems	All Questions	326 and 327
9.3 Homogeneous, Non-homogeneous system of linear equations	Theory	-	306 to 309, 311
	Concept Building Problems	All Questions	327 and 328

9.6 Adjoint of a matrix and inverse of a matrix *(Elementary transformations)*	Theory	-	320 and 321
Practice Problems	-	2 to 8, 10 to 13, 15, 16, 17, 20, 21, 23 to 35, 37, 38, 52, 61	330 to 333, 335
Problems to Ponder	-	1, 2	336
10. Vector Algebra	Theory	-	365 and 366
10.3 Scalar and vector triple products and their applications	Concept Building Problems	All Questions	370 and 371
Practice Problems	-	42 to 61, 63 to 65	375 and 376
11. Three Dimensional Geometry	Theory	-	405 to 408
11.3 Equations of a plane in different forms	Concept Building Problems	All Questions	410 to 412
11.4 Intersection of a line and a plane, Coplanar lines	Theory	-	407 to 409
	Concept Building Problems	All Questions	412 and 413
Practice Problems	-	18, 20, 23 to 48	414 to 417
Problems to Ponder	-	1, 2, 3	417
12. Mathematical Reasoning	Entire Chapter Deleted		436 to 446
JEE (Main) 2021 - 24th February (Shift - I) MCQs	-	3, 8, 9, 10, 11	447 and 448
Numerical Value Type Questions (NVT)	-	4	448
JEE (Main) 2021 - 16th March (Shift - I) MCQs	-	8, 9, 10	449 and 450
Numerical Value Type Questions (NVT)	-	3, 4, 6	450

JEE (Main) 2022 – 25th July (Shift - I) MCQs	-	7, 9, 11, 12	451 and 452
Numerical Value Type Questions (NVT)	-	2, 4	452
JEE (Main) 2023 – 24th January (Shift - II) MCQs	-	5, 6, 8, 9, 10	453

Note: i. *The above table contains the list of chapters/subtopics/question numbers that are excluded from the latest syllabus of JEE (Main) 2024.*

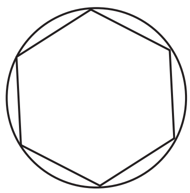
ii. *Only the concepts highlighted in italics are excluded from the latest syllabus within the specified subtopics.*

iii. *These questions are covered to give an idea about the variety and difficulty levels of questions asked in the examination over the years.*

1 Limits

1. Concept of limits:

i. Consider an n-sided polygon inscribed in a circle. Let A_n and A be the areas of the polygon and of the circle respectively. Then we make the following observations:



a. $A_n < A$

b. A_n starts approaching A as n increases indefinitely.

c. The difference between A_n and A can be made as small as we wish by taking sufficiently large n.

Mathematically, the whole instance is expressed as $\lim\limits_{n\to\infty} A_n = A$

(We read it as "As n approaches ∞, A_n approaches A".)

ii. Consider the series

$$S_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots + \frac{1}{3^{n-1}} \text{ and } S = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots \infty$$

We have $S_n = \dfrac{1 - \frac{1}{3^n}}{1 - \frac{1}{3}} = \dfrac{3}{2}\left(1 - \dfrac{1}{3^n}\right)$

and $S = \dfrac{1}{1 - \frac{1}{3}} = \dfrac{3}{2}$

We make the following observations:

a. $S_n < S$

b. S_n starts approaching S as n increases indefinitely.

c. The difference between S_n and S can be made as small as we wish by taking sufficiently large n.

Mathematically, we express it as $\lim\limits_{n\to\infty} S_n = S$

(We read it as "As n tends to ∞, S_n tends to S".)

Important Note

❖ If A_n approaches A (i.e., A_n tends to A), then A_n does not attain value A.

2. Neighbourhood of a point $x = a$ and meaning of 'x tends to a'

i. Neighbourhood of $x = a$:

A neighbourhood of $x = a$ is an open interval around $x = a$, denoted by $N_\delta(a)$ and is defined as

$N_\delta(a) = \{ x : |x - a| < \delta \}$

(where δ is a small number)

ii. Meaning of $x \to a$ (i.e., x tends to a)

$x \to a \quad \Leftrightarrow \quad x \in N_\delta(a) \quad$ **but** $x \neq a$

a. When $x < a$ and $x \in N_\delta(a) \Rightarrow x \to a^-$

(We read it as 'x approaches a from its left.')

b. When $x > a$ and $x \in N_\delta(a) \Rightarrow x \to a^+$

(We read it as 'x approaches a from its right.')

3. Limit of a function:

i. Left hand and right hand limits (L.H.L. and R.H.L.)

a. L.H.L. = a value where f approaches as $x \to a^-$

b. R.H.L. = a value where f approaches as $x \to a^+$

ii. Definition : If L.H.L. = $\lim\limits_{x\to a^-} f(x)$ and

R.H.L. = $\lim\limits_{x\to a^+} f(x)$

exist and are equal to, say, l

then l is called the limit of the function as $x \to a$ and is denoted as $\lim\limits_{x\to a} f(x)$.

e.g. $\lim\limits_{x\to\frac{\pi}{2}} \cos x = 0$, $\quad \lim\limits_{x\to\frac{\pi}{4}} \tan x = 1$,

$\lim\limits_{x\to\frac{\pi}{2}} (\tan x)$ does not exist.

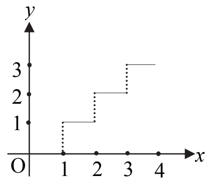
Important Notes

- Limit of a function does not exist in the following cases:
  i. One of the limits (L.H.L. or R.H.L.) does not exist or both the limits do not exist.
  ii. L.H.L. and R.H.L. exist but are unequal.
- If a is an integer, then $\lim\limits_{x \to a^-} [x] = a - 1$

  and $\lim\limits_{x \to a^+} [x] = a$

  e.g. $\lim\limits_{x \to 2^-} [x] = 1$

  and $\lim\limits_{x \to 2^+} [x] = 2$



  which can be observed from the graph.
- $\lim\limits_{x \to a} [x]$ does not exist if a is an integer.
- Limit of a function is always a finite number.

Remark: $\lim\limits_{x \to a} f(x) = \infty$ or $-\infty$ is a wrong statement.

4. Algebra of limits:

Let $\lim\limits_{x \to a} f(x) = l_1$ and $\lim\limits_{x \to a} g(x) = l_2$, then

i. $\lim\limits_{x \to a} (\alpha f(x) + \beta g(x)) = \alpha l_1 + \beta l_2$ where α, β are constants. **(Linearity property)**

ii. $\lim\limits_{x \to a} (f(x)\, g(x)) = l_1 l_2$

iii. $\lim\limits_{x \to a} \left(\dfrac{f(x)}{g(x)}\right) = \dfrac{l_1}{l_2}$ provided $l_2 \neq 0$

iv. $\lim\limits_{x \to a} (f(x))^{g(x)} = (l_1)^{l_2}$

5. Indeterminate forms:

There are certain situations in evaluating limits, where algebra of limits does not work.

e.g. $f(x) = x^2 - 3x + 2$, $g(x) = x^2 - 6x + 5$

$\lim\limits_{x \to 1} f(x) = 0$, $\lim\limits_{x \to 1} g(x) = 0$

but $\lim\limits_{x \to 1} \dfrac{f(x)}{g(x)}$ cannot be evaluated by algebra of limits.

Such limits (forms) are called indeterminate forms.

There are many indeterminate forms viz.

$\dfrac{0}{0}, \dfrac{\infty}{\infty}, 0 \times \infty, \infty - \infty, \infty^0, 0^0, 1^\infty$

Important Note

- $\dfrac{0}{0}$ and $\dfrac{\infty}{\infty}$ are just the names of the indeterminate forms. It should be clearly understood that we are not trying to divide 0 by 0 (or ∞ by ∞) as $\lim\limits_{x \to a} g(x) = 0$

  i.e. $g(x)$ approaches 0 and the value of $g(x)$ is not 0 as x tends to a.

The forms $\dfrac{0}{0}$ and $\dfrac{\infty}{\infty}$ are equivalent forms.

