Written as per the revised syllabus prescribed by the Maharashtra State Board of Secondary and Higher Secondary Education, Pune.

Std. XI Commerce
Mathematics & Statistics - I

Salient Features

• Exhaustive coverage of entire syllabus.
• Topic-wise distribution of all textual questions and practice problems at the beginning of every chapter.
• Covers answers to all textual and miscellaneous exercises.
• Precise theory for every topic.
• Neat, labelled and authentic diagrams.
• Relevant and important formulae wherever required.
• Practice problems and Multiple Choice Questions for effective preparation.

Printed at: Repro India Ltd. Mumbai

© Target Publications Pvt. Ltd.
No part of this book may be reproduced or transmitted in any form or by any means, C.D. ROM/Audio Video Cassettes or electronic, mechanical including photocopying, recording or by any information storage and retrieval system without permission in writing from the Publisher.
**Preface**

Mathematics is not just a subject that is restricted to the four walls of a classroom. Its philosophy and applications are to be looked for in the daily course of our life. The knowledge of mathematics is essential for us, to explore and practice in a variety of fields like business administration, banking, stock exchange and in science and engineering.

With the same thought in mind, we present to you “Std. XI Commerce: Mathematics and Statistics-I” a complete and thorough book with a revolutionary fresh approach towards content and thus laying a platform for an in depth understanding of the subject. This book has been written according to the revised syllabus.

At the beginning of every chapter, topic–wise distribution of all textual questions including practice problems have been provided for simpler understanding of different types of questions. Neatly labelled diagrams have been provided wherever required. We have provided answer keys for all the textual questions and miscellaneous exercises. In addition to this, we have included practice problems based upon solved exercises which not only aid students in self evaluation but also provide them with plenty of practice. We’ve also ensured that each chapter ends with a set of Multiple Choice Questions so as to prepare students for competitive examinations.

We are sure this study material will turn out to be a powerful resource for students and facilitate them in understanding the concepts of Mathematics in the most simple way.

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we’ve nearly missed something or want to applaud us for our triumphs, we’d love to hear from you.

Please write to us on: mail@targetpublications.org

*Best of luck to all the aspirants!*

Yours faithfully
Publisher

---

**Contents**

<table>
<thead>
<tr>
<th>No.</th>
<th>Topic Name</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sets, Relations and Functions</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Complex Numbers</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>Sequence and Series</td>
<td>78</td>
</tr>
<tr>
<td>4</td>
<td>Angle and its Measurement</td>
<td>120</td>
</tr>
<tr>
<td>5</td>
<td>Trigonometric Functions</td>
<td>140</td>
</tr>
<tr>
<td>6</td>
<td>Plane Co-ordinate Geometry</td>
<td>235</td>
</tr>
<tr>
<td>7</td>
<td>Circle and Conics</td>
<td>289</td>
</tr>
<tr>
<td>8</td>
<td>Equations</td>
<td>339</td>
</tr>
<tr>
<td>9</td>
<td>Determinants</td>
<td>367</td>
</tr>
<tr>
<td>10</td>
<td>Limits</td>
<td>411</td>
</tr>
<tr>
<td>11</td>
<td>Differentiation</td>
<td>447</td>
</tr>
</tbody>
</table>
### Sets, Relations and Functions

<table>
<thead>
<tr>
<th>Type of Problems</th>
<th>Exercise</th>
<th>Q. Nos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>To describe sets in Roster form</td>
<td>1.1</td>
<td>Q.1 (i., ii., iii.)</td>
</tr>
<tr>
<td></td>
<td>Practice Problems (Based on Exercise 1.1)</td>
<td>Q.1 (i., ii., iii.)</td>
</tr>
<tr>
<td>To describe sets in Set-Builder form</td>
<td>1.1</td>
<td>Q.2 (i., ii., iii.), Q.12(i. to iv.)</td>
</tr>
<tr>
<td></td>
<td>Practice Problems (Based on Exercise 1.1)</td>
<td>Q.2 (i., ii., iii.)</td>
</tr>
<tr>
<td></td>
<td>Miscellaneous</td>
<td>Q.1 (i., ii., iii.)</td>
</tr>
<tr>
<td></td>
<td>Practice Problems (Based on Miscellaneous)</td>
<td>Q.1(i., ii.)</td>
</tr>
<tr>
<td>Operations on Sets</td>
<td>1.1</td>
<td>Q.3 to Q.11</td>
</tr>
<tr>
<td></td>
<td>Practice Problems (Based on Exercise 1.1)</td>
<td>Q.3 to Q.10, Q.13 (i., ii.)</td>
</tr>
<tr>
<td></td>
<td>Miscellaneous</td>
<td>Q.2, 3, 4</td>
</tr>
<tr>
<td></td>
<td>Practice Problems (Based on Miscellaneous)</td>
<td>Q.2, 3, 4</td>
</tr>
<tr>
<td>Ordered Pairs</td>
<td>1.2</td>
<td>Q.1, 2, 6, 11</td>
</tr>
<tr>
<td></td>
<td>Practice Problems (Based on Exercise 1.2)</td>
<td>Q.1, 2, 5, 9</td>
</tr>
<tr>
<td>Cartesian product of two Sets</td>
<td>1.2</td>
<td>Q.3, 4, 5</td>
</tr>
<tr>
<td></td>
<td>Practice Problems (Based on Exercise 1.2)</td>
<td>Q.3, 4</td>
</tr>
<tr>
<td></td>
<td>Miscellaneous</td>
<td>Q.5</td>
</tr>
<tr>
<td></td>
<td>Practice Problems (Based on Miscellaneous)</td>
<td>Q.5</td>
</tr>
<tr>
<td>To find domain and range of a given relation</td>
<td>1.2</td>
<td>Q.7, 8, 9, 10</td>
</tr>
<tr>
<td></td>
<td>Practice Problems (Based on Exercise 1.2)</td>
<td>Q.6, 7</td>
</tr>
<tr>
<td></td>
<td>Miscellaneous</td>
<td>Q.6, 7</td>
</tr>
<tr>
<td></td>
<td>Practice Problems (Based on Miscellaneous)</td>
<td>Q.6, 7</td>
</tr>
<tr>
<td>Types of Functions</td>
<td>1.2</td>
<td>Q.20, 21</td>
</tr>
<tr>
<td>--------------------</td>
<td>-----</td>
<td>----------</td>
</tr>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.18, 19</td>
</tr>
<tr>
<td>(Based on Exercise 1.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miscellaneous</td>
<td></td>
<td>Q.9, 10</td>
</tr>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.9, 10</td>
</tr>
<tr>
<td>(Based on Miscellaneous)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>To find values of the given function</th>
<th>1.2</th>
<th>Q.14, 16, 17, 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.12, 13, 15, 16</td>
</tr>
<tr>
<td>(Based on Exercise 1.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miscellaneous</td>
<td></td>
<td>Q.13 to Q.17</td>
</tr>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.12 to Q.16</td>
</tr>
<tr>
<td>(Based on Miscellaneous)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operations on functions</th>
<th>1.3</th>
<th>Q.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.1 (i. to iii.)</td>
</tr>
<tr>
<td>(Based on Exercise 1.3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Composite function</th>
<th>1.3</th>
<th>Q.2 to Q.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.2, 3, 4, 5</td>
</tr>
<tr>
<td>(Based on Exercise 1.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miscellaneous</td>
<td></td>
<td>Q.18 to Q.21</td>
</tr>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.17 to Q.20</td>
</tr>
<tr>
<td>(Based on Miscellaneous)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inverse function</th>
<th>1.3</th>
<th>Q.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.6</td>
</tr>
<tr>
<td>(Based on Exercise 1.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miscellaneous</td>
<td></td>
<td>Q.11, 12</td>
</tr>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.11</td>
</tr>
<tr>
<td>(Based on Miscellaneous)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>To find domain and range of a given function</th>
<th>1.2</th>
<th>Q.12, 13, 15, 19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.10, 11, 14, 17</td>
</tr>
<tr>
<td>(Based on Exercise 1.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miscellaneous</td>
<td></td>
<td>Q.8, 23, 24</td>
</tr>
<tr>
<td>Practice Problems</td>
<td></td>
<td>Q.8</td>
</tr>
<tr>
<td>(Based on Miscellaneous)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Syllabus:

1.1 Sets
1.2 Types of sets
1.3 Algebra of sets
1.4 Intervals
1.5 Cartesian product of sets
1.6 Relations
1.7 Functions
1.8 Particular types of functions and their graphs
1.9 Composite function
1.10 Inverse function
1.11 Functions in Economics
1.12 Some more functions and their graphs

Chapter 01: Sets, Relations and Functions

Methods of Representation of Sets

There are two methods of representing a set which are as follows:

i. **Roster method (Listing method):**
   In this method all the elements are listed or tabulated. The elements are separated by commas and are enclosed within two braces (curly brackets).

   **Example:**
   The set A of all positive even integers less than 9 can be written as A = {2, 4, 6, 8}.

ii. **Set-Builder method:**
   In this method, the set is described by the characteristic property of its elements. In general, if all the elements of set A satisfy some property P, then write A in set-builder notation as A = {x/x has property P} and read it as ‘A is the set of all x such that x has the property P’.

   **Example:**
   Let B = {3, 4, 5, 6, 7, 8}
   Using the set-builder method, B can be written as B = \{x/x \in \mathbb{N}, 3 \leq x \leq 8\}
   Since B = {3, 4, 5, 6, 7, 8} can also be stated as the set of natural numbers from 3 to 8 including 3 and 8.

Some standard sets are as follows:
N = set of all natural numbers
   = \{1, 2, 3,.....\}

Z or I = set of all integers
   = \{........-3, -2, -1, 0, 1, 2, 3 ....\}

Q = set of all rational numbers
   = \{p/q / p, q \in \mathbb{Z}, q \neq 0\}

1.2 Types of sets

1. **Empty set:**
   A set which does not contain any element is called an empty set and it is denoted by \(\emptyset\) or \{\}. It is also called null set.

   **Example:**
   If A = {2, 4, 6, 8}, then 4 \in A, 7 \notin A, 8 \in A, 10 \notin A
   The set of natural numbers, whole numbers, integers, rational numbers and real numbers are denoted by N, W, I, Q and R respectively.
2. **Singleton set:**
   A set which contains only one element is called a singleton set.
   **Example:**
   \[ A = \{5\}, \]
   \[ B = \{3\}, \]
   \[ X = \{x/x \in \mathbb{N}, 1 < x < 3\} \]
   The set \( A \) = set of all integers which are neither positive nor negative is a singleton set since \( A = \{0\} \)

3. **Finite set:**
   A set which contains countable number of elements is called a finite set.
   **Example:**
   \[ A = \{a, b, c\} \]
   \[ B = \{1, 2, 3, 4, 5\} \]
   \[ C = \{a, e, i, o, u\} \]

4. **Infinite set:**
   A set which contains uncountable number of elements is called an infinite set.
   **Example:**
   \[ N = \{1, 2, 3, 4 \ldots\} \]
   \[ Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \]

5. **Subset:**
   Set \( A \) is called a subset of set \( B \), if every element of set \( A \) is also an element of set \( B \)
   i.e., if \( x \in A \), then \( x \in B \).
   We denote this relation as \( A \subseteq B \) and read it as ‘\( A \) is a subset for \( B \)’. It’s clear that
   i. Every set is a subset of itself.
   ii. Empty set \( \emptyset \) is a subset of every set.
   **Example:**
   If \( A = \{2, 4, 6, 8\} \) and \( B = \{2, 4, 6, 8, 10, 12\} \), then \( A \subseteq B \).
   If \( A \subseteq B \), then \( B \) is called a superset of \( A \), denoted by \( B \supseteq A \).

6. **Proper subset:**
   A set \( A \) is said to be a proper subset of a set \( B \) if every element of set \( A \) is also an element of the set \( B \) and \( B \) contains at least one element which is not in \( A \). We denote it by \( A \subset B \).
   **Example:**
   1. \( A = \{x/x \text{ is a natural number less than } 5\} \)
      \( \therefore \ A = \{1, 2, 3, 4\} \)
   2. \( B = \{x/x \text{ is a divisor of } 12\} \)
      \( \therefore \ B = \{1, 2, 3, 4, 6, 12\} \)
      \( \therefore \ A \subset B \)

7. **Universal set:**
   A non-empty set of which all the sets under consideration are subsets, is called a universal set.
   It is usually denoted by \( X \) or \( U \).
   **Example:**
   If \( A = \{1, 2, 3, 4\} \), \( B = \{2, 8, 13, 15\} \) and \( C = \{1, 2, 3, \ldots, 50\} \) are sets under consideration, then the set \( N \) of all natural numbers can be taken as the universal set.

**Venn diagram:**
A set is represented by any closed figure such as circle, rectangle, triangle, etc. The diagrams representing sets are called venn diagrams.
**Example:**

8. **Equal sets:**
   Two sets \( A \) and \( B \) are said to be equal if they have the same elements and we denote this as \( A = B \). From this definition it follows that “two sets \( A \) and \( B \) are equal if and only if \( A \subseteq B \) and \( B \subseteq A \)”
   **Example:**
   If \( A = \{1, 2, 3, 4\} \), \( B = \{2, 4, 1, 3\} \), then \( A = B \).

