Written as per the revised syllabus prescribed by the Maharashtra State Board of Secondary and Higher Secondary Education, Pune.

Std. XII Commerce

Mathematics and Statistics - II

Salient Features

- Precise Theory for every Topic.
- Exhaustive coverage of entire syllabus.
- Topic-wise distribution of all textual questions and practice problems at the beginning of every chapter.
- Relevant and important formulae wherever required.
- Covers answers to all Textual Questions.
- Practice problems based on Textual Exercises and Board Questions (March 08 – July 17) included for better preparation and self evaluation.
- Multiple Choice Questions at the end of every chapter.
- Two Model Question papers based on the latest paper pattern.
- Includes Board Question Papers of 2016, 2017 and March 2018.

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Unitwise Distribution of Marks
Section - II

Sr. No.	Units	Marks with Option
1.	Commercial Arithmetic: – Ratio, Proportion, Partnership – Commission, Brokerage, Discount – Insurance, Annuity	13
2.	Demography	08
3.	Bivariate Data Correlation	08
4.	Regression Analysis	07
5.	Random Variable and Probability Distribution	08
6.	Management Mathematics	14
	Total	**58**

Weightage of Objectives

Sr. No.	Objectives	Marks	Marks with Option	Percentage
1.	Knowledge	08	13	10.00
2.	Understanding	22	32	27.50
3.	Application	32	45	40.00
4.	Skill	18	26	22.50
	Total	**80**	**116**	**100.00**

Weightage of Types of Questions

Sr. No.	Types of Questions	Marks	Marks with Option	Percentage
1.	Objective Type	24	32	30
2.	Short Answer	24	36	30
3.	Long Answer	32	48	40
	Total	**80**	**116**	**100.00**

Contents

01 Ratio, Proportion and Partnership

Type of Problems	Exercise	Q. Nos.
Ratio and Proportion	1.1	Q.1 to 10
	Practice Problems (Based on Exercise 1.1)	Q.1 to 8
	Miscellaneous	Q.1 to 9
	Practice Problems (Based on Miscellaneous)	Q.1, 2, 3, 4
Partnership	1.2	Q.1 to 10
	Practice Problems (Based on Exercise 1.2)	Q.1 to 7
	Miscellaneous	Q.10 to 20
	Practice Problems (Based on Miscellaneous)	Q.5, 6, 7, 8

Syllabus:

1.1	Ratio
1.2	Proportion
1.3	Partnership

Introduction

Ratio is the comparative relationship between two quantities of same kind, expressed in same unit.
i.e., the ratio of two quantities a and b of the same kind and measured in the same units is the fraction $\frac{a}{b}$ and is written as a : b, read as 'a' is to 'b'.

For example, If height of a person 'a' is 4ft and that of another person 'b' is 6ft, then a = 4ft and b = 6ft. Here, the quantity concerned (height) is of same kind and is measured in the same unit.

$\therefore \quad \frac{a}{b} = \frac{4}{6} = \frac{2}{3}$

1.1 Ratio

Definition:
If a, b and k are non-zero real numbers such that $a = bk$ i.e., $\frac{a}{b} = k$, then k is the ratio of a to b.

Terms of a ratio:
In the ratio a : b, a is called as first term or antecedent and b is called as second term or consequent.

Ratio in the simplest form:
The ratio a : b is said to be in the simplest form if H.C.F. of a and b is 1 i.e., there is no common factor other than 1.

Properties of ratio:

1. If both the terms of the ratio are multiplied or divided by same non-zero number, then the ratio remains unchanged.
   i.e., $\frac{a}{b} = \frac{ak}{bk}$, where $k \neq 0$
   and $\frac{a}{b} = \frac{a/k}{b/k}$, where $k \neq 0$
2. **Order relation between the ratios**
   Let $\frac{a}{b}$ and $\frac{c}{d}$ be two given ratios,
   where $b > 0$, $d > 0$
   i. If $ad > bc$, then $\frac{a}{b} > \frac{c}{d}$ i.e., $a : b > c : d$
   ii. If $ad < bc$, then $\frac{a}{b} < \frac{c}{d}$ i.e., $a : b < c : d$
   iii. If $ad = bc$, then $\frac{a}{b} = \frac{c}{d}$ i.e., $a : b = c : d$