The form $\dfrac{\infty}{\infty}$ indicates that $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ do not exist but $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)}$ may exist.

i. The forms $0 \times \infty$, $\infty - \infty$ can be reduced to $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$ form.

a. In $0 \times \infty$ form, let $\lim\limits_{x \to a} f(x) = 0$ and $\lim\limits_{x \to a} g(x)$ does not exist (as $g(x)$ gets indefinitely large as $x \to a$).

$$\lim_{x \to a} (f(x) \times g(x)) = \lim_{x \to a} \frac{f(x)}{\frac{1}{g(x)}} \quad \ldots \left[\frac{0}{0} \text{ form}\right]$$

b. In $\infty - \infty$ form, $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ do not exist (as $f(x)$ and $g(x)$ get indefinitely large as $x \to a$).

$$\lim_{x \to a} (f(x) - g(x)) = \lim_{x \to a} \left(\frac{1}{\frac{1}{f(x)}} - \frac{1}{\frac{1}{g(x)}}\right)$$

$$= \lim_{x \to a} \left(\frac{\frac{1}{g(x)} - \frac{1}{f(x)}}{\frac{1}{f(x)} \cdot \frac{1}{g(x)}}\right) \quad \ldots \left[\frac{0}{0} \text{ form}\right]$$

ii. The forms ∞^0, 1^∞ etc. can also be reduced to $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$ form.

a. $\lim\limits_{x \to a} f(x)$ does not exist (as $f(x)$ gets indefinitely large as $x \to a$), $\lim\limits_{x \to a} g(x) = 0$.

But $\lim\limits_{x \to a} (f(x))^{g(x)}$ may exist.

Let $L = \lim\limits_{x \to a} f(x)^{g(x)}$. Then,

$\log L = \lim\limits_{x \to a} g(x) \log f(x) \ldots [0 \times \infty \text{ form}]$

(which can be reduced to $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$ form.)

b. $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x)$ does not exist (as $g(x)$ gets indefinitely large as $x \to a$). But $\lim_{x\to a} (f(x))^{g(x)}$ may exist.

Let $L = \lim_{x\to a} (f(x))^{g(x)}$. Then,

$\text{Log } L = \lim_{x\to a} g(x) \log(f(x))$... [$0 \times \infty$ form]

(which can be reduced to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form.)

6. Methods of evaluation of $\lim_{x\to a} f(x)$:

i. Substitution method:

Replace x by a if $\lim_{x\to a} f(x)$ is not in indeterminate form.

e.g. $\lim_{x\to 1} (x^2 + x + 1) = 3$, $\lim_{x\to 1} \frac{x^2+2}{x} = \frac{1+2}{1} = 3$

ii. Factorization method:

If $\lim_{x\to a} f(x)$ is in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form, then the numerator and denominator would surely have a factor $(x - a)$. By cancelling out the common factor, we get rid of $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form.

e.g. $\lim_{x\to 2} \frac{x^2-4x+4}{x^2-3x+2} = \lim_{x\to 2} \frac{(x-2)^2}{(x-1)(x-2)}$

$$= \lim_{x\to 2} \frac{x-2}{x-1} \quad \ldots \left(\text{not in } \frac{0}{0} \text{ form}\right)$$

$$= \frac{2-2}{2-1} = 0$$

iii. Rationalization Method:

This method is used if $\lim_{x\to a} \frac{f(x)}{g(x)}$ is in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form, where $f(x)$ and / or $g(x)$ have square roots, is to be evaluated. By rationalising we get a factor $(x - a)$ in numerator as well as denominator.

Illustration: Evaluate: $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$

***Solution*:** $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$

$$= \lim_{x\to 0} \frac{\left(\sqrt{1+x}-1\right)\left(\sqrt{1+x}+1\right)}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x\to 0} \frac{1+x-1}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{2}$$

iv. Method for evaluating $\lim_{x\to\infty} f(x)$:

Replace x by $\frac{1}{t}$. As $x \to \infty$, $t \to 0$, and proceed as discussed before.

Illustration: Evaluate: $\lim_{x\to\infty} \frac{(2x-3)(3x-4)}{(4x-5)(5x-6)}$

***Solution*:** Substitute $x = \frac{1}{t}$

$$\lim_{x\to\infty} \frac{(2x-3)(3x-4)}{(4x-5)(5x-6)}$$

$$= \lim_{t\to 0} \frac{\left(\frac{2}{t}-3\right)\left(\frac{3}{t}-4\right)}{\left(\frac{4}{t}-5\right)\left(\frac{5}{t}-6\right)} = \underbrace{\lim_{t\to 0} \frac{(2-3t)(3-4t)}{(4-5t)(5-6t)}}_{\left(\text{not in } \frac{0}{0} \text{ form}\right)}$$

$$= \frac{(2)(3)}{(4)(5)} = \frac{3}{10}$$

7. Standard formulae:

i.
- $\lim_{x\to 0} \frac{\sin x}{x} = 1$
- $\lim_{x\to 0} \frac{\sin(kx)}{x} = k$
- $\lim_{x\to 0} \frac{\tan x}{x} = 1$
- $\lim_{x\to 0} \frac{\tan kx}{x} = k$
- $\lim_{x\to 0} \frac{\sin^{-1} x}{x} = 1$
- $\lim_{x\to 0} \frac{\sin^{-1}(kx)}{x} = k$
- $\lim_{x\to\infty} \frac{\sin x}{x} = 0$
- $\lim_{x\to\infty} \frac{\sin kx}{x} = 0$
- $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right) = 1$
- $\lim_{x\to\infty} x \sin\frac{k}{x} = k$

Illustrations:

1. Evaluate: $\lim_{x\to a} \frac{\sin(x-a)}{x-a}$

***Solution*:**

Substitute $x - a = t$

$$\lim_{x\to a} \frac{\sin(x-a)}{x-a} = \lim_{t\to 0} \frac{\sin t}{t} = 1$$

2. Evaluate: $\lim_{x\to 0} \frac{\sin x^\circ}{x}$

***Solution*:**

$180^\circ = \pi$ radians

$\Rightarrow x^\circ = \frac{\pi x}{180}$ radians

$$\lim_{x\to 0} \frac{\sin x^\circ}{x} = \lim_{x\to 0} \left(\frac{\sin\left(\frac{\pi x}{180}\right)}{x}\right)$$

$$= \lim_{x\to 0} \left(\frac{\pi}{180}\right) \cdot \frac{\sin\left(\frac{\pi x}{180}\right)}{\left(\frac{\pi x}{180}\right)}$$

$$= \frac{\pi}{180}$$

ii.

- $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n = e$
- $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}} = e$
- $\lim\limits_{x\to 0}(1+f(x))^{\frac{1}{f(x)}} = e$ provided $\lim\limits_{x\to 0} f(x) = 0$

iii.

- $\lim\limits_{x\to 0}\dfrac{a^x-1}{x} = \log_e a$
- $\lim\limits_{x\to 0}\dfrac{e^x-1}{x} = \log_e e = 1$
- $\lim\limits_{x\to 0}\dfrac{a^{f(x)}-1}{f(x)} = \log_e a$ provided $\lim\limits_{x\to 0} f(x) = 0$
- $\lim\limits_{x\to 0}\dfrac{e^{f(x)}-1}{f(x)} = \log_e e = 1$ provided $\lim\limits_{x\to 0} f(x) = 0$

Illustration:

Evaluate: $\lim\limits_{x\to 0}\dfrac{\log(1+x)}{x}$

Solution:

$$\lim_{x\to 0}\frac{\log(1+x)}{x} = \lim_{x\to 0}\log(1+x)^{\frac{1}{x}}$$

$$= \log_e e \quad \ldots \left[\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\right]$$

$$= 1$$

Remark: Great mathematician Leonhard Euler (1707 – 1783) discovered the number e (an irrational number) as the limits of the sequences $a_n = \left(1+\dfrac{1}{n}\right)^n$ and $b_n = \left(1+\dfrac{1}{n}\right)^{n+1}$.

$e \simeq 2.71828$

iv. **Series expansion:**

- $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \ldots$
- $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \ldots$
- $\tan x = x + \dfrac{x^3}{3} + \dfrac{2}{15}x^5 + \ldots$
- $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$
- $e^{-x} = 1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \ldots$
- $\log(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \ldots$
- $\log(1-x) = -x - \dfrac{x^2}{2} - \dfrac{x^3}{3} - \dfrac{x^4}{4} - \ldots$
- $\sin^{-1} x = x + \dfrac{x^3}{6} + \dfrac{3}{40}x^5 + \ldots$
- $\tan^{-1} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \ldots$

v. **Sandwich Theorem:**

If $f(x) \le g(x) \le h(x)$ and $\lim\limits_{x\to a} f(x) = l = \lim\limits_{x\to a} h(x)$ then $\lim\limits_{x\to a} g(x) = l$.

Illustration:

If $1 - \dfrac{x^2}{4} \le f(x) \le 1 + \dfrac{x^2}{2}$, $x \ne 0$, find $\lim\limits_{x\to 0} f(x)$.