9. **Complement of a set:**
   Let \( A \) be a subset of a universal set \( X \) then the set of all those elements of \( X \) which do not belong to \( A \) is called the complement of set \( A \) and it is denoted by \( A' \) or \( A^c \).
   Thus, \( A' = \{x/x \in X, x \notin A\} \)

The shaded region in the above figure represents \( A' \).
Example:
Let \( X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \) be an universal set and \( A = \{1, 3, 5, 6, 8\} \).
Then \( A' = \{2, 4, 7, 9\} \)

Properties:
If \( X \) is the universal set and \( A, B \subseteq U \), then
i. \((A')' = A\)
ii. \(X = \emptyset\)
iii. \(\emptyset' = X\)
iv. \(A \cap A' = \emptyset\)
v. \(A \cup A' = X\)

### 1.3 Algebra of sets

1. **Union of sets:**
If \( A \) and \( B \) are two sets, then the set of those elements which belong to \( A \) or to \( B \) or to both \( A \) and \( B \) is called the union of the sets \( A \) and \( B \) and is denoted by \( A \cup B \).

   \[ A \cup B = \{x : x \in A \text{ or } x \in B\} \]

   The shaded portion in the below venn diagram represents \( A \cup B \).

   ![Venn Diagram for Union](image)

   **Example:**
   i. If \( A = \{1, 2, 3, 4\} \), \( B = \{2, 4, 6, 8\} \), then \( A \cup B = \{1, 2, 3, 4, 6, 8\} \)
   ii. If \( A \) is the set of all odd integers and \( B \) is the set of all even integers, then \( A \cup B \) is the set of all integers.

   **Properties:**
   If \( A, B, C \) are any three sets, then
   i. \( A \cup \emptyset = A\)
   ii. \( A \cup X = A\)
   iii. \( A \cup B = B \cup A \) (Commutative law)
   iv. \((A \cup B) \cup C = A \cup (B \cup C)\) (Associative law)
   v. \( A \cup A = A\) (Idempotent law)
   vi. If \( A \subseteq B \), then \( A \cup B = B\)
   vii. \( (A \cup B) \subseteq A, (A \cup B) \subseteq B\)

2. **Intersection of sets:**
If \( A \) and \( B \) are two sets, then the set of those elements which belong to both \( A \) and \( B \) i.e., which are common to both \( A \) and \( B \) is called the intersection of the sets \( A \) and \( B \) and is denoted by \( A \cap B \).

   \[ A \cap B = \{x : x \in A \text{ and } x \in B\} \]

   The shaded portion in the below venn diagram represents \( A \cap B \).

   ![Venn Diagram for Intersection](image)

   **Example:**
   If \( A = \{1, 2, 3, 4, 5\} \), \( B = \{1, 3, 5, 7, 9\} \), then \( A \cap B = \{1, 3, 5\} \)

   **Properties:**
   If \( A, B, C \) are any three sets, then
   i. \( A \cap \emptyset = \emptyset\)
   ii. \( A \cap X = A\)
   iii. \( A \cap B = B \cap A \) (Commutative law)
   iv. \((A \cap B) \cap C = A \cap (B \cap C)\) (Associative law)
   v. \( A \cap A = A\) (Idempotent law)
   vi. If \( A \subseteq B \), then \( A \cap B = A\)
   vii. \( (A \cap B) \subseteq A, (A \cap B) \subseteq B\)

**Distributive Properties of union and intersection**
If \( a, b, c \in R \), then
\[ a \times (b + c) = (a \times b) + (a \times c) \]
This is known as distributive property of multiplication over addition.
In set theory, the operation of union and intersection of sets are both distributive over each other i.e.,
If \( A, B, C \) are any three sets, then
i. \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C)\)
ii. \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C)\)

We verify these distributive laws using Venn diagrams shown below. The shaded portion in each figure shows the set obtained by performing the operation given below the figure.

i. \( A \cup (B \cap C) \)
   ![Venn Diagram for Distributive Law](image)

ii. \( A \cap (B \cup C) \)
   ![Venn Diagram for Distributive Law](image)
De Morgan’s laws
If A and B are two subsets of a universal set X, then
i. \((A \cup B)' = A' \cap B'\)
ii. \((A \cap B)' = A' \cup B'\)

We verify these laws using Venn diagrams shown below. The shaded portion in each figure shows the set obtained by performing the operation below the figure:

\[ \begin{align*}
(A \cup B)' & = A' \cap B' \\
(A \cap B)' & = A' \cup B'
\end{align*} \]

Disjoint sets:
Two sets A and B are said to be disjoint, if they have no element in common i.e., \(A \cap B = \emptyset\).

Example:
If A = \{2, 4, 6\} and B = \{3, 5, 7\}, then \(A \cap B = \emptyset\).

\(\therefore\) A and B are disjoint sets.

The venn diagram of the disjoint sets A and B is shown below:

3. Difference of sets:
If A and B are two sets then the set of all the elements of A which are not in B is called difference of sets A and B and is denoted by \(A - B\).

Thus, \(A - B = \{x \in A \text{ and } x \notin B\}\)

Similarly, \(B - A = \{x \in B \text{ and } x \notin A\}\)

In the below venn diagrams, shaded region represents \(A - B\) and \(B - A\).

Example:
If A = \{1, 2, 3, 4, 5, 6\}, B = \{2, 4, 6, 8\}, then \(A - B = \{1, 3, 5\}\) and \(B - A = \{8\}\)

Note:
1. The sets \(A - B\), \(A \cap B\), \(B - A\) are mutually disjoint sets i.e. the intersection of any of these two sets is the null (empty) set.
2. \(A \cup B = (A - B) \cup (A \cap B) \cup (B - A)\)

Number of elements in a set:
Let A be a set. Then the total number of elements in it is denoted by \(n(A)\).

Example:
Let \(A = \{8, 9, 10, 11, 12\}\)

\(\therefore\) \(n(A) = 5\)

The number of elements in the empty set \(\emptyset\) is zero.

\(\therefore n(\emptyset) = 0\)

Results:
For given sets A, B
1. \(n(A \cup B) = n(A) + n(B) - n(A \cap B)\)
2. When A and B are disjoint sets then
\(n(A \cup B) = n(A) + n(B)\)
3. \(n(A - B) + n(A \cap B) = n(A)\)
4. \(n(B - A) + n(A \cap B) = n(B)\)
5. \(n(A - B) + n(A \cap B) + n(B - A) = n(A \cup B)\)
6. For any sets A, B, C
\[\begin{align*}
n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) \\
&\quad - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)
\end{align*}\]

Power set:
The set of all subsets of set A is called the power set of A and it is denoted by \(P(A)\).

Example:
If A = \{a, b, c\}, then
\(P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}\)

Note:
If A contains \(n\) elements, then the power set of A i.e., \(P(A)\) contains \(2^n\) elements.

1.4 Intervals
Open interval:
If \(p, q \in \mathbb{R}\) and \(p < q\), then the set \(\{x \in \mathbb{R}, p < x < q\}\) is called open interval and is denoted by \((p, q)\). Here all the numbers between \(p\) and \(q\) except \(p\) and \(q\).

\[ (p, q) = \{x \in \mathbb{R}, p < x < q\} \]
Closed interval:
If p, q ∈ R and p < q, then the set \{x/ x ∈ R, p ≤ x ≤ q\} is called closed interval and is denoted by \([p, q]\).

Here all the numbers between p and q \([p, q]\) including p, q.

\[\therefore \quad [p, q] = \{x/ x \in R, p \leq x \leq q\}\]

Semi-closed interval:
If p, q ∈ R and p < q, then the set \{x/ x \in R, p \leq x < q\} is called semi-closed interval and is denoted by \([p, q)\).

\[\therefore \quad (p, q) = \{x/ x \in R, p < x < q\}\]

\[\therefore \quad (p, q) \text{ includes } p \text{ but excludes } q.\]

Semi-open interval:
If p, q ∈ R and p < q, then the set \{x/ x \in R, p < x \leq q\} is called semi-open interval and is denoted by \((p, q]\).

\[\therefore \quad [p, q) = \{x/ x \in R, p < x \leq q\}\]

\[\therefore \quad [p, q) \text{ excludes } p \text{ but includes } q.\]

Remarks:

i. Set of all real numbers > p
   i.e., \((p, \infty) = \{x/ x \in R, x > p\}\)

\[\therefore \quad (p, \infty) \text{ excludes } p \text{ but includes } q.\]

ii. Set of all real numbers ≥ p
    i.e., \([p, \infty) = \{x/ x \in R, x \geq p\}\)

iii. Set of all real numbers ≤ q
    i.e., \((-\infty, q] = \{x/ x \in R, x \leq q\}\)

\[\therefore \quad (-\infty, q] \text{ includes } p \text{ and } q \text{ but excludes } p.\]

Exercise 1.1
1. Describe the following sets in Roster form:

i. \{x/ x is a letter of the word ‘MARRIAGE’\}
   \[A = \{M, A, R, I, G, E\}\]

ii. \{x/ x is an integer and \(-\frac{1}{2} < x < \frac{9}{2}\}\}
   \[B = \{0, 1, 2, 3, 4\}\]

iii. \{x/ x = 2n, x \in N, n \in N\}
    \[C = \{2, 4, 6, 8, \ldots\}\]

Solution:

i. Let A = \{x/ x is a letter of the word ‘MARRIAGE’\}
   \[A = \{M, A, R, I, G, E\}\]

ii. Let B = \{x/ x is an integer and \(-\frac{1}{2} < x < \frac{9}{2}\}\}
    \[B = \{0, 1, 2, 3, 4\}\]

iii. Let C = \{x/ x = 2n, n \in N\}
    \[C = \{2, 4, 6, 8, \ldots\}\]

2. Describe the following sets in Set-Builder form:

i. \{0\}

ii. \{0, ± 1, ± 2, ± 3\}

iii. \{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\}\]

Solution:

i. Let A = \{0\}
   0 is a whole number but it is not a natural number
   \[A = \{x/ x \in W, x \notin N\}\]

ii. Let B = \{0, ± 1, ± 2, ± 3\}
    B is the set of elements which belongs to Z from -3 to 3
    \[B = \{x/ x \in Z, -3 \leq x \leq 3\}\]

iii. Let C = \{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\}\]
    In the given set C, numerators are natural numbers from 1 to 7 and denominator = (numerator)² + 1
    \[C = \left\{\frac{n}{n^2 + 1}, n \in N, n \leq 7\right\}\]
3. If $A = \{x / 6x^2 + x - 15 = 0\}$
   $B = \{x / 2x^2 - 5x - 3 = 0\}$
   $C = \{x / 2x^2 - x - 3 = 0\}$

   Find
   i. $(A \cup B \cup C)$
   ii. $(A \cap B \cap C)$

   **Solution:**
   $A = \left\{ x / 6x^2 + x - 15 = 0 \right\}$
   $\therefore 6x^2 + x - 15 = 0$
   $\therefore 6x^2 + 10x - 9x - 15 = 0$
   $\therefore 2x(3x+5) - 3(3x+5) = 0$
   $\therefore (3x+5)(2x-3) = 0$
   $\therefore x = \frac{-5}{3}$ or $x = \frac{3}{2}$
   $\therefore A = \left\{ \frac{-5}{3}, \frac{3}{2} \right\}$

   $B = \{x/2x^2 - 5x - 3 = 0\}$
   $\therefore 2x^2 - 5x - 3 = 0$
   $\therefore 2x^2 - 6x + x - 3 = 0$
   $\therefore 2x(x-3) + 1(x-3) = 0$
   $\therefore (x-3)(2x+1) = 0$
   $\therefore x = 3$ or $x = \frac{-1}{2}$
   $\therefore B = \left\{ 3, \frac{-1}{2}, 3 \right\}$

   $C = \{x/2x^2 - x - 3 = 0\}$
   $\therefore 2x^2 - x - 3 = 0$
   $\therefore 2x^2 - 3x + 2x - 3 = 0$
   $\therefore x(2x-3) + 1(2x-3) = 0$
   $\therefore (2x-3)(x+1) = 0$
   $\therefore x = \frac{3}{2}$ or $x = -1$
   $\therefore C = \left\{ \frac{-1}{2}, \frac{3}{2} \right\}$

   Thus,
   i. $A \cup B \cup C = \left\{ \frac{-5}{3}, \frac{3}{2}, \frac{-1}{2}, \frac{3}{2} \right\}$

   ii. $A \cap B \cap C = \{ \}$

4. $A$, $B$, $C$ are the sets of the letters in the words ‘college’, ‘marriage’ and ‘luggage’ respectively, verify that

   $[A – (B \cup C)] = [(A – B) \cap (A – C)]$.