Properties of equal ratios:

1. **Invertendo:** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$
2. **Alternendo:** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$
3. **Componendo:** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$
   This property is generalized as
   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+mb}{b} = \frac{c+md}{d}$
   Where, m is a positive integer.
4. **Dividendo:**
   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$
   This property is generalized as
   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-mb}{b} = \frac{c-md}{d}$
   Where, m is a positive integer.
5. **Componendo-Dividendo:**
   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$
   This property is generalized as
   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+mb}{a-mb} = \frac{c+md}{c-md}$
   Where, m is a positive integer.

Theorem on Equal ratios:

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$
In general this theorem is written as
If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots\ldots\ldots$
and if l, m, n ……… are non-zero numbers such that $lb + md + nf + \ldots \neq 0$, then
$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots = \frac{la + mc + ne + \ldots..}{lb + md + nf + \ldots..}$

Percentage (%):
It is the numerator of the ratio of two numbers, where the denominator is always 100.
Percent means per hundred (cent).

For example,

$\frac{40}{100} = 40\%$

i.e., 40 percent (40%) means 40 per 100.

Note: A fraction can be converted into percentage on multiplication by 100.

e.g., $\frac{4}{5}$ means $\frac{4}{5} \times 100 = 80\%$

1.2 Proportion

An equality of two ratios is called a proportion.
i.e., if two ratios are equal then the terms are said to be in proportion.

If $\frac{a}{b} = \frac{c}{d}$, then the terms a, b, c and d are in proportion and it is expressed as a : b : : c : d

For example,

If $\frac{2}{6} = \frac{3}{9}$, then 2, 6, 3 and 9 are in proportion and it is expressed as 2 : 6 : : 3 : 9

Note:
Here,
1. a and d are called extremes.
2. b and c are called means or middle terms.
3. If a, b, c, d are in proportion, then ad = bc

Continued Proportion:

Three numbers say a, b, c are said to be in continued proportion if a : b = b : c.

Remark:
Since, a : b = b : c

∴ $\frac{a}{b} = \frac{b}{c}$

∴ $b^2 = ac$

∴ $b = \sqrt{ac}$

∴ the numbers a, $\sqrt{ac}$, c are always in continued proportion.

In general the numbers a, b, c, d, e, f, …. are in continued proportion,

if $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \ldots\ldots$

Note:
1. If a : b :: c : d then a, b, c and d are called first, second, third and fourth proportions respectively.
2. If a, b, c, are in continued proportion, then $b = \sqrt{ac}$ is called geometric mean of a and c.
   b is also called mean proportion of a and c.

Exercise 1.1

1. The ratio of number of boys and girls in a school is 3 : 2. If 20% of the boys and 30% of the girls are scholarship holders. Find the percentage of students who are not scholarship holders. [Mar 15; 17]

Solution:

Let x be the proportionality constant.
Since, the ratio of number of boys and girls in the school is 3 : 2.

∴ the number of boys and girls are $3x$ and $2x$ respectively.

∴ total number of students = $3x + 2x = 5x$

Now, 20% of the boys are scholarship holders.

∴ the number of boys who are scholarship holders = 20% of $3x$

$= \frac{20}{100} \times 3x = \frac{3x}{5}$

Also, 30% of the girls are scholarship holders.

∴ the number of girls who are scholarship holders = 30% of $2x$

$= \frac{30}{100} \times 2x = \frac{3x}{5}$

∴ The number of students who are not scholarship holders

$= \text{Total number of students} - \text{Number of students who are scholarship holders}$

$= 5x - \left(\frac{3x}{5} + \frac{3x}{5}\right) = 5x - \frac{6x}{5} = \frac{19x}{5}$

Now, percentage of students who are not scholarship holders

$= \frac{\text{Number of students who are not scholarship holders}}{\text{Total number of students}} \times 100$

$= \frac{\frac{19x}{5}}{5x} \times 100 = \frac{19x}{25x} \times 100 = 76\%$

2. If the numerator of a fraction is increased by 20% and its denominator be diminished by 10%, the value of the fraction is $\frac{16}{21}$. Find the original fraction.