Solution:

Let $g(x) = 1 - \dfrac{x^2}{4}$ and $h(x) = 1 + \dfrac{x^2}{2}$

$\Rightarrow \lim\limits_{x\to 0} g(x) = 1$ and $\lim\limits_{x\to 0} h(x) = 1$

Given $1 - \dfrac{x^2}{4} \le f(x) \le 1 + \dfrac{x^2}{2}$,

by Sandwich theorem $\lim\limits_{x\to 0} f(x) = 1$

Concept Building Problems

1.1 Evaluation by Factorization

1. $\lim\limits_{x\to 1}\dfrac{(\sqrt{x}-1)(\sqrt[3]{x}-1)}{x-1}$

   (A) is 1 (B) is – 1
   (C) is 0 (D) does not exist

2. The value of $\lim\limits_{x\to 3}\dfrac{x^3-x^2-18}{x-3}$ is

   (A) 3 (B) 9
   (C) 18 (D) 21

3. If $f(x) = \dfrac{2}{x-3}$, $g(x) = \dfrac{x-3}{x+4}$ and $h(x) = -\dfrac{2(2x+1)}{x^2+x-12}$, then $\lim\limits_{x\to 3}[f(x)+g(x)+h(x)]$ is

   (A) – 2 (B) – 1
   (C) $-\dfrac{2}{7}$ (D) 0

4. $\lim\limits_{x\to 2}\dfrac{x^4-4x^3+8x^2-16x+16}{x^3-3x^2+4} =$

   (A) $\dfrac{4}{3}$ (B) $\dfrac{8}{3}$
   (C) $\dfrac{5}{2}$ (D) $\dfrac{7}{2}$

5. $\lim\limits_{x\to 1}\dfrac{x^8-2x+1}{x^4-2x+1}$ equals

   (A) 3 (B) 0
   (C) – 3 (D) 1

6. If $\lim\limits_{x\to a}\dfrac{(x+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{x-a} = \dfrac{m}{n}(a+2)^{\frac{p}{q}}$, then $(m+p)-(n+q)$ is

   (A) 2 (B) –1
   (C) 0 (D) 1

7. $\lim\limits_{\theta \to \frac{\pi}{6}} \dfrac{\cot^2\theta - 3}{\operatorname{cosec}\theta - 2} =$

(A) 2 (B) 4
(C) 6 (D) 0

1.2 Evaluation by Rationalization

1. $\lim\limits_{y \to 0} \dfrac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

[JEE (Main) Jan 2019]

(A) does not exist
(B) exists and equals $\dfrac{1}{2\sqrt{2}}$
(C) exists and equals $\dfrac{1}{4\sqrt{2}}$
(D) exists and equals $\dfrac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

2. If $\lim\limits_{x \to a} \dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} = \dfrac{\sqrt{p}}{q\sqrt{r}}$, $a \neq 0$, then $p\,(q - r)$ equals

(A) 10 (B) 0
(C) 9 (D) 7

3. If $\mathrm{G}(x) = -\sqrt{25 - x^2}$, then $\lim\limits_{x \to 1} \dfrac{\mathrm{G}(x) - \mathrm{G}(1)}{x - 1} =$

(A) $\dfrac{1}{24}$ (B) $\dfrac{1}{5}$
(C) $-\sqrt{24}$ (D) $\dfrac{1}{\sqrt{24}}$

4. Let $f(x) = \dfrac{x\sqrt{x - 2\sqrt{x-1}}}{\sqrt{x-1} - 1}$, then

(A) $\lim\limits_{x \to 2^-} f(x) = 2$
(B) $\lim\limits_{x \to 2^+} f(x) = -2$
(C) $\lim\limits_{x \to 2} f(x)$ does not exist.
(D) $\lim\limits_{x \to 2} f(x) = 2$

5. Let $\mathrm{L} = \lim\limits_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2} - \dfrac{x^2}{4}}{x^4}$, $a > 0$. Given that L is finite, then

(A) $a = 2$, $\mathrm{L} = \dfrac{1}{32}$
(B) $a = 2$, $\mathrm{L} = \dfrac{1}{64}$
(C) $a = 4$, $\mathrm{L} = \dfrac{1}{64}$
(D) $a = 4$, $\mathrm{L} = \dfrac{1}{32}$

1.3 Standard Trigonometric Limits

1. $\lim\limits_{x \to 0} \dfrac{\sin(2+x) - \sin(2-x)}{x} =$

(A) sin 2 (B) 2 sin 2
(C) 2 cos 2 (D) 2

2. $\lim\limits_{x \to 0} \dfrac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to

[JEE (Main) 2015]

(A) 4 (B) 3
(C) 2 (D) $\dfrac{1}{2}$

3. $\lim\limits_{x \to 0} \dfrac{\sin(\pi \cos^2 x)}{x^2}$ is equal to **[JEE (Main) 2014]**

(A) $-\pi$ (B) π
(C) $\dfrac{\pi}{2}$ (D) 1

4. $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$ equals

[JEE (Main) April 2019]

(A) $4\sqrt{2}$ (B) $\sqrt{2}$
(C) $2\sqrt{2}$ (D) 4

5. If $\lim\limits_{x \to 0} \dfrac{\tan x - \sin x}{x^3} = \dfrac{m}{p!}$, then mp is

(A) 2 (B) 3
(C) 4 (D) 1

6. If $\lim\limits_{x \to 0} \dfrac{\tan^3 x - \sin^3 x}{x^5} = \dfrac{p}{q}$,

where G.C.D. of $(p, q) = 1$, then

(A) $|p - q| = 1$ (B) $p + 2q = 5$
(C) $|2q - p| = 3$ (D) $p + q = 4$

7. $\lim\limits_{x \to 0} \dfrac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ equals

(A) 0 (B) $\dfrac{1}{2}$
(C) 2 (D) 1

8. The value of $\lim\limits_{x \to 0} \dfrac{\cos\left(\dfrac{\pi}{2\cos x}\right)}{\sin(\sin x^2)}$ is

(A) $\dfrac{\pi}{4}$ (B) $-\dfrac{\pi}{4}$
(C) $\dfrac{\pi}{2}$ (D) $-\dfrac{\pi}{2}$

9. If $\lim\limits_{x \to 0} \dfrac{\sin 2x + a \sin x}{x^3} = b$, where a and b are constants, then which of the following is NOT correct?

(A) $a + b + 3 = 0$ (B) $ab = 2$
(C) $b - a = 1$ (D) $b = 2a$

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To see complete chapter buy **Target Notes** or **Target E-Notes**

Practice Problems

1. If the function f is defined by

$$f(x) = \frac{5x}{|x| + 7x}, \quad x \neq 0$$
$$= 0, \quad x = 0$$

then the limit of the function as x approaches 0

(A) is $\frac{5}{8}$ (B) is $\frac{5}{6}$
(C) is $\frac{5}{7}$ (D) does not exist

2. If $f(x) = |x| + |x - 1|$ and $l_1 = \lim\limits_{x \to 0} f(x)$, $l_2 = \lim\limits_{x \to 1} f(x)$, then $l_1 + l_2$ equals

(A) 1 (B) 0
(C) 2 (D) -1

3. If $f: R \to R$ is defined by $f(x) = [x - 3] + |x - 4|$ for $x \in R$, where [ ] represents the greatest integer function, then $\lim\limits_{x \to 3^-} f(x)$ is equal to

(A) -2 (B) -1
(C) 0 (D) 1

4. For what value of p, does $\lim\limits_{x \to 1} f(x)$ exist if

$$f(x) = 2px + 3, \quad x < 1$$
$$= 1 - px^2, \quad x \geq 1$$

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$
(C) $-\frac{3}{2}$ (D) $-\frac{2}{3}$

5. $\lim\limits_{x \to 0} \frac{\log(1 + \{x\})}{\{x\}}$, where $\{x\}$ represents the fractional part of x,

(A) is e (B) is 1
(C) is 0 (D) does not exist

6. $\lim\limits_{x \to 0} \left(\frac{1}{x} - \frac{n!}{x(x+1)(x+2)\ldots(x+n)} \right)$ is

(A) $n!$
(B) $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$
(C) $\frac{1}{n!}$
(D) $\frac{n(n+1)}{2}$

7. $\lim\limits_{x \to 1} \frac{x + x^2 + \ldots + x^{100} - 100}{x - 1}$ equals

(A) 4950 (B) 5050
(C) 5000 (D) 5100

8. $\lim\limits_{x \to 2} \frac{|x^2 - 5x + 6|}{(x-2)(x-3)}$

(A) is 1 (B) is -1
(C) does not exist (D) is 0

9. $\lim\limits_{x \to 5^+} \left(\frac{x^2 - 9x + 20}{x - [x]} \right) - \lim\limits_{x \to 4^-} \left(\frac{x^2 - 9x + 20}{x - [x]} \right)$, where [ ] represents the greatest integer function, equals

(A) 5 (B) -4
(C) 9 (D) 1

10. Let $f(x) = \frac{\sin\{x - 10\}}{\{10 - x\}}$, where { } represents the fractional part. Which of the following is incorrect?

(A) $\lim\limits_{x \to 8^+} f(x) = 0$
(B) $\lim\limits_{x \to 8^-} f(x)$ does not exist.
(C) $\lim\limits_{x \to 8} f(x)$ exists.
(D) $\lim\limits_{x \to 8} f(x)$ does not exist.