   **Solution:**
   $A = \{c, o, l, g, e\}$
   $B = \{m, a, r, i, g, e\}$
   $C = \{l, u, g, a, e\}$
   $B \cup C = \{m, a, r, i, g, e, l, u\}$
   $A \cup (B \cup C) = \{c, o\}$
   $A \cup (B \cup C) = \{c, o\}$
   $A \cup (B \cup C) = \{(A \cap B) \cap (A \cap C)\}$

5. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{4, 5, 6, 7, 8\}$ and universal set $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ verify the following:

   i. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   ii. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   iii. $A \cup B = (A \cap B) \cup (A \cap C)$
   iv. $A \cap (B \cup C) = (A \cap B) \cap (A \cap C)$
   v. $A = (A \cap B) \cup (A \cap C)$
   vi. $A \cap (B \cup C) = (A \cap B) \cap (A \cap C)$
   vii. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

   **Solution:**
   $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{4, 5, 6, 7, 8\}$
   $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
   i. $(B \cap C) = \{4, 5, 6\}$
   $A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\}$
   $A \cap (B \cup C) = \{1, 2, 3, 4, 5, 6\}$
   $A \cap (B \cup C) \cup (A \cap C) = \{1, 2, 3, 4, 5, 6\}$
   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   ii. $(B \cup C) = \{3, 4, 5, 6\}$
   $A \cap (B \cup C) = \{3, 4\}$
   $A \cap (B \cup C) = \{3, 4\}$
   $A \cap (B \cup C) = \{3, 4\}$
   $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$

   iii. $A \cup B = \{1, 2, 3, 4, 5, 6\}$
   $(A \cup B)' = \{7, 8, 9, 10\}$
   $A' = \{5, 6, 7, 8, 9, 10\}$
   $B' = \{1, 2, 7, 8, 9, 10\}$
   $A' \cap B' = \{7, 8, 9, 10\}$
   $(A \cup B)' = A' \cap B'$
iv. \( A \cap B = \{3, 4\} \)
\[ (A \cap B)' = \{1, 2, 5, 6, 7, 8, 9, 10\} \]
\[ A' = \{5, 6, 7, 8, 9, 10\} \]
\[ B' = \{1, 2, 7, 8, 9, 10\} \]

\[ A' \cup B' = \{1, 2, 5, 6, 7, 8, 9, 10\} \]
\[ (A \cap B)' = A' \cup B' \]

v. \( A \cap B = \{3, 4\} \)
\[ A \cap B = \{1, 2\} \]
\[ (A \cap B)' = \{3, 4, 5, 6\} \]
\[ A' = \{5, 6, 7, 8, 9, 10\} \]
\[ B' = \{1, 2, 7, 8, 9, 10\} \]

\[ (A \cap B) \cup (A' \cap B) = \{3, 4, 5, 6\} \]
\[ B = (A \cap B) \cup (A' \cap B) \]

vi. \( A \cap B = \{3, 4\} \)
\[ A' \cap B = \{5, 6\} \]
\[ (A \cap B) \cup (A' \cap B) = \{3, 4, 5, 6\} \]
\[ B = (A \cap B) \cup (A' \cap B) \]

vii. \( A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\} \)
\[ A \cap B = \{3, 4\} \]
\[ A \cup B = \{1, 2, 3, 4, 5, 6\} \]

\[ n(A) = 4, n(B) = 4, n(A \cap B) = 2, n(A \cup B) = 6 \]
\[ n(A) + n(B) - n(A \cap B) = 4 + 4 - 2 = 6 \]
\[ n(A \cup B) = n(A) + n(B) - n(A \cap B) \]

6. If \( A \) and \( B \) are subsets of the universal set \( X \) and \( n(X) = 50 \), \( n(A) = 35 \), \( n(B) = 20 \), \( n(A' \cap B') = 5 \), find

i. \( n(A \cup B) \)  
ii. \( n(A \cap B) \)  
iii. \( n(A' \cap B) \)  
iv. \( n(A \cap B') \)

**Solution:**
\[ n(X) = 50, n(A) = 35, n(B) = 20, n(A' \cap B') = 5 \]

i. \( n(A \cup B) = n(X) - [n(A \cap B')] \)
\[ = n(X) - n(A' \cap B') \]
\[ = 50 - 5 \]
\[ = 45 \]

ii. \( n(A \cap B) = n(A) + n(B) - n(A \cup B) \)
\[ = 35 + 20 - 45 \]
\[ = 10 \]

iii. \( n(A' \cap B) = n(B) - n(A \cap B) \)
\[ = 20 - 10 \]
\[ = 10 \]

iv. \( n(A \cap B') = n(A) - n(A \cap B) = 35 - 10 = 25 \)

7. In a class of 200 students who appeared in a certain examinations, 35 students failed in MHT-CET, 40 in AIEEE, 40 in IIT, 20 failed in MHT-CET and AIEEE, 17 in AIEEE and IIT, 15 in MHT-CET and IIT and 5 failed in all three examinations. Find how many students
i. did not fail in any examination.
ii. failed in AIEEE or IIT.

**Solution:**
\[ n(X) = 200, n(A) = 35, n(B) = 40, n(C) = 40, \]
\[ n(A \cap B) = 20, n(B \cap C) = 17, n(A \cap C) = 15, \]
\[ n(A \cap B) = 5 \]

i. \( n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \)
\[ = 35 + 40 + 40 - 20 - 17 - 15 + 5 \]
\[ = 68 \]

\[ \therefore \text{No. of students who did not fail in any exam} = n(X) - n(A \cup B \cup C) = 200 - 68 = 132 \]

ii. No. of students who failed in AIEEE or IIT
\[ n(B \cup C) = n(B) + n(C) - n(B \cap C) \]
\[ = 40 + 40 - 17 \]
\[ = 63 \]

8. From amongst 2000 literate individuals of a town, 70% read Marathi newspapers, 50% read English newspapers and 32.5% read both Marathi and English newspapers. Find the number of individuals who read
i. at least one of the newspapers.
ii. neither Marathi nor English newspaper
iii. only one of the newspapers.

**Solution:**
\[ n(X) = 2000, n(M) = \frac{70}{100} \times 2000 = 1400 \]
\[ n(E) = \frac{50}{100} \times 2000 = 1000 \]
\[ n(M \cap E) = \frac{32.5}{100} \times 2000 = 650 \]
n(M ∪ E) = n(M) + n(E) − n(M ∩ E)
= 1400 + 1000 − 650 = 1750

i. No. of individuals who read at least one of the newspapers = n(M ∪ E) = 1750.

ii. No. of individuals who read neither Marathi nor English newspaper
= n(M' ∩ E')
= n(M ∪ E)'
= n(X) − n(M ∪ E)
= 2000 − 1750 = 250

iii. No. of individuals who read only one of the newspaper = n(M ∩ E') + n(M' ∩ E)
= n(M ∪ E) − n(M ∩ E)
= 1750 − 650 = 1100

9. In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 students take both tea and coffee. 8 students take both milk and coffee. None of them take tea and milk both and everyone takes at least one beverage, find the number of students in the hostel.

Solution:
Let T = set of students who take tea
C = set of students who take coffee
M = set of students who take milk

n(T) = 25, n(C) = 20, n(M) = 15,
n(T ∩ C) = 10, n(M ∩ C) = 8, n(T ∩ M) = 0,
n(T ∩ M ∩ C) = 0

No. of students in the hostel
= n(T ∪ C ∪ M)
= n(T) + n(C) + n(M) − n(T ∩ C) − (M ∩ C) − (T ∩ M) + n(T ∩ M ∩ C)
= 25 + 20 + 15 − 10 − 8 − 0 + 0
= 42

10. There are 260 persons with a skin disorder. If 150 had been exposed to the chemical A, 74 to the chemical B, and 36 to both chemicals A and B, find the number of persons exposed to
i. Chemical A but not Chemical B
ii. Chemical B but not Chemical A
iii. Chemical A or Chemical B.

Solution:
Let A = set of persons exposed to chemical A
B = set of persons exposed to chemical B
X = set of all persons

n(X) = 260, n(A) = 150, n(B) = 74, n(A ∩ B) = 36

i. No. of persons exposed to chemical A but not to chemical B
= n(A ∩ B’)
= n(A) − n(A ∩ B)
= 150 − 36
= 114

ii. No. of persons exposed to chemical B but not to chemical A
= n(B) − n(A ∩ B)
= 74 − 36
= 38

iii. No. of persons exposed to chemical A or chemical B
= n(A ∪ B)
= n(A) + n(B) − n(A ∩ B)
= 150 + 74 − 36
= 188

11. If A = {1, 2, 3}, write down all possible subsets of A i.e., the power set of A.

Solution:
A = {1, 2, 3}
The power set of A is given by
P(A) = {φ, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}}

12. Write the following intervals in Set-Builder form:

i. (−3, 0) ii. [6, 12] iii. (6, 12] iv. [−23, 5).

Solution:

i. Let A = (−3, 0)
A = {x/x ∈ R and −3 < x < 0}

ii. Let B = [6, 12]
B = {x/x ∈ R and 6 ≤ x ≤ 12}

iii. Let C = (6, 12]
C = {x/x ∈ R and 6 < x ≤ 12}

iv. Let D = [−23, 5]
D = {x/x ∈ R and −23 ≤ x ≤ 5}
13. In the Venn diagram below shade
i. \((A \cup B)'\)  
ii. \(A' \cap B\)  
iii. \(A' \cup B'\)  
iv. \(A' - B'\)

**Solution:**

i. \((A \cup B)'

ii. \(A' \cap B\)

iii. \(A' \cup B'\)

iv. \(A' - B'\)

14. In the Venn-diagram below, shade
i. \(A' \cap (B \cup C)\)  
ii. \((A \cup B) \cap (A' \cup C')\)

**Solution:**

i. \(A' \cap (B \cup C)\)

ii. \((A \cup B) \cap (A' \cup C')\)
Ordered Pair:

If \((a, b)\) is a pair of numbers then the order in which the numbers appear is important, is called an ordered pair. Ordered pairs \((a, b)\) and \((b, a)\) are different. Two ordered pairs \((a, b)\) and \((c, d)\) are equal, if and only if \(a = c\) and \(b = d\).

Also, \((a, b) = (b, a)\) if and only if \(a = b\).

1.5 Cartesian product of two sets

Let \(A\) and \(B\) be any two non-empty sets. The set of all ordered pairs \((a, b)\) such that \(a \in A\) and \(b \in B\) is called the cartesian product of \(A\) and \(B\) and is denoted by \(A \times B\).

Thus, \(A \times B = \{(a, b) / a \in A, b \in B\}\)

where \(a\) is called the first element and \(b\) is called the second element of the ordered pair \((a, b)\).

If \(A \neq B\), then \(A \times B \neq B \times A\)

If \(A = \phi\) or \(B = \phi\) or both \(A\) and \(B\) are empty sets, then \(A \times B = \phi\).

Example:

If \(A = \{1, 2, 3\}, B = \{a, b\}\), then

\[A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}\]

1.6 Relations

Consider the following statements:

i. Ram is taller than Shyam.

ii. Harshal and Ravi have shirts of same colour.

iii. 25 is the square of 5.

iv. 2 and 4 are even integers.

Here we can say that Ram is related to Shyam by the relation “is taller than”. Harshal is related to Ravi by the relation “have shirts of same colour”. 25 is related to 5 by the relation “is the square of” and 2 is related to 4 by the relation “are even integers”.

Definition:

If \(A\) and \(B\) are two non-empty sets, then any subset of \(A \times B\) is called relation from \(A\) to \(B\) and is denoted by capital letters \(P, Q, R, \ldots\).

Consider the following illustration:

Let us consider \(A = \{a, b, c\}, B = \{l, m, n, o, p\}\)

\[A \times B = \{(a, l), (a, m), (a, n), (a, o), (a, p), (b, l), (b, m), (b, n), (b, o), (b, p), (c, l), (c, m), (c, n), (c, o), (c, p)\}\]

In the above figure, the arrow starting from the element ‘\(a\)’ and pointing to the elements ‘\(l\)’ and ‘\(n\)’ indicates that ‘\(a\)’ is related to ‘\(l\)’ and ‘\(n\)’. Similarly, ‘\(b\)’ is related to ‘\(m\)’ and ‘\(o\)’ and ‘\(c\)’ is related to ‘\(p\)’.

This relation is also represented by set of ordered pairs, \(R = \{(a, l), (a, n), (b, m), (b, o), (c, p)\}\)

This relation \(R\) is a subset of \(A \times B\). Thus, relation from set \(A\) to \(B\) is a subset of \(A \times B\).

If \(R\) is a relation and \((x, y) \in R\), then it is denoted by \(xRy\).

\(y\) is called image of \(x\) under \(R\) and \(x\) is called pre-image of \(y\) under \(R\).

Domain:
The set of all first components of the ordered pairs in a relation \(R\) is called the domain of the relation \(R\).

\[\text{domain } (R) = \{a / (a, b) \in R\}\]

Range:
The set of all second components of the ordered pairs in a relation \(R\) is called the range of the relation \(R\).

\[\text{range } (R) = \{b / (a, b) \in R\}\]

Co-domain:

If \(R\) is a relation from \(A\) to \(B\), then set \(B\) is called the co-domain of the relation \(R\).

Binary relation on a set:

Let \(A\) be non-empty set then every subset of \(A \times A\) is binary relation on \(A\).

Types of Relation

i. One-One relation:

If every element of \(A\) has at most one image in \(B\) and distinct elements in \(A\) have distinct images in \(B\), then a relation \(R\) from \(A\) to \(B\) is said to be one-one.

Example:

Let \(A = \{3, 4, 5, 6\}, B = \{4, 5, 6, 7, 9\}\)

and \(R = \{(3, 5), (4, 6), (5, 7)\}\)
Then R is a one-one relation from A to B. Here, domain of R = \{3, 4, 5\} and range of R = \{5, 6, 7\}

\section*{ii. Many-one relation:}
If two or more than two elements in A have same image in B, then a relation R from A to B is said to be many-one.

\textbf{Example:}
Let A = \{1, 2, 3, 4\}, B = \{2, 3, 4, 5, 6, 7\} and R = \{(1, 4), (3, 7), (4, 4)\}

Then R is a many-one relation from A to B. Here, domain of R = \{1, 3, 4\} and range of R = \{4, 7\}

\section*{iii. Into relation:}
If there exists at least one element in B which has no pre-image in A, then a relation R from A to B is said to be into relation.

\textbf{Example:}
Let A = \{-2, 1, 0, 1, 2, 3\}, B = \{0, 1, 2, 3, 4\} and R = \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}

Then R is into relation from A to B. Here, domain of R = \{-2, -1, 0, 1, 2\} and range of R = \{0, 1, 4\}

\section*{iv. Onto relation:}
If every element of B is the image of some element of A, then a relation R from A to B is said to be onto relation.