Solution:

Let the numerator of the fraction be x and the denominator be y.

∴ the fraction is $\frac{x}{y}$

Given, numerator of the fraction is increased by 20%.

$\therefore$ numerator becomes x + 20% of x

$= x + \frac{20}{100} \times x$

$= x + \frac{1}{5}x = \frac{6x}{5}$

and denominator of the fraction is diminished by 10%.

$\therefore$ denominator becomes y – 10% of y

$= y - \frac{10}{100} \times y$

$= y - \frac{1}{10}y = \frac{9y}{10}$

Also, value of the new fraction is given to be $\frac{16}{21}$

i.e. $\frac{\frac{6x}{5}}{\frac{9y}{10}} = \frac{16}{21}$

$\therefore \frac{6x}{5} \times \frac{10}{9y} = \frac{16}{21}$

$\therefore \frac{4x}{3y} = \frac{16}{21}$

$\therefore \frac{x}{y} = \frac{16}{21} \times \frac{3}{4}$

$\therefore \frac{x}{y} = \frac{4}{7}$

$\therefore$ the original fraction is $\frac{4}{7}$.

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3. The ratio of incomes of Salim and Jawed was 20:11. Three years later income of Salim has increased by 20% and income of Jawed was increased by ₹ 500. Now ratio of their incomes become 3:2. Find original incomes of Salim and Jawed.

Solution:

Let x be the proportionality constant.

Since, the ratio of incomes of Salim and Jawed was 20:11.

$\therefore$ The original incomes of Salim and Jawed were ₹ $20x$ and ₹ $11x$ respectively.

Given, three years later, income of Salim has increased by 20%.

$\therefore$ income of Salim becomes $20x$ + 20% of $20x$

$= 20x + \frac{20}{100} \times 20x$

$= 20x + 4x =$ ₹ $24x$

and income of Jawed was increased by ₹ 500.

$\therefore$ income of Jawed becomes ₹ $(11x + 500)$.

Also, the ratio of their new incomes is given to be 3:2

$\therefore \frac{24x}{11x + 500} = \frac{3}{2}$

$\therefore 2 \times 24x = 3(11x + 500)$

$\therefore 48x = 33x + 1500$

$\therefore 48x - 33x = 1500$

$\therefore 15x = 1500$

$\therefore x = \frac{1500}{15} = 100$

$\therefore$ Original income of Salim = ₹ $20x$ = ₹ 20×100 = ₹ 2000.

and Original income of Jawed = ₹ $11x$ = ₹ 11×100 = ₹ 1100.

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4. In a class, 60% students are boys and 40% are girls. By admitting 16 boys and 8 girls, the ratio of number of boys and girls becomes 8:5. What must be the number of boys and number of girls originally in the class? [July 17]

Solution:

Let the total number of students be x.

Given, 60% of the students are boys.

$\therefore$ total number of boys= 60% of $x = \frac{60}{100} \times x = \frac{3x}{5}$

and 40% of the students are girls.

$\therefore$ total number of girls = 40% of $x = \frac{40}{100} \times x = \frac{2x}{5}$

Now, 16 boys and 8 girls are admitted in the class.

$\therefore$ total number of boys in the class becomes $\frac{3x}{5} + 16 = \frac{3x + 80}{5}$

and total number of girls in the class becomes $\frac{2x}{5} + 8 = \frac{2x + 40}{5}$

Also, after the admission the ratio of number of boys to number of girls becomes 8 : 5.

$\therefore \frac{\frac{3x + 80}{5}}{\frac{2x + 40}{5}} = \frac{8}{5}$

$\therefore \frac{3x + 80}{2x + 40} = \frac{8}{5}$

$\therefore$ $5(3x + 80) = 8(2x + 40)$

$\therefore$ $15x + 400 = 16x + 320$

$\therefore$ $400 - 320 = 16x - 15x$

$\therefore$ $x = 80$

$\therefore$ total number of boys that were originally present in the class = 60% of 80 = $\frac{60}{100} \times 80 = 48$

and total number of girls that were originally present in the class = 40% of 80 = $\frac{40}{100} \times 80 = 32$

$\therefore$ 48 boys and 32 girls were originally present in the class.