11. The value of $\lim\limits_{x \to 0} \frac{11 + 2^{\frac{1}{x}}}{3 + 8^{\frac{1}{x}}}$

(A) is $\frac{11}{3}$ (B) is 1
(C) is 0 (D) does not exist

12. If $\lim\limits_{x \to \infty} \left(\sqrt{ax^2 + bx + c} - x \right) = -\frac{1}{2}$, where a, b, c are constants, then

(A) $a = 1, b = -1, c \in R$
(B) $a = 0, b = 1, c = 0$
(C) $a = 1, b = 1, c \in R$
(D) none of these

13. $\lim\limits_{x \to 0} \frac{\sin|x|}{x}$

(A) is 0 (B) is -1
(C) is 1 (D) does not exist

14. $\lim\limits_{x \to \frac{\pi}{2}} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x} =$

(A) $\log a$ (B) $\log 2$
(C) a (D) $\log x$

15. $\lim\limits_{x \to -1} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x + 1}} =$

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{\pi}}$
(C) $\frac{1}{\sqrt{2\pi}}$ (D) $\frac{1}{2\sqrt{\pi}}$

16. If $\lim_{t\to 0} k\ t$ cosec $t = \lim_{t\to 0} t$ cosec $k\ t$, then k equals
(A) 1 (B) -1
(C) 1 or -1 (D) a number $\neq \pm 1$

17. $\lim_{x\to\frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x} =$
(A) $5\sqrt{2}$ (B) $3\sqrt{2}$
(C) $\sqrt{2}$ (D) $4\sqrt{2}$

18. If $\lim_{x\to 0} \frac{((a-n)nx-\tan x)\sin nx}{x^2} = 0$, where $n \neq 0$ and a are constants, then a equals
(A) 0 (B) $\frac{n+1}{n}$
(C) n (D) $n + \frac{1}{n}$

19. If $\lim_{x\to\frac{1}{\sqrt{2}}} \left(\frac{x-\cos(\sin^{-1}x)}{1-\tan(\sin^{-1}x)}\right) = p$, then $\sqrt{2}\,p$ is
(A) 1 (B) -1
(C) $2\sqrt{2}$ (D) $\frac{1}{2}$

20. $\lim_{x\to-\infty} \frac{x^4\sin\left(\frac{1}{x}\right)+x^2}{1+|x|^3}$ equals
(A) -1 (B) 1 (C) 0 (D) 2

21. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{\frac{2}{x}}$ equals
(A) e^{-4} (B) e^2
(C) e^4 (D) e^{-2}

22. If $\lim_{x\to 0} \frac{(3(p-2)x-\tan 3x)}{x^2}$ sin $2x = 0$, then the value of p is
(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) 2

23. The value of $\lim_{x\to 0} \frac{\cos 6x-\cos 10x}{x^2}$ is
(A) 36 (B) 64 (C) 32 (D) 18

24. The value of $\lim_{x\to\infty} 5^{x-3}\tan\left(\frac{b}{5^x}\right)$ is
(A) $\frac{b}{5}$ (B) $\frac{b}{125}$ (C) $\frac{125}{b}$ (D) $5b$

25. The value of $\lim_{x\to 0} \frac{(\sqrt{x^2+1}-1)x}{\sqrt{x^2+1}\,(\tan^{-1}x)^3}$ is
(A) $\frac{1}{2}$ (B) 2 (C) 1 (D) 0

26. The value of $\lim_{x\to\infty} x\left(\tan^{-1}\frac{x+2}{x+3}-\frac{\pi}{4}\right)$ is
(A) $\frac{1}{2}$ (B) 2 (C) $-\frac{1}{2}$ (D) -2

27. The value of $\lim_{x\to 1} \frac{8x^3-x^2\log x+\log x-8}{x^2-1}$ is
(A) 24 (B) 12
(C) $12 - \log 2$ (D) $\log 2$

28. Let $f(x) = \frac{\sqrt{1-e^{-x^2}}}{x}$. If x approaches 0, then which of the following is a correct statement?
(A) L. H. L. and R. H. L. exist and are unequal.
(B) L. H. L. and R.H. L. exist and both are equal.
(C) L. H. L. does not exist.
(D) R. H. L. does not exist.

29. The value of $\lim_{x\to 0} \frac{27^x-9^x-3^x+1}{\sqrt{5}-\sqrt{4+\cos x}}$ is
(A) $\sqrt{5}(\log 3)^2$ (B) $8\sqrt{5}\log 3$
(C) $16\sqrt{5}\log 3$ (D) $8\sqrt{5}(\log 3)^2$

30. $\lim_{x\to-1}(2+3x+3x^2+x^3)^{\frac{2}{(x+1)^3}}$
(A) is e (B) is e^2
(C) is $\sqrt{e}$ (D) does not exist

31. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b are
(A) $a = 1, b = 2$ (B) $a = 1, b \in \mathrm{R}$
(C) $a \in \mathrm{R}, b = 2$ (D) $a \in \mathrm{R}, b \in \mathrm{R}$

32. If $\lim_{n\to\infty} \frac{1-(10)^n}{1+(10)^{n+1}} = -\frac{\alpha}{10}$, then the value of α is
(A) 0 (B) -1
(C) 1 (D) 2

33. Let $a_n = 1 + 2 + 3 + \ldots + n$ and
$\mathrm{L}_n = \frac{a_2}{a_2-1}\cdot\frac{a_3}{a_3-1}\cdot\frac{a_4}{a_4-1}\ldots\frac{a_n}{a_n-1}$, where $n \in \mathrm{N}$ $(n \geq 2)$. Then $\lim_{n\to\infty} \mathrm{L}_n$ equals
(A) $\frac{3}{2}$ (B) 2 (C) 0 (D) 3

34. The value of $\lim_{n\to\infty}\left(\frac{1^2}{n^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}+\ldots+\frac{n^2}{n^3}\right)$ is
(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{1}{3}$

35. The value of $\lim\limits_{n\to\infty} \dfrac{5^{n+1}+3^{n+2}}{7.5^n - 3^{n-1}}$ is

(A) $\dfrac{7}{5}$ (B) $\dfrac{5}{7}$ (C) 1 (D) 0

36. The value of $\lim\limits_{x\to 2} \dfrac{4^x + 4^{3-x} - 20}{4^{3-x} - 4^{\frac{x}{2}}}$ equals

(A) 2 (B) 0 (C) 1 (D) – 2

37. If p, q, r, s all > 0, then $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{p+qx}\right)^{r+sx}$ is

(A) $e^{\frac{r}{p}}$ (B) $e^{\frac{s}{q}}$

(C) $\dfrac{s}{q}$ (D) $e^{\frac{q}{s}}$

38. If $f(x) = \begin{cases} \dfrac{1}{1+e^{-\frac{1}{x}}} & , \quad x \neq 0 \\ = 0 & , \quad x = 0 \end{cases}$

then at $x = 0$

(A) right hand limit of $f(x)$ exists but not left hand limit.

(B) left hand limit of $f(x)$ exists but not right hand limit.

(C) both limits exist and are unequal.

(D) both limits exist and are equal.

39. If $x_n = \dfrac{(1-2+3-4+5-6+\ldots)-2n}{\sqrt{n^2+1}+\sqrt{4n^2-1}}$,

then $\lim\limits_{n\to\infty} x_n$ is equal to

(A) $-\dfrac{1}{3}$ (B) $-\dfrac{2}{3}$ (C) $\dfrac{2}{3}$ (D) 1

40. The value of $\lim\limits_{x\to\infty} \dfrac{\sqrt[3]{1+x^3}-\sqrt[4]{1+x^5}}{\sqrt[5]{1+x^5}-\sqrt[7]{1+x^6}}$ is

(A) 1 (B) 2 (C) 0 (D) $\dfrac{1}{2}$

41. The value of $\lim\limits_{x\to-\infty} (\sqrt{36x^2-5x}+6x)$ is

(A) 12 (B) 5 (C) $\dfrac{5}{12}$ (D) $\dfrac{12}{5}$

42. The value of $\lim\limits_{x\to 0} \dfrac{e^{\tan x}-e^x}{x-\tan x}$

(A) is 0 (B) is – 1

(C) is 2 (D) does not exist

43. The value of $\lim\limits_{x\to 0} \dfrac{e^{4+x}-\tan x - e^4}{x}$ is

(A) e^4 (B) 1

(C) $e^4 + 1$ (D) $e^4 - 1$

44. The value of $\lim\limits_{n\to\infty} \dfrac{(4(n+1))!}{(n+1)^4\,(4n)!}$ is

(A) 16 (B) 0

(C) 4 (D) 256

45. The value of $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x}}-\sqrt{x-\sqrt{x}}\right)$ is

(A) 0 (B) 1

(C) 2 (D) –1

46. If the value of $\lim\limits_{x\to 0^+}\left(\dfrac{\frac{5}{x}+1}{\frac{5}{x}-1}\right)^{\frac{1}{x}}$ can be expressed in the form of $e^{\frac{p}{q}}$, where p and q are prime numbers, then $p+q$ is equal to