\textbf{Example:}
Let A = \{-2, -1, 1, 3, 4\}, B = \{1, 4, 9\} and R = \{(-2, 4), (-1, 1), (1, 1), (3, 9)\}

Then R is onto relation from A to B. Here, domain of R = \{-2, -1, 1, 3\} and range of R = \{1, 4, 9\}

\textbf{Note:}
\begin{enumerate}
  \item \(\emptyset \subseteq A \times A\) Here, \(\emptyset\) is a relation on A and is called the empty or void relation on A.
  \item \(A \times A \subseteq A \times A\) Here, \(A \times A\) is a relation on A called the universal relation on A. i.e. \(R = A \times A\)
  \item The total number of relations that can be defined from a set A to set B is the number of possible subsets of \(A \times B\).
\end{enumerate}

\subsection*{1.7 Functions}

\textbf{Definition:}
A function from set A to the set B is a relation which associates every element of a set A to unique element of set B and is denoted by \(f: A \rightarrow B\). If \(f\) is a function from A to B and \((x, y) \in f\), then we write it as \(y = f(x)\)

\textbf{Example:}
Let A = \{1, 2, 3, 4\}, B = \{2, 3, 4, 5, 6, 7, 8\}

Let a relation from A to B be given as “twice of” then we observe that every element \(x\) of set A is related to one and only one element of set B. Hence this relation is a function from set A to set B. In this
case \( f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8 \) are the values of function \( f(x) = 2x \) at \( x = 1, 2, 3, 4 \) respectively.

The set of all values of function \( f \) is \{2, 4, 6, 8\}. This set is called range of the function \( f \).

**Range:**

If \( f \) is a function from set \( A \) to set \( B \), then the set of all values of the function \( f \) is called the range of the function \( f \).

Thus the range set of the function \( f: A \to B \) is \( \{f(x) / x \in A\} \)

Note that the range set is a subset of co-domain. This subset may be proper or improper.

Consider the function \( f: A \to B \) represented by the following arrow diagram

In this case, range is equal to co-domain = \{l, m, n\}

Hence \( f: A \to B \) is onto function.

3. **Into function:**

If the function \( f: A \to B \) is such that there exists at least one element in \( B \) which is not the image of any element in \( A \), then \( f \) is said to be into function. In this case, the range of a function \( f \) is a proper subset of its co-domain.

Consider the function \( f: A \to B \) represented by the following arrow diagram.

In this case range = \{1, 5, 7, 9\} is a proper subset of co-domain \{1, 5, 7, 9, 13\}

Hence, \( f: A \to B \) is into function.

4. **Many-one function:**

If the function \( f: A \to B \) is such that two or more elements in a set \( A \) have the same image in set \( B \) i.e. there is at least one element in \( B \) which has more than one pre-image in \( A \) then the function \( f \) is called many-one function.

The function \( f: A \to B \) represented by the following arrow diagram is such that the co-domain \( B \) contains 1, 4 and 9 each of which is the value of the function \( f \) at two distinct elements of the domain set \( A \).
Chapter 01: Sets, Relations and Functions

Representation of functions

1. **Arrow diagram:**
   In this diagram, we use arrows. Arrows start from the element of domain and point out its value.

   ![Arrow Diagram](image)

2. **Starting the rule: (In terms of formula):**
   This is the most usual way of exhibiting function.

   Let \( A = \{1, 2, 3, 4, 5\} \), \( B = \{5, 7, 9, 11, 13\} \) and \( f: A \rightarrow B \) is a function represented by arrow diagram.

   In this case we observe that, if we take any element \( x \) of the set \( A \), then the element of the set \( B \) related to \( x \) is obtained by adding 3 to twice of \( x \). Applying this rule we get in general \( f(x) = 2x + 3 \), for all \( x \in A \).

   This is the formula which exhibits the function \( f \).

   If we denote the value of \( f \) at \( x \) by \( y \), then we get \( y = 2x + 3 \), for all \( x \in A \).

3. **Function as a set of ordered pairs:**
   Consider the function \( f(x) = 3x + 5 \), where \( A = \{0, 1, 2, 3\} \), \( B = \{5, 8, 11, 14\} \) we can form set of ordered pairs, as
   \[ \{(0, 5), (1, 8), (2, 11), (3, 14)\} \]

   In each of the ordered pairs, first component is an element of set \( A \) and second component is an element of set \( B \).

   \[ f = \{(0, 0), (1, 8), (2, 11), (3, 14)\} \]

   function \( f \) is subset of \( A \times B \).

   Here, we observed that no two pairs of this set have the same first component.

4. **Tabular form:**
   If the sets \( A \) and \( B \) are finite and contain very few elements, then a function \( f: A \rightarrow B \) can be exhibited by means of a table of corresponding elements.

   Let \( f = \{(1, 7), (2, 9), (3, 11), (4, 13)\} \)

   We can represent the above function in tabular form as follows:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>13</td>
</tr>
</tbody>
</table>

   **Real valued function:**
   A function whose co-domain is a set of real numbers \( R \), is called a real valued function.

   Henceforth, we will deal with only real valued functions.

**Exercise 1.2**

1. If \( (x - 1, y + 4) = (1, 2) \), find the values of \( x \) and \( y \).

   **Solution:**
   By the definition of equality of ordered pairs, we have
   \[ (x - 1, y + 4) = (1, 2) \]
   \[ \therefore x - 1 = 1 \text{ and } y + 4 = 2 \]
   \[ \therefore x = 2 \text{ and } y = -2 \]

2. \[ \left( x + \frac{1}{3}, y - 1 \right) = \left( \frac{1}{2}, \frac{3}{2} \right) \], find \( x \) and \( y \).

   **Solution:**
   By the definition of equality of ordered pairs, we have
   \[ \left( x + \frac{1}{3}, y - 1 \right) = \left( \frac{1}{2}, \frac{3}{2} \right) \]
   \[ \therefore x + \frac{1}{3} = \frac{1}{2} \text{ and } y - 1 = \frac{3}{2} \]
   \[ \therefore x = \frac{1}{6} \text{ and } y = 5 \]

3. If \( A = \{a, b, c\}, B = \{x, y\} \), find \( A \times B, B \times A, A \times A, B \times B \).

   **Solution:**
   \[ A = \{a, b, c\}, B = \{x, y\} \]
   \[ A \times B = \{(a, x), (a, y), (b, x), (b, y), (c, x), (c, y)\} \]
   \[ B \times A = \{(x, a), (x, b), (x, c), (y, a), (y, b), (y, c)\} \]
   \[ A \times A = \{(a, a), (a, b), (a, c) (b, a), (b, b), (b, c), \}
   \[ (c, a), (c, b) (c, c)\} \]
   \[ B \times B = \{(x, x), (x, y), (y, x), (y, y)\} \]
4. If \( P = \{1, 2, 3\} \) and \( Q = \{4\} \), find sets \( P \times Q \) and \( Q \times P \).

**Solution:**

\( P = \{1, 2, 3\} \), \( Q = \{4\} \)

\[ \therefore P \times Q = \{(1, 4), (2, 4), (3, 4)\} \]

and \( Q \times P = \{(4, 1), (4, 2), (4, 3)\} \)

5. Let \( A = \{1, 2, 3, 4\} \), \( B = \{4, 5, 6\} \), \( C = \{5, 6\} \)
Find

i. \( A \times (B \cap C) \)

ii. \( (A \times B) \cap (A \times C) \)

iii. \( A \times (B \cup C) \)

iv. \( (A \times B) \cup (A \times C) \)

**Solution:**

\( A = \{1, 2, 3, 4\} \), \( B = \{4, 5, 6\} \), \( C = \{5, 6\} \)

i. \( B \cap C = \{5, 6\} \)

\[ \therefore A \times (B \cap C) = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\} \]

ii. \( A \times B = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\} \)

\( A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\} \)

\[ \therefore (A \times B) \cap (A \times C) = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\} \]

iii. \( B \cup C = \{4, 5, 6\} \)

\( A \times (B \cup C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\} \)

iv. \( (A \times B) \cup (A \times C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\} \)

6. Express \( \{(x, y) \mid x^2 + y^2 = 100\} \), where \( x, y \in \mathbb{W} \) as a set of ordered pairs.

**Solution:**

\( \{(x, y) \mid x^2 + y^2 = 100\} \), where \( x, y \in \mathbb{W} \)

We have, \( x^2 + y^2 = 100 \)

When \( x = 0, y = 10 \Rightarrow x^2 + y^2 = 0^2 + 10^2 = 100 \)

When \( x = 6, y = 8 \Rightarrow x^2 + y^2 = 6^2 + 8^2 = 100 \)

When \( x = 8, y = 6 \Rightarrow x^2 + y^2 = 8^2 + 6^2 = 100 \)

When \( x = 10, y = 0 \Rightarrow x^2 + y^2 = 10^2 + 0^2 = 100 \)

\[ \therefore \text{Set of ordered pairs} = \{(0, 10), (6, 8), (8, 6), (10, 0)\} \]

7. Write the domain and range of the following relations:

i. \( \{(a, b) \mid a \text{ is a natural number less than 6 and } b = 4\} \)

ii. \( \{(a, b) \mid a \text{ and } b \text{ are natural numbers and } a + b = 12\} \)

iii. \( \{(2, 4), (2, 5), (2, 6), (2, 7)\} \)

**Solution:**

i. Let \( R_1 = \{(a, b) \mid a \in \mathbb{N}, a < 6 \text{ and } b = 4\} \)

Set of values of ‘a’ are domain and set of values of ‘b’ are range

\( a \in \mathbb{N} \) and \( a < 6 \)

\[ \therefore \text{Domain } (R_1) = \{1, 2, 3, 4\} \]

\( \text{Range } (R_1) = \{4\} \)

ii. Let \( R_2 = \{(a, b) \mid a, b \in \mathbb{N} \text{ and } a + b = 12\} \)

Now, \( a, b \in \mathbb{N} \) and \( a + b = 12 \)

When \( a = 1, b = 11 \)

When \( a = 2, b = 10 \)

When \( a = 3, b = 9 \)

When \( a = 4, b = 8 \)

When \( a = 5, b = 7 \)

When \( a = 6, b = 6 \)

When \( a = 7, b = 5 \)

When \( a = 8, b = 4 \)

When \( a = 9, b = 3 \)

When \( a = 10, b = 2 \)

When \( a = 11, b = 1 \)

\[ \therefore \text{Domain } (R_2) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\} \]

\( \text{Range } (R_2) = \{11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1\} \)

iii. Let \( R_3 = \{(2, 4), (2, 5), (2, 6), (2, 7)\} \)

\( \text{Domain } (R_3) = \{2\} \)

\( \text{Range } (R_3) = \{4, 5, 6, 7\} \)

8. Let \( A = \{6, 8\} \) and \( B = \{1, 3, 5\} \).

Let \( R = \{(a, b) \mid a \in A, b \in B, a - b \text{ is an even}\} \)

Show that \( R \) is an empty relation from \( A \) to \( B \).

**Solution:**

\( A = \{6, 8\} \), \( B = \{1, 3, 5\} \)

\( R = \{(a, b) \mid a \in A, b \in B, a - b \text{ is an even}\} \)

\( a \in A \quad \therefore a = 6, 8 \)

\( b \in B \quad \therefore b = 1, 3, 5 \)

When \( a = 6 \) and \( b = 1, a - b = 5 \) which is odd

When \( a = 6 \) and \( b = 3, a - b = 3 \) which is odd

When \( a = 6 \) and \( b = 5, a - b = 1 \) which is odd

When \( a = 8 \) and \( b = 1, a - b = 7 \) which is odd

When \( a = 8 \) and \( b = 3, a - b = 5 \) which is odd

When \( a = 8 \) and \( b = 5, a - b = 3 \) which is odd

Thus, no set of values of \( a \) and \( b \) gives \( a - b \) even

\[ \therefore R \text{ is an empty relation from } A \text{ to } B. \]
9. Determine the domain and range of the following:
   i. \(R_1 = \{(a, a^2) / a \text{ is a prime number less than 15}\}\)
   ii. \(R_2 = \left\{ \left( a, \frac{1}{a} \right) / 0 < a \leq 5, a \in \mathbb{N} \right\}\)

Solution:
   i. \(R_1 = \{(a, a^2) / a \text{ is a prime number less than 15}\}\)
      \(\implies a = 2, 3, 5, 7, 11, 13\)
      \(\implies a^2 = 4, 9, 25, 49, 121, 169\)
      \(\implies R_1 = \{(2, 4), (3, 9), (5, 25), (7, 49), (11, 121), (13, 169)\}\)
      \(\implies \text{Domain (R}_1\text{)} = \{a/a \text{ is a prime number less than 15}\}\)
      \(= \{2, 3, 5, 7, 11, 13\}\)
      \(\text{Range (R}_1\text{)} = \{a^2/a \text{ is a prime number less than 15}\}\)
      \(= \{4, 9, 25, 49, 121, 169\}\)
   ii. \(R_2 = \left\{ \left( a, \frac{1}{a} \right) / 0 < a \leq 5, a \in \mathbb{N} \right\}\)
      \(\implies a = 1, 2, 3, 4, 5\)
      \(\implies \frac{1}{a} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\)
      \(\implies R_2 = \left\{ \left(1,1\right), \left(2,\frac{1}{2}\right), \left(3,\frac{1}{3}\right), \left(4,\frac{1}{4}\right), \left(5,\frac{1}{5}\right)\right\}\)
      \(\implies \text{Domain (R}_2\text{)} = \{a/0 < a \leq 5, a \in \mathbb{N}\}\)
      \(= \{1, 2, 3, 4, 5\}\)
      \(\text{Range (R}_2\text{)} = \left\{ \frac{1}{a} / 0 < a \leq 5, a \in \mathbb{N}\right\}\)
      \(= \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}\)