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5. Incomes of Mr. Shah, Mr. Patel and Mr. Mehta are in the ratio 1:2:3, while their expenditure are in the ratio 2:3:4. If Mr. Shah saves 20% of his income, find the ratio of their savings. [Oct 14]

Solution:

Let x and y be the proportionality constants.

Since, incomes of Mr. Shah, Mr. Patel and Mr. Mehta are in the ratio 1:2:3.

$\therefore$ their incomes are ₹ x, ₹ $2x$ and ₹ $3x$ respectively.

Also, their expenditures are in the ratio 2:3:4.

$\therefore$ their expenditures are ₹ $2y$, ₹ $3y$ and ₹ $4y$ respectively.

$\therefore$ the savings of Mr. Shah is ₹ $(x - 2y)$, Mr. Patel is ₹ $(2x - 3y)$ and that of Mr. Mehta is ₹ $(3x - 4y)$

Given, Mr. Shah saves 20% of his income.

$\therefore$ $x - 2y = 20\%$ of x

$\therefore$ $x - 2y = \frac{20}{100} \times x$

$\therefore$ $x - 2y = \frac{x}{5}$ $\quad\therefore$ $x - \frac{x}{5} = 2y$

$\therefore$ $\frac{4x}{5} = 2y$

$\therefore$ $y = \frac{2x}{5}$ ….(i)

Now, saving of Mr. Shah = $x - 2y$

$= x - 2\left(\frac{2x}{5}\right)$ ….[From (i)]

$= x - \frac{4x}{5} = ₹\,\frac{x}{5}$

Saving of Mr. Patel $= 2x - 3y$

$= 2x - 3\left(\frac{2x}{5}\right)$ ….[From (i)]

$= 2x - \frac{6x}{5} = ₹\,\frac{4x}{5}$

and Saving of Mr. Mehta = $3x - 4y$

$= 3x - 4\left(\frac{2x}{5}\right)$ ….[From (i)]

$= 3x - \frac{8x}{5} = ₹\,\frac{7x}{5}$

$\therefore$ The ratio of their savings is $\frac{x}{5} : \frac{4x}{5} : \frac{7x}{5}$

i.e., in the ratio $x : 4x : 7x$

i.e., in the ratio 1 : 4 : 7.

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6. What must be subtracted from each of the numbers 5, 7 and 10, so that the resulting numbers are in continued proportion?

Solution:

Let x be the number which is to be subtracted from each of the numbers 5, 7 and 10.

$\therefore$ the required numbers are $5 - x$, $7 - x$, $10 - x$

Since these numbers are in continued proportion.

$\therefore$ $\frac{5-x}{7-x} = \frac{7-x}{10-x}$

$\therefore$ $(7 - x)^2 = (5 - x)(10 - x)$

$\therefore$ $49 - 14x + x^2 = 50 - 5x - 10x + x^2$

$\therefore$ $49 - 14x = 50 - 15x$

$\therefore$ $15x - 14x = 50 - 49$

$\therefore$ $x = 1$

$\therefore$ 1 must be subtracted from each of the numbers 5, 7 and 10, so that the resulting numbers are in continued proportion.

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7. The employees of a firm have maintained their standard of living in such a manner, that they all have identical percentage of saving from their salaries. Amina and Sabina are two employees of the firm. Amina spends ₹ 12,800 per month from her salary of ₹ 35,000 per month. What would be Sabina's saving per month from her salary of ₹ 48,000 per month?

Solution:

Given, Amina's expenditure = ₹ 12,800 p.m. and her salary = ₹ 35,000 p.m.

$\therefore$ Amina's saving = 35,000 – 12,800 = ₹ 22,200 p.m.

$\therefore$ Percentage of Amina's savings

$= \frac{\text{Amina's Saving}}{\text{Amina's Total salary}} \times 100$

$= \frac{22{,}200}{35{,}000} \times 100 = \frac{444}{7}\,\%$