(A) 5 (B) 8

(C) 7 (D) 9

47. $\lim\limits_{x\to 0} \dfrac{(5+\cos x)(1-\cos 4x)}{x \tan 8x}$ equals

(A) 4 (B) 6

(C) 8 (D) 12

48. Let a convergent sequence $< b_n >$ of real numbers satisfy the recurrence relation: $b_{n+1} = \dfrac{1}{3}\left(2b_n + \dfrac{125}{b_n^{\,2}}\right)$, $b_n \neq 0$, then $\lim\limits_{n\to\infty} b_n =$

(A) is 0 (B) does not exist

(C) is 5 (D) $\dfrac{2}{3}$

49. If $\lim\limits_{x\to\infty}\left(\dfrac{x^2+1}{x+1}-ax-b\right)=0$, where a, b are constants, then (a, b) is

(A) (1, – 1) (B) (– 1, 1)

(C) (1, 0) (D) (0, – 1)

50. The value of $\lim\limits_{x\to 0} \dfrac{e^x+\log(1+x)-(1-x)^{-2}}{x^2}$

(A) is 0 (B) is –3

(C) is –1 (D) does not exist

51. Let n be an odd number and S (n) denote the sum of the unordered products of all the pairs of positive integers whose sum = n. Then $\lim\limits_{n\to\infty} \dfrac{S(n)}{n^3}$ is

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{3}$

(C) $\dfrac{1}{12}$ (D) $\dfrac{1}{24}$

52. If $\lim_{x\to\infty} \dfrac{30 + 4\sqrt{x} + 7\sqrt[3]{x}}{2 + \sqrt{4x-7} + \sqrt[3]{6x-2}} = \dfrac{p}{q}$, then

(A) $q = 2p$ (B) $q = 3p$
(C) $p = 2q$ (D) $p = 3q$

53. Let $f : \mathrm{R} \to \mathrm{R}$ be a positive increasing function with $\lim_{x\to\infty} \dfrac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \dfrac{f(2x)}{f(x)} =$

(A) $\dfrac{3}{2}$ (B) 3
(C) 1 (D) $\dfrac{2}{3}$

54. $\lim_{n\to\infty} \dfrac{7^n}{n!}$ equals

(A) 7 (B) 0
(C) 1 (D) none of these

55. $\lim_{x\to\infty} \left[\sqrt{x+\sqrt{x+\sqrt{x}}} - \sqrt{x}\right]$ is equal to

(A) 0 (B) $\dfrac{1}{2}$
(C) log 2 (D) e^4

56. $\lim_{x\to\infty} \dfrac{(x+1)^{10} + (x+2)^{10} + ... + (x+100)^{10}}{x^{10} + 10^{10}}$ is equal to

(A) 0 (B) 1
(C) 10 (D) 100

57. If $f(x) = \lim_{n\to\infty} \dfrac{x^{3n}\sin x + \cos x}{x^{3n} + 2}$, then $f\left(\dfrac{\pi}{6}\right) + f\left(\dfrac{\pi}{3}\right)$ is

(A) $2\sqrt{3}$ (B) $\dfrac{\sqrt{3}}{2}$
(C) $\dfrac{3\sqrt{3}}{4}$ (D) $\dfrac{2\sqrt{3}+1}{4}$

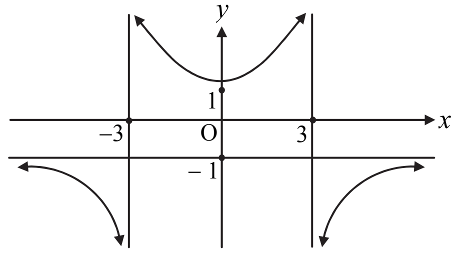
58. The value of $\lim_{x\to\frac{1}{4}} \dfrac{a^{\sin 2\pi x - 1} - 1}{\tan^2 4\pi x}$ is

(A) $\dfrac{1}{8}$ (B) $-\dfrac{\log a}{8}$
(C) $\dfrac{\log a}{8}$ (D) $-\dfrac{1}{8}$

59. If $S_1 = \Sigma n$, $S_2 = \Sigma n^2$, $S_3 = \Sigma n^3$, then the value of $\lim_{n\to\infty} \dfrac{S_1\left(1+\dfrac{S_3}{8}\right)}{S_2^{\,2}}$ is equal to

(A) $\dfrac{3}{32}$ (B) $\dfrac{3}{64}$
(C) $\dfrac{9}{32}$ (D) $\dfrac{9}{64}$

60.



Consider the graph of $y = f(x)$ and the following statements:

I : The domain of f is R.

II : The range of f is R – [– 1, 1).

III : $\lim_{x\to 3^+} f(x)$ does not exist.

IV : $\lim_{x\to\infty} f(x) = -1$

Which of the statements is / are correct?

(A) II, III, IV only (B) I, II, III, IV
(C) III and IV only (D) I only

61. Consider a decreasing sequence (x_n) as $\tan^{-1} 2 = x_1 > x_2 > x_3 > ... > x_n > ... \infty$, of strictly positive terms such that $\sin(x_{n+1} - x_n) + 2^{-(n+1)} \sin x_n \sin x_{n+1} = 0$ for all $n \geq 1$. Then

(A) $\cot x_n = \dfrac{7}{8} \Rightarrow n > 3$

(B) $\lim_{n\to\infty} x_n = \dfrac{\pi}{4}$

(C) $\cot x_n > 1$ for all n

(D) $\cot x_n$ is not rational for all n.

62. Let f and g be two functions defined as
$f(x) = -1 + |x-1|$, $-1 \leq x \leq 3$
$g(x) = 2 - |x+1|$, $-2 \leq x \leq 2$
Then $\lim_{x\to 1} (gof)(x)$ equals

(A) 2 (B) 3
(C) – 2 (D) 0

63. If m and n are positive integers, then $\lim_{x\to 1}\left(\dfrac{mx^m}{x^m - 1} - \dfrac{nx^n}{x^n - 1}\right)$ equals

(A) $\dfrac{m+n}{2}$ (B) $\dfrac{m-n}{2}$
(C) $-\left(\dfrac{m+n}{2}\right)$ (D) $-\dfrac{m-n}{2}$

64. The value of $\lim_{x\to 0} \dfrac{(1+x)^{\frac{1}{x}} - e + \frac{1}{2}ex}{x^2}$ is

(A) $\dfrac{11e}{24}$ (B) $-\dfrac{11e}{24}$
(C) $\dfrac{e}{24}$ (D) $-\dfrac{e}{24}$

65. The expression $\lim\limits_{x\to 0}\left\{\dfrac{\sin x}{x}\right\}^{\frac{1}{\left\{\frac{\tan x}{x}\right\}}}$, where { } represents the fractional part, is

(A) e (B) $\sqrt{e}$

(C) $\dfrac{1}{e}$ (D) $\dfrac{1}{\sqrt{e}}$

66. Let $f(x)$ = [sin [x]], where [ ] represents the greatest integer function. Which of the following statement is NOT correct?

(A) $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 3^+} f(x)$

(B) $\lim\limits_{x\to 4^+} f(x) < \lim\limits_{x\to 3^-} f(x)$

(C) $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 3^-} f(x)$

(D) $\lim\limits_{x\to 0^-} f(x) > \lim\limits_{x\to 5^-} f(x)$

67. Let $f(x) = e^{\left\{e^{|x|}sgn\, x\right\}}$, $g(x) = e^{\left[e^{|x|}sgn\, x\right]}$, $x \in$ R,

where $sgn\,(x) = -1 \quad , \quad x < 0$

$= 0 \quad , \quad x = 0$

$= 1 \quad , \quad x > 0$

and, { } and [ ] represent fractional part and greatest integer function respectively. If $h(x) = \log f(x) + \log g(x)$, then which of the following statement is correct?

(A) $\lim\limits_{x\to 0^-} h(x) = 1$

(B) $\lim\limits_{x\to 0^+} \dfrac{h(x)-1}{x} = 1$

(C) $\lim\limits_{x\to 0^+} h(x) = h(0)$

(D) $\lim\limits_{x\to 0^+} h(x)$ does not exist.

68. Let f be an odd function such that $1 + f(x) = f(x+1)$ for all $x\in$R. Then

(A) $\lim\limits_{x\to 1^-} \dfrac{f(x)-1}{x} = 1$

(B) $\lim\limits_{x\to 3^-} \dfrac{(f(x))^2-4}{x-[x]} = 5$

(C) $\lim\limits_{x\to 2} \dfrac{f(x)-2}{x-2}$ does not exist.