10. The domain of the relation
    \(R = \{(a, b) / b = a + 1, a \in 1, 0 < a < 5\}\).
    Find the range of R.

Solution:
    \(R = \{(a, b) / b = a + 1, a \in 1, 0 < a < 5\}\)
    \(\implies a = 1, 2, 3, 4\)
    \(\implies b = 2, 3, 4, 5\)
    \(\implies \text{Range (R)} = \{2, 3, 4, 5\}\)

11. Write the following relations as sets of ordered pairs and find which of them are functions.
    i. \(\{(x, y) / y = 3x, x \in \{1, 2, 3\}, y \in \{3, 6, 9, 12\}\}\)
    ii. \(\{(x, y) / y > x + 1, x \in \{1, 2\} \text{ and } y \in \{2, 4, 6\}\}\)
    iii. \(\{(x, y) / x + y = 3, x, y \in \{0, 1, 2, 3\}\}\)

Solution:
    i. \(f(x) = x^2\)
    ii. \(f(x) = \sqrt{(x-1)(3-x)}\)
    iii. \(f(x) = \frac{3-x}{x-3}\)
    iv. \(f(x) = \sqrt{9-x^2}\)

12. Find the domain and range of the following functions:
    i. \(f(x) = x^2\)
    ii. \(f(x) = \sqrt{(x-1)(3-x)}\)
    iii. \(f(x) = \frac{3-x}{x-3}\)
    iv. \(f(x) = \sqrt{9-x^2}\)

Solution:
    i. \(f(x) = x^2\)
    \(\implies \text{Domain } = \text{set of all real numbers}\)
    \(\text{Range } = \{x / x \in \mathbb{R} \text{ and } x \geq 0\}\)
ii. \( f(x) = \sqrt{(x-1)(3-x)} \)

For this to exist

\( (x-1)(3-x) \geq 0 \)

\( \therefore (x-1) \geq 0 \) and \( 3-x \geq 0 \)

\( \therefore x \geq 1 \) and \( 3 \geq x \)

\( \therefore x \geq 1 \) and \( x \leq 3 \)

\( \therefore 1 \leq x \leq 3 \)

\( \therefore x \in [1, 3] \)

or \( x - 1 \leq 0 \) and \( 3 - x \leq 0 \)

\( \therefore x \leq 1 \) and \( 3 \leq x \)

\( \therefore x \leq 1 \) and \( x \geq 3 \)

Which is not possible

\( \therefore \) Domain is \([1, 3]\) and

For range

Let \( y = f(x) = \sqrt{(x-1)(3-x)} \)

\( \therefore y^2 = (x-1)(3-x) \)

\( \therefore y^2 = -x^2 + 4x - 3 \)

\( \therefore x^2 - 4x + (3 + y^2) = 0 \)

Disc \( \geq 0 \) \( \quad \ldots \) (\( \because x \) is real)

\( (-4)^2 - 4(1)(3+y^2) \geq 0 \)

\( 16 - 12 - 4y^2 \geq 0 \)

\( 4y^2 \leq 4 \)

\( y^2 \leq 1 \)

\( -1 \leq y \leq 1 \)

\( -1 \leq f(x) \leq 1 \)

\( \therefore \) Range is \([-1, 1]\)

iii. \( f(x) = \frac{3-x}{x-3} \)

\( f(x) \) is not defined, when \( x - 3 = 0 \) i.e., when \( x = 3 \)

\( \therefore \) Domain of \( f = \mathbb{R} - \{3\} \)

Let \( y = f(x) = \frac{3-x}{x-3} \)

\( \therefore xy - 3y = 3 - x \)

\( \therefore xy + x = 3 + 3y \)

\( \therefore x(y+1) = 3 + 3y \)

\( \therefore x = \frac{3+3y}{y+1} \)

which is not defined, when \( y+1 = 0 \)

i.e., when \( y = -1 \)

\( \therefore \) Range of \( f = \mathbb{R} - \{-1\} \)

iv. \( f(x) = \sqrt{9-x^2} \)

\( f(x) \) is defined, when \( 9 - x^2 \geq 0 \)

\( \therefore 9 \geq x^2 \)

\( \therefore x^2 \leq 9 \)

\( \therefore x \leq 3 \) \( \text{and} \) \( x \geq -3 \)

\( \therefore -3 \leq x \leq 3 \)

\( \therefore \) Domain of \( f = [-3, 3] \)

Now, \( -3 \leq x \leq 3 \)

\( \therefore 0 \leq x^2 \leq 9 \)

\( \therefore 0 \geq -x^2 \geq -9 \)

\( \therefore 0 + 9 \geq 9 - x^2 \geq 9 - 9 \)

\( \therefore 0 \geq 9 - x^2 \leq 9 \)

\( \therefore -3 \leq \sqrt{9-x^2} \leq 3 \)

\( \therefore -3 \leq f(x) \leq 3 \)

\( \therefore \) Range of \( f = [-3, 3] \)

13. Find the range of each of the following functions:

i. \( f(x) = 3x - 4, \) for \(-1 \leq x \leq 3 \)

As \( -1 \leq x \leq 3 \)

\( \therefore -3 \leq 3x \leq 9 \)

\( \therefore -3 - 4 \leq 3x - 4 \leq 9 - 4 \)

\( \therefore -7 \leq 3x - 4 \leq 5 \)

\( \therefore -7 \leq f(x) \leq 5 \)

\( \therefore \) Range of \( f \) is \([-7, 5]\).

ii. \( f(x) = 9 - 2x^2, \) for \(-5 \leq x \leq 3 \)

As \(-5 \leq x \leq 3 \)

\( \therefore 0 \leq x^2 \leq 25 \)

\( \therefore 0 \leq 2x^2 \leq 50 \)

\( \therefore 0 \geq -2x^2 \geq -50 \)

\( \therefore 0 + 9 \geq 9 - 2x^2 \geq 9 - 50 \)

\( \therefore 9 \geq 9 - 2x^2 \geq -41 \)

\( \therefore 9 \geq f(x) \geq -41 \)

\( \therefore \) Range of \( f \) is \([-41, 9]\)

iii. \( f(x) = x^2 - 6x + 11, \) for all \( x \in \mathbb{R} \)

\( = (x^2 - 6x + 9) + 2 \)

\( = (x - 3)^2 + 2 \)

But \((x - 3)^2 \geq 0, \) for all \( x \in \mathbb{R} \)

\( \therefore (x - 3)^2 + 2 \geq 0 + 2 \)

\( \therefore f(x) \geq 2 \)

\( \therefore \) Range = \([2, \infty)\)
14. Solve the following:

i. \( f(x) = \frac{x^3 + 1}{x^2 + 1} \), find \( f(-3) \), \( f(-1) \)

\( f(-3) = \frac{(-3)^3 + 1}{(-3)^2 + 1} = \frac{-27 + 1}{9 + 1} = \frac{-26}{10} = -2.6 \)

\( f(-1) = \frac{(-1)^3 + 1}{(-1)^2 + 1} = \frac{-1 + 1}{1 + 1} = 0 \)

ii. \( f(x) = (x - 1)(2x + 1) \), find \( f(1) \), \( f(2) \), \( f(-3) \)

\( f(1) = (1 - 1)[2(1) + 1] = 0 \)

\( f(2) = (2 - 1)[2(2) + 1] = 5 \)

\( f(-3) = (-3 - 1)[2(-3) + 1] = 20 \)

iii. \( f(x) = 2x^2 - 3x - 1 \)

\( f(x + 2) = 2(x + 2)^2 - 3(x + 2) - 1 \)
\( = 2(x^2 + 4x + 4) - 3x - 6 - 1 \)
\( = 2x^2 + 8x + 8 - 3x - 7 \)
\( = 2x^2 + 5x + 1 \)

15. Which of the following relations are functions? Give reason, if it is a function. Determine its domain and range. Also express the function by a formula.

i. \( \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\} \)

ii. \( \{(2, 1), (3, 1), (5, 2)\} \)

iii. \( \{(2, 3), (3, 2), (2, 5), (5, 2)\} \)

iv. \( \{(0,0), (1, 1), (1,-1), (4, 2), (4,-2), (9, 3), (9, -3), (16, 4), (16, -4)\} \)

Solution:

i. Let \( f = \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\} \)

From fig. every element of set A is associated with unique element of set B

\( \therefore \) It is a function

Domain = \{2, 4, 6, 8, 10, 12, 14\}

Range = \{1, 2, 3, 4, 5, 6, 7\}

Also, each element of domain is half of the corresponding element of co-domain.

\( \therefore \) Function is \( y = \frac{x}{2} \)

ii. Let \( f = \{(2,1), (3, 1), (5, 2)\} \)

From fig. every element of set A is associated with unique element of set B

\( \therefore \) It is a function

Domain = \{2, 3, 5\} and Range = \{1, 2\}

The function cannot be expressed in formula.

iii. Let \( f = \{(2, 3), (3, 2), (2, 5), (5, 2)\} \)

Since ‘2’ has two images i.e. 3 and 5, therefore it is not a function

iv. Let \( f = \{(0,0), (1, 1), (1,-1), (4, 2), (4,-2), (9, 3), (9, -3), (16, 4), (16, -4)\} \)

\( \therefore \) ‘1’, ‘4’, ‘9’, ‘16’ have two images

\( \therefore \) It is not a function
16. Find $a$, if $f(x) = ax + 5$ and $f(1) = 8$.
Solution:
\[ f(x) = ax + 5 \text{ and } f(1) = 8 \]
\[ \therefore f(1) = a(1) + 5 \]
\[ \therefore 8 = a + 5 \]
\[ \therefore a = 3 \]

17. If $f(x) = f(3x - 1)$, for $f(x) = x^2 - 4x + 11$, find $x$.
Solution:
\[ f(x) = x^2 - 4x + 11 \]
Also, $f(x) = f(3x - 1)$
\[ \therefore x^2 - 4x + 11 = (3x - 1)^2 - 4(3x - 1) + 11 \]
\[ \therefore x^2 - 4x + 9x^2 - 6x + 1 = 12x + 4 \]
\[ \therefore 8x^2 - 14x + 5 = 0 \]
\[ \therefore 8x^2 - 4x - 10x + 5 = 0 \]
\[ \therefore 4x(2x - 1) - 5(2x - 1) = 0 \]
\[ \therefore (2x - 1)(4x - 5) = 0 \]
\[ \therefore x = \frac{1}{2} \text{ or } x = \frac{5}{4} \]

18. If $f(x) = x^3 - 3x + 4$, then find the value of $x$ satisfying $f(x) = f(2x + 1)$.
Solution:
\[ f(x) = x^3 - 3x + 4 \]
Also, $f(x) = f(2x + 1)$
\[ \therefore x^3 - 3x + 4 = (2x + 1)^3 - 3(2x+1) + 4 \]
\[ \therefore x^3 - 3x + 4 = 8x^3 + 4x + 1 - 6x + 3 + 4 \]
\[ \therefore 3x^3 + x - 2 = 0 \]
\[ \therefore 3x^3 + 3x - 2x - 2 = 0 \]
\[ \therefore 3x(x + 1) - 2(x + 1) = 0 \]
\[ \therefore (x + 1)(3x - 2) = 0 \]
\[ \therefore x = -1 \text{ or } x = \frac{2}{3} \]

19. Let $A = \{1, 2, 3, 4\}$ and $Z$ be the set of integers. Define $f: A \rightarrow Z$ by $f(x) = 3x + 7$.
Show that $f$ is a function from $A$ to $Z$. Also find the range of $f$.
Solution:
\[ A = \{1, 2, 3, 4\} \]
\[ f(x) = 3x + 7 \]
when $x = 1$, $f(1) = 3(1) + 7 = 10$
when $x = 2$, $f(2) = 3(2) + 7 = 13$
when $x = 3$, $f(3) = 3(3) + 7 = 16$
when $x = 4$, $f(4) = 3(4) + 7 = 19$
\[ \therefore f = \{(1, 10), (2, 13), (3, 16), (4, 19)\} \]
It is a function because each element in $A$ has one and only one image in $Z$
\[ \therefore \text{Range of } f = \{10, 13, 16, 19\} \]

20. Find whether following functions are one-one, onto or not:
\[ \text{i. } f: R \rightarrow R \text{ given by } f(x) = x^3 + 5 \text{ for all } x \in R \]
\[ \text{ii. } f: Z \rightarrow Z \text{ given by } f(x) = x^2 + 4 \text{ for all } x \in Z \]
Solution:
\[ \text{i. } \text{Let } f: R \rightarrow R \text{ given as } f(x) = x^3 + 5 \text{ for all } x \in R. \]
First we have to show that $f$ is one-one function
For this we have to show that
if $f(x_1) = f(x_2)$, then $x_1 = x_2$
Here, $f(x) = x^3 + 5$
Let $f(x_1) = f(x_2)$
\[ \therefore x_1^3 + 5 = x_2^3 + 5 \]
\[ \therefore x_1^3 = x_2^3 \]
\[ \therefore x_1 = x_2 \]
\[ \therefore f \text{ is one-one function} \]
Now we have to show that $f$ is onto.
For that we have to prove that for any $y \in \text{co-domain } R$, there exist an element $x \in \text{domain } R$ such that $f(x) = y$.
Let $y \in R$ be such that
\[ y = f(x) \]
\[ \therefore y = x^3 + 5 \]
\[ \therefore x^3 = y - 5 \]
\[ \therefore x = \sqrt[y-5]{y} \in R \]
\[ \therefore \text{for any } y \in \text{co-domain } R, \text{ there exist an element } x = \sqrt[y-5]{y} \in \text{domain } R \text{ such that } f(x) = y. \]
\[ \therefore f \text{ is onto function.} \]
\[ \therefore f \text{ is one-one onto function.} \]
\[ \text{ii. } \text{Let } f: Z \rightarrow Z \text{ given by } f(x) = x^2 + 4 \text{ for all } x \in Z \]
Let $x_1, x_2 \in R$ be such that $f(x_1) = f(x_2)$
\[ \therefore x_1^2 + 4 = x_2^2 + 4 \]
\[ \therefore x_1^2 = x_2^2 \]
\[ \therefore x_1 = \pm x_2 \]
\[ \therefore f \text{ is not one-one function.} \]
Here $0 \in \text{co-domain } Z$, but there does not exist $x \in \text{domain } Z$ such that $f(x) = 0$
\[ \therefore f \text{ is not onto function.} \]
\[ \therefore f \text{ is not one - one onto function.} \]
21. Find which of the functions are one-one onto, many-one onto, one-one into, many-one into. Justify your answer.

i.  \( f: \mathbb{R} \rightarrow \mathbb{R} \) given as \( f(x) = 3x + 7 \) for all \( x \in \mathbb{R} \)

ii. \( f: \mathbb{R} \rightarrow \mathbb{R} \) given as \( f(x) = x^2 \) for all \( x \in \mathbb{R} \)

iii. \( f = \{(1, 3), (2, 6), (3, 9), (4, 12)\} \) defined from \( A \) to \( B \) where \( A = \{1, 2, 3, 4\} \), \( B = \{3, 6, 9, 12, 15\} \).