(D) $\lim\limits_{x\to 1} \dfrac{\sqrt{f(x)}-1}{x-1} = \dfrac{1}{\sqrt{2}}$

69. If $\lim\limits_{x\to 0} \dfrac{axe^x - b\log(1+x)}{x^2} = 3$, then the values of a, b are respectively

(A) 2, 2 (B) 1, 2

(C) 2, 1 (D) 2, 0

70. The value of $\lim\limits_{n\to\infty} \dfrac{\left[1^3 x\right]+\left[2^3\ x\right]+\ldots+\left[n^3 x\right]}{n^4}$, where [x] denotes the greatest integral part of x, is

(A) $\dfrac{x}{4}$ (B) $\dfrac{1}{4}$

(C) 4 (D) $\dfrac{4}{x}$

71. If $S_n = \dfrac{1}{1.3} + \dfrac{2}{1.3.5} + \ldots + \dfrac{n}{1.3.5\ldots(2n-1)}$,

$S_n' = \dfrac{3}{1^2 \cdot 2^2} + \dfrac{5}{2^2\ 3^2} + \ldots + \dfrac{(2n+1)}{n^2(n+1)^2}$,

$L_1 = \lim\limits_{n\to\infty} S_n$ and $L_2 = \lim\limits_{n\to\infty} S_n'$,

then $\dfrac{L_1}{L_2}$ is

(A) $\dfrac{1}{2}$ (B) 2

(C) $\dfrac{4}{3}$ (D) $\dfrac{3}{4}$

Problems To Ponder

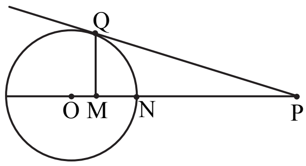
1. Consider a regular polygon of n sides inscribed in a circle of radius r. Let P(n) and A(n) denote its perimeter and area respectively. Let $l_1 = \lim\limits_{n\to\infty}$ P (n) and

$l_2 = \lim\limits_{n\to\infty}$ A(n). Evaluate

i. P(n) ii. A(n)

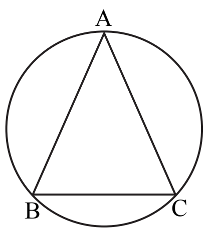
iii. l_1 iv. l_2

2. P is any point on an extended diameter of a circle (centre O). PQ is a tangent at Q. M is the projection of Q on OP. Let $\angle$OPQ = θ.



Prove that $\lim\limits_{\theta\to\frac{\pi}{2}} \dfrac{|PM|}{|MN|} = 2$.

3. ABC is a triangle inscribed in a circle of radius r. | AB | = | AC | and the altitude from A has length h. Show that



i. the perimeter of triangle is

$P = 2\left(\sqrt{2rh-h^2} + \sqrt{2rh}\right)$

ii. the area of the triangle is $\Delta = h\sqrt{2rh-h^2}$

iii. $\lim\limits_{h\to 0}\left(\dfrac{\Delta}{P^3}\right) = \dfrac{1}{128r}$

Answers to MCQs

Concept Building Problems

1.1 :	1.	(C)	2.	(D)	3.	(C)	4.	(B)	5.	(A)	6.	(C)	7.	(B)						
1.2 :	1.	(C)	2.	(B)	3.	(D)	4.	(C)	5.	(B)										
1.3 :	1.	(C)	2.	(C)	3.	(B)	4.	(A)	5.	(A)	6.	(A)	7.	(B)	8.	(B)	9.	(D)	10.	(C)
	11.	(B)	12.	(C)	13.	(C)	14.	(B)	15.	(B)	16.	(C)	17.	(C)						
1.4 :	1.	(C)	2.	(A)	3.	(B)	4.	(A)	5.	(A)	6.	(B)	7.	(B)	8.	(C)	9.	(D)		
1.5 :	1.	(D)	2.	(B)	3.	(D)	4.	(C)	5.	(D)	6.	(A)	7.	(B)	8.	(A)	9.	(A)	10.	(C)
	11.	(B)	12.	(B)	13.	(B)	14.	(B)	15.	(A)	16.	(B)	17.	(C)	18.	(D)				
1.6 :	1.	(B)	2.	(A)	3.	(A)	4.	(B)	5.	(D)										
1.7 :	1.	(C)	2.	(C)	3.	(A)	4.	(B)	5.	(B)										

Practice Problems

1.	(D)	2.	(C)	3.	(C)	4.	(D)	5.	(D)	6.	(B)	7.	(B)	8.	(C)	9.	(D)	10.	(C)
11.	(D)	12.	(A)	13.	(D)	14.	(A)	15.	(C)	16.	(C)	17.	(A)	18.	(D)	19.	(B)	20.	(A)
21.	(C)	22.	(C)	23.	(C)	24.	(B)	25.	(A)	26.	(C)	27.	(B)	28.	(A)	29.	(D)	30.	(B)
31.	(B)	32.	(C)	33.	(D)	34.	(D)	35.	(B)	36.	(D)	37.	(B)	38.	(C)	39.	(B)	40.	(C)
41.	(C)	42.	(B)	43.	(D)	44.	(D)	45.	(B)	46.	(C)	47.	(B)	48.	(C)	49.	(A)	50.	(B)
51.	(C)	52.	(C)	53.	(C)	54.	(B)	55.	(B)	56.	(D)	57.	(C)	58.	(B)	59.	(D)	60.	(A)
61.	(B)	62.	(A)	63.	(B)	64.	(A)	65.	(D)	66.	(D)	67.	(B)	68.	(B)	69.	(A)	70.	(A)
71.	(A)																		

Problems To Ponder

1. i. $2\,n\,r\sin\left(\dfrac{\pi}{n}\right)$ ii. $\dfrac{1}{2}\,n\,r^2\sin\left(\dfrac{2\pi}{n}\right)$ iii. $2\pi r$ iv. πr^2

Hints to MCQs

Concept Building Problems

1.1 Evaluation by Factorization

1. $$\lim_{x\to1}\frac{(\sqrt{x}-1)(\sqrt[3]{x}-1)}{x-1}=\lim_{x\to1}\frac{(\sqrt{x}-1)(\sqrt[3]{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)}$$
$$=\lim_{x\to1}\left(\frac{\sqrt[3]{x}-1}{\sqrt{x}+1}\right)=0$$

2. $$\lim_{x\to3}\frac{x^3-x^2-18}{x-3}=\lim_{x\to3}\frac{(x-3)(x^2+2x+6)}{x-3}$$
$$=\lim_{x\to3}(x^2+2x+6)$$
$$=9+6+6=21$$

3. $$f(x)+g(x)+h(x)=\frac{2}{x-3}+\frac{x-3}{x+4}-\frac{2(2x+1)}{x^2+x-12}$$
$$=\frac{x^2-8x+15}{x^2+x-12}$$
$$=\frac{(x-3)(x-5)}{(x-3)(x+4)}$$
$$\lim_{x\to3}[f(x)+g(x)+h(x)]=\lim_{x\to3}\frac{(x-3)(x-5)}{(x-3)(x+4)}$$
$$=\lim_{x\to3}\frac{x-5}{x+4}$$
$$=-\frac{2}{7}$$

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As L_2 exists and $L_2 \neq 0$

$\Rightarrow m$ cannot be greater or less than 5.

$\Rightarrow m = 5$, and that gives $L_2 = \dfrac{3}{40}$

$\Rightarrow m - n = 2$

1.7 Use of Sandwich Theorem

1. Using $x - 1 < [x] \le x$, we have

$$\frac{n}{5} - 1 < \left[\frac{n}{5}\right] \le \frac{n}{5}, \quad \text{... (i)}$$

$$\frac{n}{7} - 1 < \left[\frac{n}{7}\right] \le \frac{n}{7}, \quad \text{... (ii)}$$

$$\frac{n}{20} - 1 < \left[\frac{n}{20}\right] \le \frac{n}{20}$$

$$\Rightarrow -\frac{n}{20} \le -\left[\frac{n}{20}\right] < 1 - \frac{n}{20} \quad \text{... (iii)}$$

Adding (i), (ii) and (iii), we get

$$\frac{\frac{n}{5}+\frac{n}{7}-\frac{n}{20}-2}{n} < \frac{\left[\frac{n}{5}\right]+\left[\frac{n}{7}\right]-\left[\frac{n}{20}\right]}{n} \le \frac{\frac{n}{5}+\frac{n}{7}-\frac{n}{20}+1}{n}$$