**Solution:**

i. \( f: \mathbb{R} \rightarrow \mathbb{R} \) given as \( f(x) = 3x + 7 \) for all \( x \in \mathbb{R} \)

First we have to show that \( f \) is one-one. For this we have to show that if \( f(x_1) = f(x_2) \) then \( x_1 = x_2 \)

Let \( f(x_1) = f(x_2) \)

\[ 3x_1 + 7 = 3x_2 + 7 \]

\[ x_1 = x_2 \]

\( \therefore \) \( f \) is one-one function.

Now we have to show that \( f \) is onto function. For that we have to prove that for any \( y \in \text{co-domain } \mathbb{R} \), there exist an element \( x \in \text{domain } \mathbb{R} \) such that \( f(x) = y \).

Let \( y \in \mathbb{R} \) be such that \( y = f(x) \)

\[ y = 3x + 7 \]

\[ y - 7 = 3x \]

\[ x = \frac{y - 7}{3} \in \mathbb{R} \]

\( \therefore \) for any \( y \in \text{co-domain } \mathbb{R} \), there exist an element \( x = \frac{y - 7}{3} \in \mathbb{R} \) such that \( f(x) = y \).

\( \therefore \) \( f \) is onto function.

\( \therefore \) \( f \) is one-one onto function.

ii. \( f: \mathbb{R} \rightarrow \mathbb{R} \) given as \( f(x) = x^2 \) for all \( x \in \mathbb{R} \)

To find whether it is one-one or many-one

Let \( f(x_1) = f(x_2) \)

\[ x_1^2 = x_2^2 \]

\[ x_1 = \pm x_2 \]

\( \therefore \) \( f \) is not one-one function.

\( \therefore \) \( f \) is many one function

Now we have to show that \( f \) is onto function. For that we have to prove that for any \( y \in \text{co-domain } \mathbb{R} \), there exist an element \( x \in \text{domain } \mathbb{R} \) such that \( f(x) = y \).

Let \( y \in \mathbb{R} \) be such that \( y = f(x) \)

\( \therefore \) \( y = x^2 \)

\( \therefore \) \( x = \pm \sqrt{y} \notin \mathbb{R} \) if \( y < 0 \)

\( \therefore \) for any \( y \in \text{co-domain } \mathbb{R} \), there does not exist an element \( x \in \text{domain } \mathbb{R} \) such that \( f(x) = y \).

\( \therefore \) \( f \) is into function

Hence \( f \) is many-one into function.

iii. \( f = \{(1, 3), (2, 6), (3, 9), (4, 12)\} \) defined from \( A \) to \( B \) where \( A = \{1, 2, 3, 4\} \), \( B = \{3, 6, 9, 12, 15\} \)

\( f \) is defined from \( A \) to \( B \)

\( f: A \rightarrow B \)

Here each and every element of \( A \) have their distinct images in \( B \)

\( \therefore \) \( f \) is one-one function.

Also element 15 in the co-domain \( B \) don’t have any pre-image in the domain \( A \).

\( \therefore \) \( f \) is into function.

\( \therefore \) \( f \) is one-one into function.

1.8 Particular types of functions and their graphs

1. **Constant function:**

A function \( f \) defined by \( f(x) = k \), for all \( x \in \mathbb{R} \), where \( k \) is a constant, is called a constant function. The graph of a constant function is a line parallel to the X-axis, intersecting Y-axis at \((0, k)\).

For example, \( f(x) = 5 \) is a constant function.

2. **Identity function:**

The function \( f(x) = x \), where \( x \in \mathbb{R} \) is called an identity function. The graph of the identity function is the line which bisects the first and the third quadrants. Observe the following table of some values of \( f \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>(-3)</td>
<td>(-2)</td>
<td>(-1)</td>
<td>(0)</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
</tbody>
</table>
3. Polynomial function:
A function of the form
\[ f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n, \]
where \( n \) is a non-negative integer and \( a_0, a_1, a_2, \ldots, a_n \in \mathbb{R} \) is called a polynomial function.

Example:
\[ f(x) = x^2 - 2x - 3 \text{ for } x \in \mathbb{R} \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^2 - 2x - 3 )</td>
<td>5</td>
<td>0</td>
<td>-3</td>
<td>-4</td>
<td>-3</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

4. Rational function:
The function of the type \( \frac{f(x)}{g(x)} \), where \( f(x) \) and \( g(x) \) are polynomial functions of \( x \), defined in a domain, where \( g(x) \neq 0 \) is called a rational function.

5. Modulus function:
Let \( f: \mathbb{R} \rightarrow \mathbb{R} \), the function \( f(x) = |x| \) such that

\[ |x| = \begin{cases} 
  x, & \text{for } x \geq 0 \\
  -x, & \text{for } x < 0 
\end{cases} \]

is called modulus or absolute value function. The graph of the absolute value function consist of two rays having common end point origin and bisecting the first and second quadrant.

Consider table of same values of \( f(x) = |x| \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

6. Even function:
A function \( f \) is said to be an even function, if
\[ f(-x) = f(x) \text{ for all } x \in \mathbb{R} \]
Let \( f: \mathbb{R} \rightarrow \mathbb{R} : f(x) = x^2 \text{ for all } x \in \mathbb{R} \)
Domain of \( f = \mathbb{R} \), range of \( f = \{x / x \in \mathbb{R}, x \geq 0\} \)
Chapter 01: Sets, Relations and Functions

7. **Odd function:**

A function $f$ is called an odd function, if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$

Let $f: \mathbb{R} \to \mathbb{R}: f(x) = x^3$ for all $x \in \mathbb{R}$

Then, domain of $f = \mathbb{R}$ and range of $f = \mathbb{R}$.

We have

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x^3$</td>
<td>$-8$</td>
<td>$-1$</td>
<td>$0$</td>
<td>$1$</td>
<td>$8$</td>
</tr>
</tbody>
</table>

8. **Exponential function:**

Let $f: \mathbb{R} \to \mathbb{R}^+$. The function $f$ is defined by

$f(x) = a^x$, where $a > 0$, $a \neq 1$ is called an exponential function.

For example, $f(x) = 2^x$ is an exponential function.

9. **Logarithmic function:**

Let $a$ be a positive real number with $a \neq 1$, if $a^y = x$, $x \in \mathbb{R}$ then $y$ is called the logarithm of $x$ with base ‘$a$’ and we write it as $y = \log_a x$.

i.e. A function $f : \mathbb{R}^+ \to \mathbb{R}$ defined by $f(x) = \log_a x$ is called logarithmic function.

$\therefore f = \{(x, \log_a x)/x \in \mathbb{R}, a > 0, a \neq 1\}$
If \( f: A \rightarrow B \) and \( g: B \rightarrow C \) are two functions then the composite function of \( f \) and \( g \) is the function \( gof: A \rightarrow C \) given by \( (gof)(x) = g(f(x)) \), for all \( x \in A \).

Let \( z = g(y) \) then \( z = g(y) = g[f(x)] \in C \).

This shows that every element \( x \) of the set \( A \) is related to one and only one element \( z = g[f(x)] \) of \( C \). This gives rise to a function from the set \( A \) to the set \( C \). This function is called the composite of \( f \) and \( g \).

Note that \( (gof)(x) \neq (gof)(y) \).

### Inverse function

If a function \( f: A \rightarrow B \) is one-one and onto function defined by \( y = f(x) \), then the function \( g: B \rightarrow A \) defined by \( g(y) = x \) is called the inverse of \( f \) and is denoted by \( f^{-1} \).

Thus \( f^{-1}: B \rightarrow A \) is defined by \( x = f^{-1}(y) \).

We also write if \( y = f(x) \) then \( x = f^{-1}(y) \).

Note that if the function is not one-one nor onto, then its inverse does not exist.

The geometrical representation of the function is called graph of the function.

We know that a function can be expressed as a set of ordered pairs.

Let \( f: A \rightarrow B \) be a function, where \( A \) and \( B \) are non empty subsets of \( R \). Let \( (x, y) \) be an element of \( f \), where \( x \in A, y \in B \).

Since \( x, y \) are real numbers, we can plot the point \((x, y)\) in a plane by choosing a suitable co-ordinate system. On plotting all such ordered pairs from the set representing \( f \) we get a geometrical representation of the function. This is called graph of the function \( f \).
\[ 4x + 6 + 9x - 6 = 13x = x \]
\[ 6x + 9 - 6x + 4 = x \]
\[ \therefore \quad \text{fof}(x) = x \]
\[ \therefore \quad \text{fof is an identity function.} \]

4. If \( f(x) = \frac{3x + 2}{4x - 1}, \) \( x \neq \frac{1}{4} \) and \( g(x) = \frac{x + 2}{4x - 3}, \)
\( x \neq \frac{3}{4} \) prove that \( (gof)(x) = (fog)(x) = x. \)

Solution:
\[ (gof)(x) = g[f(x)] = \frac{3x + 2}{4x - 1} + 2 \]
\[ = \frac{3x + 2 + 8x - 2}{12x + 8 - 12x + 3} = \frac{11x}{11} = x \]
\[ (fog)(x) = f[g(x)] = f\left(\frac{x + 2}{4x - 3}\right) = \frac{3x + 2}{4x - 3} + 2 \]
\[ = \frac{3x + 6 + 8x - 6}{4x + 8 - 4x + 3} = \frac{11x}{11} = x \]
\[ \therefore \quad \text{(gof)(x) = (fog)(x) = x} \]

5. If \( f = \{(2, 4), (3, 6), (4, 8), (5, 10), (6, 12)\}, \)
\( g = \{(4, 13), (6, 19), (8, 25), (10, 31), (12, 37)\}, \)
find \( (gof). \)

Solution:
\( f = \{(2, 4), (3, 6), (4, 8), (5, 10), (6, 12)\} \)
\( g = \{(4, 13), (6, 19), (8, 25), (10, 31), (12, 37)\} \)
Let \( A = \{2, 3, 4, 5, 6\}, B = \{4, 6, 8, 10, 12\} \) and \( C = \{13, 19, 25, 31, 37\} \)
\[ \therefore \quad f(x) = 2x \quad \text{and} \quad g(x) = 3x + 1 \]
\[ \therefore \quad \text{(gof)(x) = g[f(x)] = 9(2x)} \]
\[ = 3(2x) + 1 = 6x + 1 \]
\[ \therefore \quad \text{(gof)(2) = 6(2) + 1 = 13} \]
\[ \text{(gof)(3) = 6(3) + 1 = 19} \]
\[ \text{(gof)(4) = 6(4) + 1 = 25} \]
\[ \text{(gof)(5) = 6(5) + 1 = 31} \]
\[ \text{(gof)(6) = 6(6) + 1 = 37} \]
\[ \therefore \quad \text{(gof) = \{(2, 13), (3, 19), (4, 25), (5, 31), (6, 37)\}} \]

6. Show that \( f: \mathbb{R} \to \mathbb{R} \) given by \( f(x) = 3x - 4 \) is one-one and onto. Find its inverse function.