Taking limit $n \to \infty$ and using Sandwich theorem, $\lim\limits_{n\to\infty} \dfrac{\left[\frac{n}{5}\right]+\left[\frac{n}{7}\right]-\left[\frac{n}{20}\right]}{n} = \dfrac{1}{5} + \dfrac{1}{7} - \dfrac{1}{20}$

$$= \frac{28+20-7}{140} = \frac{41}{140}$$

2. $x \le f(x) \le \sqrt{6-x}$, $\quad x \in [1, 2)$

$1 + \dfrac{2}{x} \le f(x) \le \sqrt{6-x}$, $\quad x \in [2, 3)$

$L_1 = \lim\limits_{x\to 2^-} f(x)$, $L_2 = \lim\limits_{x\to 2^+} f(x)$

Also $\lim\limits_{x\to 2^+} \sqrt{6-x} = 2$ and $\lim\limits_{x\to 2^+} \left(1 + \dfrac{2}{x}\right) = 2$

$\Rightarrow$ Using Sandwich theorem,

$L_1 = 2$ and $L_2 = 2$

$\Rightarrow L_1 = L_2$

3. $\dfrac{1}{n^2+1} < \dfrac{1}{n^2}, \dfrac{2}{n^2+2} < \dfrac{2}{n^2}$ etc., on addition, lead to

$$\sum_{r=1}^{n} \frac{r}{n^2+r} < \frac{1+2+3+...+n}{n^2}$$

$$\Leftrightarrow \sum_{r=1}^{n} \frac{r}{n^2+r} < \frac{n(n+1)}{2n^2} \quad \text{... (i)}$$

Also, $\dfrac{1}{n^2+1} > \dfrac{1}{n^2+n}, \dfrac{2}{n^2+2} > \dfrac{2}{n^2+n}$ etc.,

on addition, lead to

$$\sum_{r=1}^{n} \frac{r}{n^2+r} > \frac{1+2+3+...+n}{n^2+n}$$

$$\Leftrightarrow \sum_{r=1}^{n} \frac{r}{n^2+r} > \frac{n(n+1)}{2(n^2+n)} \quad \text{... (ii)}$$

Taking limit $n \to \infty$ in (i), (ii) and using Sandwich theorem, $\lim\limits_{n\to\infty} u_n = \dfrac{1}{2}$

4. Using $x - 1 < [x] \le x$, we have

$$\left.\begin{aligned} 1^2x - 1 &< [1^2x] \le 1^2x \\ 2^2x - 1 &< [2^2x] \le 2^2x \\ &\vdots \\ n^2x - 1 &< [n^2x] \le n^2x \end{aligned}\right\} \text{Adding all, we get } x\left(\sum_{r=1}^{n} r^2\right) - n < \sum_{r=1}^{n} [r^2x] \le \sum_{r=1}^{n} r^2 x$$

$$\Leftrightarrow \frac{n(n+1)(2n+1)x}{6} - n < \sum_{r=1}^{n} [r^2x] < \frac{n(n+1)(2n+1)x}{6}$$

Taking limit $n \to \infty$ and using Sandwich theorem,

$$\lim_{n\to\infty} \frac{\sum_{r=1}^{n} [r^2x]}{n^3} = \frac{2x}{6} = \frac{x}{3}$$

5. $\dfrac{1}{x} - 1 < \left[\dfrac{1}{x}\right] \le \dfrac{1}{x}$

$\dfrac{2}{x} - 1 < \left[\dfrac{2}{x}\right] \le \dfrac{2}{x}$

.

.

.

$$\sum_{r=1}^{15}\left(\frac{r}{x} - 1\right) < \sum_{r=1}^{15}\left[\frac{r}{x}\right] \le \sum_{r=1}^{15}\frac{r}{x}$$

$$120 < \lim_{x\to 0^+} x\left(\sum_{r=1}^{15}\left[\frac{r}{x}\right]\right) \le 120$$

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right) = 120$$

Practice Problems

1. $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \dfrac{5x}{-x+7x}$ $\quad$ if $x < 0, |x| = -x$

$= \lim\limits_{x\to 0^-} \dfrac{5x}{6x} = \dfrac{5}{6}$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \dfrac{5x}{x+7x}$ $\quad$ if $x > 0, |x| = x$

$= \lim\limits_{x\to 0^+} \dfrac{5x}{8x} = \dfrac{5}{8}$

$\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x)$

$\Rightarrow$ Limit does not exist.

2. $f(x) = 1 - 2x$, $\quad x < 0$

$= 1$, $\quad 0 \le x < 1$

$= 2x - 1$, $\quad x \ge 1$

$$\left.\begin{aligned} \lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-} (1 - 2x) = 1 \\ \lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+} (1) = 1 \end{aligned}\right\} \Rightarrow \lim_{x\to 0} f(x) = 1 = l_1$$

$\left.\begin{array}{l}\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-} (1) = 1 \\ \lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+} (2x-1) = 1\end{array}\right\} \Rightarrow \lim\limits_{x\to1} f(x) = 1 = l_2$

$\Rightarrow l_1 + l_2 = 2$

3. $\lim\limits_{x\to3^-} f(x) = \lim\limits_{x\to3^-} [x-3] + \lim\limits_{x\to3^-} |x-4|$

$= \lim\limits_{x\to3^-} ([x]-3) + \lim\limits_{x\to3^-} (4-x)$

$= (2-3) + (4-3) \quad \ldots \left[\lim\limits_{x\to a^-} [x] = a-1\right]$

$= -1 + 1 = 0$

4. $\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^+} f(x)$

$\Leftrightarrow \lim\limits_{x\to1^-} (2px+3) = \lim\limits_{x\to1^+} (1-px^2)$

$\Leftrightarrow 2p + 3 = 1 - p$

$\Leftrightarrow p = \dfrac{-2}{3}$

5. $\lim\limits_{x\to0} \dfrac{\log(1+\{x\})}{\{x\}} = \lim\limits_{x\to0} \dfrac{\log(1+x-[x])}{x-[x]}$

L.H.L. $= \lim\limits_{x\to0^-} \dfrac{\log(1+x-(-1))}{x-(-1)}$

$= \lim\limits_{x\to0^-} \dfrac{\log(2+x)}{x+1} = \log 2$

R.H.L. $= \lim\limits_{x\to0^+} \dfrac{\log(1+x-0)}{x-0}$

$= \lim\limits_{x\to0^+} \log(1+x)^{\frac{1}{x}} = \log_e e = 1$

L.H.L. $\neq$ R.H.L. $\Rightarrow$ Limit does not exist.

6. $\lim\limits_{x\to0} \left(\dfrac{1}{x} - \dfrac{n!}{x(x+1)(x+2)\ldots(x+n)}\right)$

$= \lim\limits_{x\to0} \dfrac{(x+1)(x+2)\ldots(x+n)-n!}{x(x+1)(x+2)\ldots(x+n)}$

$= \lim\limits_{x\to0} \dfrac{x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x}{x(x+1)(x+2)\ldots(x+n)}$,

where $a_{n-1} = (2 \cdot 3 \cdot 4 \cdot \ldots n) + (1 \cdot 3 \cdot 4 \ldots n)$
$+ (1 \cdot 2 \cdot 4 \ldots n) + \ldots + (1 \cdot 2 \cdot 3 \cdot \ldots (n-1))$

$= \lim\limits_{x\to0} \dfrac{a_{n-1} + \text{terms containing } x \text{ and higher degree}}{(x+1)(x+2)\ldots(x+n)}$

$= \dfrac{a_{n-1}}{n!} = 1 + \dfrac{1}{2} + \dfrac{1}{3} + \ldots + \dfrac{1}{n}$

7. $\lim\limits_{x\to1} \dfrac{x + x^2 + \ldots + x^{100} - 100}{x-1}$

$= \lim\limits_{x\to1} \dfrac{(x-1) + (x^2-1) + \ldots + (x^{100}-1)}{x-1}$

$= \lim\limits_{x\to1} [1 + (x+1) + (x^2+x+1) + \ldots$
$+ (x^{99} + x^{98} + \ldots + 1)]$

$= 1 + 2 + 3 + \ldots + 100 = \dfrac{(100)(101)}{2} = 5050$

8. L.H.L. $= \lim\limits_{x\to2^-} \dfrac{|x^2-5x+6|}{(x-2)(x-3)}$

$= \lim\limits_{x\to2^-} \dfrac{(x^2-5x+6)}{(x-2)(x-3)} = 1$

R.H.L. $= \lim\limits_{x\to2^+} \dfrac{|x^2-5x+6|}{(x-2)(x-3)}$

$= \lim\limits_{x\to2^+} \dfrac{-(x^2-5x+6)}{(x-2)(x-3)} = -1$

L.H.L. $\neq$ R.H.L. $\Rightarrow$ Limit does not exist.