Also find \( f^{-1}(9) \) and \( f^{-1}(-2). \)

Solution:
\( f: \mathbb{R} \to \mathbb{R} \) given by \( f(x) = 3x - 4 \)
Let \( x_1, x_2 \in \mathbb{R} \) be such that
\[ f(x_1) = f(x_2) \]
\[ \therefore \quad 3x_1 - 4 = 3x_2 - 4 \]
\[ \therefore \quad x_1 = x_2 \]
\[ \therefore \quad f \text{ is one-one function.} \]
Now we have to show that \( f \) is onto function.
Let \( y \in \mathbb{R} \) be such that \( y = f(x) \)
\[ \therefore \quad y = 3x - 4 \]
\[ \therefore \quad x = \frac{y + 4}{3} \in \mathbb{R} \]
\[ \therefore \quad \text{for any } y \in \text{ co-domain } \mathbb{R}, \text{ there exist an element} \]
\[ x = \frac{y + 4}{3} \in \mathbb{R} \text{ such that } f(x) = y. \]
\[ \therefore \quad f \text{ is onto function.} \]
\[ \therefore \quad f \text{ is a one-one onto function.} \]
\[ \therefore \quad f^{-1} \text{ exists} \]
\[ \therefore \quad f^{-1}(y) = \frac{y + 4}{3} \]
\[ \therefore \quad f^{-1}(x) = \frac{x + 4}{3} \]
\[ f^{-1}(9) = \frac{9 + 4}{3} = \frac{13}{3} \]
\[ f^{-1}(-2) = \frac{-2 + 4}{3} = \frac{2}{3} \]

1.11 Functions in Economics

1. Demand Function:
According to demand law, quantity of commodity \( X \) demanded is inversely related to the price of \( X \), where other things remain the same.

i.e., for higher price, the demand is less and for lower price, the demand is more.

Demand \( D \) is function of the price \( p \)

i.e., \( D = f(p) \)

According to marshall law, the price \( P \) is function of demand \( D \)

i.e., \( p = g(D) \)

Example:
\[ p = 10 - D^2, p = \frac{200}{D+5} - 6, \quad p = \frac{50}{D}, \text{ etc.} \]
The graph of Demand and price or quantity demanded X is as follows:

\[ Y \]

\[ X' \quad O \quad X \]

\[ Y' \]

Quantity Demanded of X

It shows the diagrammatic representation of the functional relationship between the price of quantity and demand of quantity.
Also, it shows an inverse or negative relationship between price and demand of quantity.

2. Supply Function:
Supply is also related to price like demand. If p is the price and S is supply for the good X, then price p is the function of supply S i.e., \( p = g(S) \)
Example:
\( p = 9 + 4S, \ p = 3S + S^2, \ etc. \)
The graph of supply and price for a good X is as follows:

\[ Y \]

\[ X' \quad O \quad X \]

\[ Y' \]

Quantity Supplied of X

Price and supply increase or decrease together i.e., supply S increases (decreases) with the price p. Thus, the supply curves are sloping upwards from left to right as shown in graph above.

3. Total Revenue Function:
The total revenue function is obtained from the demand and price.
If p is price and D is the demand for the goods. Then the total revenue (R) is given by
\[ R = pD \]

If Demand D is the function of price p i.e., \( D = f(p) \), then total revenue function can be expressed as a function of p.
i.e., \( R = p. f(p) \)
If price p is the function of Demand D i.e., \( p = g(D) \), then total revenue function can be expressed as a function of D
i.e., \( R = D. g(D) \)
Example:
If p is a linear function of D i.e. if \( p = 500 + 7D \)
Then the total revenue function R is given by
\[ R = pD \]
\[ = (500 + 7D)D \]
\[ R = 500D + 7D^2 \]
The graph of total revenue function is as shown below:

\[ Y \]

\[ X' \quad O \quad X \]

\[ Y' \]

Demand D

Total revenue curve is parabolic in nature.

4. Total Cost Function:
The total cost function is the combination of fixed cost and variable cost, for a quantity \( x \) of a certain good. Fixed cost does not depend on the quantity \( x \) of that good. It may be due to rent of the premises, expenses on the research laboratory, etc.
Variable cost depends on \( x \)
Cost function may be of the following type
\[ C = ax + b, \ C = ax^2 + bx + c, \] where a is positive.

5. Profit Function:
Profit function is the difference between the total revenue function and total cost function.
If R is the total revenue function and C is the total cost function, then the profit function (\( \pi \)) is given as
\[ \pi = R - C \]
1.12 Some more functions and their graphs

1. **Graph of Exponential Function:**
   Let \( f : \mathbb{R} \rightarrow \mathbb{R}^+ \). The function \( f \) is defined by \( f(x) = a^x \), where \( a > 0, a \neq 1 \) is called an exponential function.

   For example, \( f(x) = 2^x \) is an exponential function.

2. **Graph of \( y = \log_a x \):**
   Let \( a \) be a positive real number with \( a \neq 1 \), if \( a^y = x, x \in \mathbb{R} \) then \( y \) is called the logarithm of \( x \) with base ‘\( a \)’ and we write it as \( y = \log_a x \).

   i.e. A function \( f : \mathbb{R}^+ \rightarrow \mathbb{R} \) defined by \( f(x) = \log_a x \) is called logarithmic function.

   \[ f = \{ (x, \log_a x) | x \in \mathbb{R}, a > 0, a \neq 1 \} \]

3. **Graph of \( y = \sin x \):**
   The values of \( x \) and \( y = \sin x \) are given in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>( \pi/6 )</th>
<th>( \pi/4 )</th>
<th>( \pi/3 )</th>
<th>( \pi/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>0.5</td>
<td>0.71</td>
<td>0.87</td>
<td>1</td>
</tr>
</tbody>
</table>

   In the above table, we have assumed 
   \( \sqrt{2} = 1.42 \) and \( \frac{\sqrt{3}}{2} = 0.87 \)

   \[ \therefore \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 1.42 \times 0.71 = 0.71 \]

   Using the result \( \sin (-x) = -\sin x \), the table for the values of \( x \) between \( -\pi \) and 0 is obtained as follows:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -\pi )</th>
<th>( -5\pi/6 )</th>
<th>( -3\pi/4 )</th>
<th>( -2\pi/3 )</th>
<th>( -\pi/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>-0.5</td>
<td>-0.71</td>
<td>-0.87</td>
<td>-1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -\pi/3 )</th>
<th>( -\pi/4 )</th>
<th>( -\pi/6 )</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-0.87</td>
<td>-0.71</td>
<td>-0.5</td>
<td>0</td>
</tr>
</tbody>
</table>

   The graph corresponding to these points is as given below:

   Extension of the graph of \( \sin x \):
   Since \( \sin (\pi + x) = -\sin x \), one can also extend the graph of \( y = \sin x \) as shown below.
4.  **Graph of** \( y = \cos x \):

The values of \( x \) and \( y = \cos x \) are given in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>( \pi/6 )</th>
<th>( \pi/4 )</th>
<th>( \pi/3 )</th>
<th>( \pi/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>0.87</td>
<td>0.71</td>
<td>0.5</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2\pi/3 )</th>
<th>( 3\pi/4 )</th>
<th>( 5\pi/6 )</th>
<th>( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-0.5</td>
<td>-0.71</td>
<td>-0.87</td>
<td>-1</td>
</tr>
</tbody>
</table>

Using the result \( \cos (-x) = \cos x \), the table for the values of \( x \) between \(-\pi \) and 0 is obtained as follows:

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-\pi )</th>
<th>(-5\pi/6 )</th>
<th>(-3\pi/4 )</th>
<th>(-2\pi/3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td>-0.87</td>
<td>-0.71</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-\pi/2 )</th>
<th>(-\pi/3 )</th>
<th>(-\pi/4 )</th>
<th>(-\pi/6 )</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>0.5</td>
<td>0.71</td>
<td>0.87</td>
<td>1</td>
</tr>
</tbody>
</table>

The graph corresponding to these points is as given below:

**Extension of the graph of** \( \cos x \):

5.  **Graph of** \( y = \tan x \):

\( \tan x \) does not exist for \( x = \frac{\pi}{2} \) but as \( x \) increases from 0 to \( \frac{\pi}{2} \).

i.  \( \sin x \) increases from 0 to 1 and

ii. \( \cos x \) decreases from 1 to 0.

\[ \therefore \tan x = \frac{\sin x}{\cos x} \] will increase indefinitely as \( x \) approaches \( \frac{\pi}{2} \).

Similarly as \( x \) (starting from the value 0) approaches \( -\frac{\pi}{2} \), \( \tan x \) decreases indefinitely.

The corresponding value of \( x \) and \( y = \tan x \) are given in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-\pi/3 )</th>
<th>(-\pi/4 )</th>
<th>(-\pi/6 )</th>
<th>0</th>
<th>( \pi/6 )</th>
<th>( \pi/4 )</th>
<th>( \pi/3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1.73</td>
<td>-1</td>
<td>-0.58</td>
<td>0</td>
<td>0.58</td>
<td>1</td>
<td>1.73</td>
</tr>
</tbody>
</table>

The graph of \( y = \tan x \)

**Extension of the graph of** \( \tan x \):

When the graph of \( \tan x \) is extended to values beyond \( \frac{\pi}{2} \), the entire curve shown in below figure repeats completely for the intervals \( \left( \frac{\pi}{2}, \frac{3\pi}{2} \right) \), \( \left( \frac{3\pi}{2}, \frac{5\pi}{2} \right) \) etc. as well as for intervals \( \left( \frac{\pi}{2}, -\frac{3\pi}{2} \right) \), \( \left( -\frac{5\pi}{2}, -\frac{3\pi}{2} \right) \) etc.
Chapter 01: Sets, Relations and Functions

The extended graph is shown below:

Miscellaneous Exercise - 1

1. Write down the following sets in set-builder form:
   i. \( \{10, 20, 30, 40, 50\} \)
   ii. \( \{a, e, i, o, u\} \)
   iii. \( \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\} \)

   **Solution:**
   i. Let \( A = \{10, 20, 30, 40, 50\} \)
     \(\Rightarrow\) \( A = \{x / x = 10n, n \in \mathbb{N} \text{ and } n \leq 5\} \)
   ii. Let \( B = \{a, e, i, o, u\} \)
     \(\Rightarrow\) \( B = \{x / x \text{ is a vowel of alphabets}\} \)
   iii. Let \( C = \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\} \)
     \(\Rightarrow\) \( C = \{x / x \text{ represents days of a week}\} \)

2. If \( U = \{x / x \in \mathbb{N}, 1 \leq x \leq 12\} \)
   \( A = \{1, 4, 7, 10\}, B = \{2, 4, 6, 7, 11\} \)
   \( C = \{3, 5, 8, 9, 12\} \)

   Write down the sets
   i. \( (A \cup B) \)
   ii. \( (B \cap C) \)
   iii. \( A - B \)
   iv. \( B \cap C' \)
   v. \( A \cup B \cup C \)
   vi. \( A \cap (B \cup C) \)

   **Solution:**
   \( U = \{x / x \in \mathbb{N}, 1 \leq x \leq 12\} = \{1, 2, 3, \ldots, 12\} \)
   \( A = \{1, 4, 7, 10\}, B = \{2, 4, 6, 7, 11\} \)
   \( C = \{3, 5, 8, 9, 12\} \)
   i. \( A \cup B = \{1, 2, 4, 6, 7, 10, 11\} \)
   ii. \( B \cap C = \{\} = \phi \)
   iii. \( A - B = \{1, 10\} \)
   iv. \( C' = \{1, 2, 4, 6, 7, 10, 11\} \)
   v. \( A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \)
   vi. \( B \cup C = \{2, 3, 4, 5, 6, 7, 8, 9, 11, 12\} \)
   \(\Rightarrow\) \( A \cap (B \cup C) = \{4, 7\} \)

3. In a survey of 425 students in a school, it was found that 115 drink apple juice, 160 drink orange juice and 80 drink both apple as well as orange juice. How many students drink neither apple juice nor orange juice?

   **Solution:**
   Let \( A = \text{set of students who drink apple juice} \)
   \( B = \text{set of students who drink orange juice} \)
   \( X = \text{set of all students} \)
   \(\Rightarrow\) \( n(X) = 425, n(A) = 115, n(B) = 160, n(A \cap B) = 80 \)
   No. of students who neither drink apple juice nor orange juice
   \(= n(A' \cap B') = n(A \cup B)' \)
   \(= n(X) - n(A \cup B) \)
   \(= 425 - (115 + 160 - 80) \)
   \(= 230 \)

4. In a school there are 20 teachers who teach Mathematics or Physics. Of these, 12 teach Mathematics and 4 teach both Physics and Mathematics. How many teach Physics?