9. $\lim\limits_{x\to5^+} \left(\dfrac{x^2-9x+20}{x-[x]}\right) - \lim\limits_{x\to4^-} \left(\dfrac{x^2-9x+20}{x-[x]}\right)$

$= \lim\limits_{x\to5^+} \dfrac{(x-5)(x-4)}{x-5} - \lim\limits_{x\to4^-} \dfrac{(x-5)(x-4)}{x-3}$

$= \lim\limits_{x\to5^+} (x-4) - \lim\limits_{x\to4^-} \dfrac{(x-5)(x-4)}{x-3}$

$= (5-4) + 0 = 1$

10. L. H. L. $= \lim\limits_{x\to8^-} f(x) = \lim\limits_{x\to8^-} \dfrac{\sin\{x-10\}}{\{10-x\}}$

$= \lim\limits_{x\to8^-} \dfrac{\sin(x-10-[x-10])}{10-x-[10-x]}$

$= \lim\limits_{x\to8^-} \dfrac{\sin(x-10+3)}{10-x-2}$

$= \lim\limits_{x\to8^-} \dfrac{\sin(x-7)}{8-x}$ does not exist

R. H. L. $= \lim\limits_{x\to8^+} f(x) = \lim\limits_{x\to8^+} \dfrac{\sin(x-10-[x-10])}{10-x-[10-x]}$

$= \lim\limits_{x\to8^+} \dfrac{\sin(x-10+2)}{10-x-1} = 0$

11. L.H.L. $= \lim\limits_{x\to0^-} \dfrac{11+(2)^{\frac{1}{x}}}{3+(8)^{\frac{1}{x}}} = \dfrac{11}{3}$

R.H.L. $= \lim\limits_{x\to0^+} \dfrac{11+2^{\frac{1}{x}}}{3+8^{\frac{1}{x}}}$

$= \lim\limits_{y\to\infty} \dfrac{11+2^y}{3+8^y} \quad \ldots \left[\dfrac{1}{x} = y\right]$

$= \lim\limits_{y\to\infty} \dfrac{11(2^{-y})+1}{3(2^{-y})+2^{2y}} = 0$

$\Rightarrow$ L.H.L. $\neq$ R.H.L.
$\Rightarrow$ Limit does not exist

12. $\lim\limits_{x\to\infty} \left(\sqrt{ax^2+bx+c} - x\right) = -\dfrac{1}{2}$

$\Leftrightarrow \lim\limits_{x\to\infty} \left(\dfrac{(ax^2+bx+c)-x^2}{\sqrt{ax^2+bx+c}+x}\right) = -\dfrac{1}{2}$

$\Leftrightarrow \lim\limits_{x\to\infty} \dfrac{(a-1)x^2+bx+c}{\sqrt{ax^2+bx+c}+x} = -\dfrac{1}{2} \quad \ldots \text{(i)}$

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$$= \frac{1}{2}\sum_{r=1}^{n} \frac{2r+1-1}{1\cdot3\cdot5\cdot\ldots\cdot(2r+1)}$$

$$= \frac{1}{2}\sum_{r=1}^{n}\left[\frac{1}{1\cdot3\cdot5\ldots(2r-1)} - \frac{1}{1\cdot3\cdot5\ldots(2r+1)}\right]$$

$$= \frac{1}{2}\left[\left(1-\frac{1}{1\cdot3}\right)+\left(\frac{1}{1\cdot3}-\frac{1}{1\cdot3\cdot5}\right)\right.$$

$$\left. + \ldots + \frac{1}{1\cdot3\cdot5\ldots(2n-1)} - \frac{1}{1\cdot3\cdot5\ldots(2n+1)}\right]$$

$$= \frac{1}{2}\left[1-\frac{1}{1\cdot3\cdot5\ldots(2n+1)}\right]$$

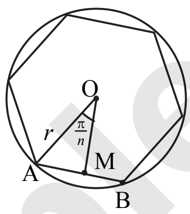
$$S_n' = \sum_{r=1}^{n}\frac{2r+1}{r^2(r+1)^2} = \sum_{r=1}^{n}\frac{(r+1)^2-r^2}{r^2(r+1)^2}$$

$$= \sum_{r=1}^{n}\left[\frac{1}{r^2}-\frac{1}{(r+1)^2}\right] = 1-\frac{1}{(n+1)^2}$$

$$L_1 = \lim_{n\to\infty} S_n = \frac{1}{2}, \quad L_2 = \lim_{n\to\infty} S_n' = 1$$

$$\Rightarrow \frac{L_1}{L_2} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

Problems To Ponder



1. $|AB| = 2\,|AM|$

$$= 2\cdot r\sin\left(\frac{\pi}{n}\right)$$

i. Perimeter $= P(n) = n\,|AB|$

$$= 2\,n\,r\sin\left(\frac{\pi}{n}\right)$$

ii. $|OM| = r\cos\dfrac{\pi}{n}$

$$\text{Area} = A(n) = \left(\frac{1}{2}|AB|\cdot|OM|\right)\cdot n$$

$$= \frac{n}{2}\cdot 2r\sin\left(\frac{\pi}{n}\right)\cdot r\cos\left(\frac{\pi}{n}\right)$$

$$= \frac{1}{2}nr^2\cdot\left(2\sin\frac{\pi}{n}\cos\frac{\pi}{n}\right)$$

$$= \frac{1}{2}nr^2\sin\left(\frac{2\pi}{n}\right)$$

iii. $l_1 = \displaystyle\lim_{n\to\infty} 2rn\sin\left(\frac{\pi}{n}\right) = \lim_{n\to\infty} 2r\pi\cdot\frac{\sin\left(\frac{\pi}{n}\right)}{\frac{\pi}{n}}$

$\Rightarrow l_1 = 2\,\pi\,r$

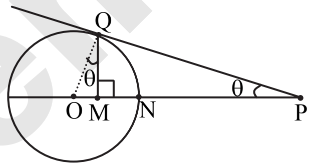
... [as $\displaystyle\lim_{n\to\infty}\frac{\pi}{n} = 0$ and hence $\displaystyle\lim_{n\to\infty}\frac{\sin\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)} = 1$]

iv. $l_2 = \displaystyle\lim_{n\to\infty}\frac{1}{2}nr^2\sin\left(\frac{2\pi}{n}\right)$

$$= \lim_{n\to\infty}\frac{1}{2}r^2(2\pi)\cdot\frac{\sin\left(\frac{2\pi}{n}\right)}{\left(\frac{2\pi}{n}\right)}$$

$\Rightarrow l_2 = \pi r^2$

... [as $\displaystyle\lim_{n\to\infty}\left(\frac{2\pi}{n}\right) = 0$ and hence $\displaystyle\lim_{n\to\infty}\frac{\sin\left(\frac{2\pi}{n}\right)}{\left(\frac{2\pi}{n}\right)} = 1$]



2. $|MN| = |ON| - |OM|$

$$= r - r\sin\theta$$

$|PM| = |OP| - |OM|$

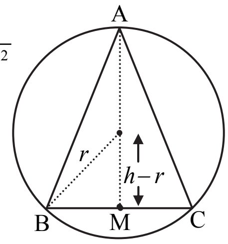
$$= \frac{r}{\sin\theta} - r\sin\theta$$

$$= \frac{r(1-\sin^2\theta)}{\sin\theta}$$

$$\therefore \frac{|PM|}{|MN|} = \frac{r(1-\sin^2\theta)}{\sin\theta}\times\frac{1}{r(1-\sin\theta)}$$

$$= \frac{1+\sin\theta}{\sin\theta}$$

$$\therefore \lim_{\theta\to\frac{\pi}{2}}\frac{|PM|}{|MN|} = \lim_{\theta\to\frac{\pi}{2}}\left(\frac{1+\sin\theta}{\sin\theta}\right) = \frac{1+1}{1} = 2$$



3. $|BM| = \sqrt{r^2-(h-r)^2} = \sqrt{2hr-h^2}$

$$|AB|^2 = \left(\sqrt{2hr-h^2}\right)^2 + h^2$$

$$= 2hr$$

i. Perimeter $P = 2\sqrt{2hr} + 2\sqrt{2hr-h^2}$ and

ii. area $\Delta = \dfrac{1}{2}h\left(2\sqrt{2hr-h^2}\right)$

$$= h\sqrt{2hr-h^2}$$

iii. $\dfrac{\Delta}{P^3} = \dfrac{h\sqrt{2hr-h^2}}{8\left(\sqrt{2hr}+\sqrt{2hr-h^2}\right)^3}$

$$= \frac{\sqrt{2hr-h^2}}{8\sqrt{h}\left(\sqrt{2r}+\sqrt{2r-h}\right)^3} = \frac{\sqrt{2r-h}}{8\left(\sqrt{2r}+\sqrt{2r-h}\right)^3}$$

$$\therefore \lim_{h\to0}\frac{\Delta}{P^3} = \frac{\sqrt{2r}}{8\left(\sqrt{2r}+\sqrt{2r}\right)^3}$$

$$= \frac{\sqrt{2r}}{8\times8\,(2\sqrt{2})\,(r\sqrt{r})} = \frac{1}{128r}$$

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