   **Solution:**
   Let \( A = \text{set of teachers who teach mathematics} \)
   \( B = \text{set of teachers who teach physics} \)
   \(\Rightarrow\) \( n(A \cup B) = 20, n(A) = 12, n(A \cap B) = 4 \)
   But \( n(A \cup B) = n(A) + n(B) - n(A \cap B) \)
   \(\Rightarrow\) \( 20 = 12 + n(B) - 4 \)
   \(\Rightarrow\) \( 12 = n(B) \)
   \(\therefore\) No. of teachers who teach physics = 12

5. If \( A = \{1, 2, 3\} \) and \( B = \{2, 4\} \),
   Write down following sets:
   \( A \times A, A \times B, B \times A, B \times B, (A \times B) \cap (B \times A) \)

   **Solution:**
   \( A = \{1, 2, 3\} \) and \( B = \{2, 4\} \)
   \( A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\} \)
   \( A \times B = \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)\} \)
   \( B \times A = \{(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)\} \)
   \( B \times B = \{(2, 2), (2, 4), (4, 2), (4, 4)\} \)
   \( (A \times B) \cap (B \times A) = \{(2, 2)\} \)
6. If A = \{1, 2, 3\} and B = \{4, 5, 6\}, which of the following are relations from A to B:
   i. \( R_1 = \{(1, 4), (1, 5), (1, 6)\} \)
   ii. \( R_2 = \{(1, 5), (2, 4), (3, 6)\} \)
   iii. \( R_3 = \{(1, 4), (1, 5), (3, 6), (2, 6), (3, 4)\} \)
   iv. \( R_4 = \{(4, 2), (2, 6), (5, 1), (2, 4)\} \)

**Solution:**
A = \{1, 2, 3\}, B = \{4, 5, 6\} 
\[ \therefore A \times B = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\} \]
   i. \( R_1 \subseteq A \times B \)
   \( \therefore R_1 \) is a relation from A to B
   ii. \( R_2 \subseteq A \times B \)
   \( \therefore R_2 \) is a relation from A to B
   iii. \( R_3 \subseteq A \times B \)
   \( \therefore R_3 \) is a relation from A to B
   iv. \( R_4 \nsubseteq A \times B \)
   \( \therefore R_4 \) is not a relation from A to B

7. Determine the domain and range of the following relations:
   i. \( R = \{(a, b) / a \in N, a < 5, b = 4\} \)
   ii. \( S = \{(a, b) / b = |a - 1|, a \in Z, |a| \leq 3\} \)

**Solution:**
   i. \( \text{Domain (R)} = \{1, 2, 3, 4\} \)
   \( \text{Range (R)} = \{4\} \)
   ii. \( \text{a} \leq 3 \text{ and } \text{a} \geq -3 \)
   \( -3 \leq \text{a} \leq 3 \)
   \( \text{a} = -3, -2, -1, 0, 1, 2, 3 \)
   \( \text{When a} = -3, \text{ b} = 4 \)
   \( \text{When a} = -2, \text{ b} = 3 \)
   \( \text{When a} = -1, \text{ b} = 2 \)
   \( \text{When a} = 0, \text{ b} = 1 \)
   \( \text{When a} = 1, \text{ b} = 0 \)
   \( \text{When a} = 2, \text{ b} = 1 \)
   \( \text{When a} = 3, \text{ b} = 2 \)
   \( \text{Domain (S)} = \{-3, -2, -1, 0, 1, 2, 3\} \)
   \( \text{Range (S)} = \{4, 3, 2, 1, 0\} \)

8. Which of the following relations are functions? If it is a function, determine its domain and range:
   i. \( \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\} \)
   ii. \( \{(0, 0), (1, 1), (1, -1), (4, 2), (4, -2), (9, 3), (9, -3), (16, 4), (16, -4)\} \)
   iii. \( \{(2, 1), (3, 1), (5, 2)\} \)

**Solution:**
   i. Refer Ex. 1.2 Q. 15 - i
   ii. Refer Ex. 1.2 Q. 15 - iv
   iii. Refer Ex. 1.2 Q. 15 - ii

9. Find whether following functions are one-one or not:
   i. \( f: R \rightarrow R \text{ defined by } f(x) = x^2 + 5 \)
   ii. \( f: R - \{3\} \rightarrow R \text{ defined by } f(x) = \frac{5x + 3}{x - 3} \)
   \( \text{for } x \in R - \{3\} \)

**Solution:**
   i. To prove that \( f \) is one-one we have to prove that if \( f(x_1) = f(x_2) \) then \( x_1 = x_2 \)
   Here \( f(x) = x^2 + 5 \)
   Let \( f(x_1) = f(x_2) \)
   \( \therefore x_1^2 + 5 = x_2^2 + 5 \)
   \( \therefore x_1^2 = x_2^2 \)
   \( \therefore x_1 = \pm x_2 \)
   \( \therefore f \) is not one-one function.
   ii. To prove that \( f \) is one-one we have to prove that if \( f(x_1) = f(x_2) \) then \( x_1 = x_2 \)
   Here, \( f(x) = \frac{5x + 3}{x - 3} \)
   Let \( f(x_1) = f(x_2) \)
   \( \therefore \frac{5x_1 + 3}{x_1 - 3} = \frac{5x_2 + 3}{x_2 - 3} \)
   \( \therefore (5x_1 + 3)(x_2 - 3) = (5x_2 + 3)(x_1 - 3) \)
   \( \therefore 5x_1x_2 - 15x_1 + 3x_2 - 9 = 5x_1x_2 - 15x_2 + 3x_1 - 9 \)
   \( \therefore -15x_1 + 3x_2 = -15x_2 + 3x_1 \)
   \( \therefore 18x_2 = 18x_1 \)
   \( \therefore x_1 = x_2 \)
   \( \therefore f \) is one-one function.
10. Find whether the following functions are onto or not:
   i.  \( f: \mathbb{Z} \to \mathbb{Z} \) defined by \( f(x) = 6x - 7 \) for all \( x \in \mathbb{Z} \).
   ii.  \( f: \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = x^2 + 3 \) for all \( x \in \mathbb{R} \).

Solution:
   i.  \( f: \mathbb{Z} \to \mathbb{Z} \) defined by \( f(x) = 6x - 7 \) for all \( x \in \mathbb{Z} \)
   We want to find whether \( f \) is a onto function.
   For that we have to prove that for any \( y \in \) co-domain \( \mathbb{Z} \), there exist an element \( x \in \) domain \( \mathbb{Z} \) such that \( f(x) = y \)
   Let \( y \in \mathbb{Z} \) be such that \( y = f(x) \)
   \[ y = 6x - 7 \]
   \[ 6x = y + 7 \]
   \[ x = \frac{y + 7}{6} \notin \mathbb{Z} \]
   For any \( y \in \) co-domain \( \mathbb{Z} \), there does not exist an element \( x \in \) domain \( \mathbb{Z} \) such that \( f(x) = y \)
   \( f \) is not onto function.

   ii.  \( f: \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = x^2 + 3 \) for all \( x \in \mathbb{R} \)
   \[ x^2 + 3 \geq 3 \]
   \[ f(x) \geq 3 \]
   Value of all element of domain \( \mathbb{R} \) is greater than or equal to 3
   This means that in co-domain for all the elements which are less than 3 will not have their pre-image in the domain.
   \( f \) is not onto function.

11. Let \( f: \mathbb{R} \to \mathbb{R} \) be a function defined by \( f(x) = 5x^3 - 8 \) for all \( x \in \mathbb{R} \), show that \( f \) is one-one and onto. Hence, find \( f^{-1} \).

Solution:
   \( f: \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = 5x^3 - 8 \)
   First, we have to prove that \( f \) one-one function
   For that we have to prove that if \( f(x_1) = f(x_2) \) then \( x_1 = x_2 \)
   \[ f(x_1) = 5x_1^3 - 8 \]
   \[ f(x_2) = 5x_2^3 - 8 \]
   \[ 5x_1^3 - 8 = 5x_2^3 - 8 \]
   \[ x_1^3 = x_2^3 \]
   \[ x_1 = x_2 \]
   \( f \) is one-one function
   Now we have to show that \( f \) is onto function.
   Let \( y \in \mathbb{R} \) be such that \( y = f(x) \)
   \[ y = 5x^3 - 8 \]
   \[ 5x^3 = y + 8 \]
   \[ x = \sqrt[3]{\frac{y + 8}{5}} \in \mathbb{R} \]
   for any \( y \in \) co-domain \( \mathbb{R} \), there exist an element \( x \in \) domain \( \mathbb{R} \) such that \( f(x) = y \)
   \( f \) is onto function.
   \( f \) is one-one onto function.
   \( f^{-1} \) exists
   \[ f^{-1}(y) = \frac{5(y - 2)}{3} \]
   \[ f^{-1}(x) = \frac{5(x - 2)}{3} \]
13. A function \( f \) is defined as follows:
\[ f(x) = 4x + 5, \text{ for } -4 \leq x < 0 \]
Find the values of \( f(-1), f(-2), f(0) \), if exist.

**Solution:**
\[ f(x) = 4x + 5, \text{ for } x \in [-4, 0) \]
\[ -1 \in [-4, 0) \]
\[ f(-1) = 4(-1) + 5 = 1 \]
\[ -2 \in [-4, 0) \]
\[ f(-2) = 4(-2) + 5 = -3 \]
But \( 0 \not\in [-4, 0) \)
\[ f(0) \text{ does not exist.} \]

14. A function \( f \) is defined as follows:
\[ f(x) = 5 - x \text{ for } 0 \leq x \leq 4 \]
Find the value of \( x \) such that
i. \( f(x) = 3 \) and
ii. \( f(x) = 5 \).

**Solution:**
\[ f(x) = 5 - x \text{ for } 0 \leq x \leq 4 \]
\[ \text{i. When } f(x) = 3, \]
\[ 5 - x = 3 \]
\[ x = 2 \]
\[ \text{ii. When } f(x) = 5, \]
\[ 5 - x = 5 \]
\[ x = 0 \]

15. If \( f(x) = 3x^4 - 5x^2 + 7 \), find \( f(x-1) \).

**Solution:**
\[ f(x) = 3x^4 - 5x^2 + 7 \]
\[ f(x-1) = 3(x-1)^4 - 5(x-1)^2 + 7 \]
\[ = 3(x^4 - 4x^3 + 6x^2 - 4x + 1) - 5(x^2 - 2x + 1) + 7 \]
\[ = 3x^4 - 12x^3 + 18x^2 - 12x + 3 - 5x^2 + 10x - 5 + 7 \]
\[ = 3x^4 - 12x^3 + 13x^2 - 2x + 5 \]

16. If \( f(x) = 3x + a \) and \( f(1) = 7 \), find \( a \) and \( f(4) \).

**Solution:**
\[ f(x) = 3x + a \]
\[ f(1) = 7 \]
\[ \therefore f(1) = 3(1) + a \]
\[ 7 = 3 + a \]
\[ a = 4 \]
\[ \therefore f(x) = 3x + 4 \]
\[ f(4) = 3(4) + 4 = 16 \]

17. If \( f(x) = ax^2 + bx + 2 \) and \( f(1) = 3 \), \( f(4) = 42 \), find \( a \) and \( b \).

**Solution:**
\[ f(x) = ax^2 + bx + 2 \]
\[ f(1) = 3 \]
\[ \therefore a(1)^2 + b(1) + 2 = 3 \]
\[ a + b + 2 = 3 \]
\[ a = 1 - b \] \( \ldots(i) \)

18. Find composite of \( f \) and \( g \) and express it by formula

i. \( f = \{(1, 1), (2, 4), (3, 4), (4, 3)\} \)
\( g = \{(1, 1), (2, 8), (3, 27), (4, 64)\} \)

**Solution:**
\[ f = \{(1, 1), (2, 4), (3, 4), (4, 3)\} \]
\[ g = \{(1, 1), (2, 8), (3, 27), (4, 64)\} \]
Let \( A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\} \) and \( C = \{6, 8, 10, 12\} \)
\[ \therefore f(x) = x + 2 \text{ and } g(x) = 2x \]
\[ (gof) (x) = g[f(x)] = g(x + 2) = 2(x + 2) = 2x + 4 \]
\[ (gof) (1) = 2(1) + 4 = 6 \]
\[ (gof) (2) = 2(2) + 4 = 8 \]
\[ (gof) (3) = 2(3) + 4 = 10 \]
\[ (gof) (4) = 2(4) + 4 = 12 \]
\[ \therefore \text{gof} = \{(1, 6), (2, 8), (3, 10), (4, 12)\} \]

ii. \( f = \{(1, 1), (2, 4), (3, 4), (4, 3)\} \)
\[ g = \{(1, 1), (2, 8), (3, 27), (4, 64)\} \]
Let \( A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\} \) and \( C = \{1, 8, 27, 64\} \)
Here, \( f(1) = 1, f(2) = 4, f(3) = 4, f(4) = 3 \)
\[ g(1) = 1, g(2) = 8, g(3) = 27, g(4) = 64 \]
\[ (gof) (1) = g[f(1)] = g(1) = 1 \]
\[ (gof) (2) = g[f(2)] = g(4) = 64 \]
\[ (gof) (3) = g[f(3)] = g(4) = 64 \]
\[ (gof) (4) = g[f(4)] = g(3) = 27 \]
\[ \therefore \text{gof} = \{(1, 1), (2, 64), (3, 64), (4, 27)\} \]
\[ \therefore \text{This composite function cannot be expressed by formula.} \]
19. Find fog and gof if,

i. \( f(x) = x^2 + 5 \), \( g(x) = x - 8 \)

ii. \( f(x) = 3x - 2 \), \( g(x) = x^2 \)

iii. \( f(x) = 256x^4 \), \( g(x) = \sqrt{x} \).

**Solution:**

i. \( f(x) = x^2 + 5 \), \( g(x) = x - 8 \)

\[
\text{fog}(x) = f[g(x)] = f(x - 8) = (x - 8)^2 + 5 = x^2 - 16x + 64 + 5 = x^2 - 16x + 69
\]

\[
gof(x) = g[f(x)] = g(x^2 + 5) = x^2 + 5 - 8 = x^2 - 3
\]

ii. \( f(x) = 3x - 2 \), \( g(x) = x^2 \)

\[
\text{fog}(x) = f[g(x)] = f(x^2) = 3x^2 - 2
\]

\[
gof(x) = g[f(x)] = [f(x)]^2 = (3x - 2)^2 = 9x^2 - 12x + 4
\]

iii. \( f(x) = 256x^4 \), \( g(x) = \sqrt{x} \)

\[
\text{fog}(x) = f[g(x)] = f(\sqrt{x}) = 256(\sqrt{x})^4 = 256x^2
\]

\[
gof(x) = g[f(x)] = g(256x^4) = \sqrt{256x^4} = 16x^2
\]

20. If \( f(x) = \frac{2x + 1}{5x - 2} \), \( x \neq \frac{2}{5} \), show that \( (fog)(x) = x \).

**Solution:**

\[
f(x) = \frac{2x + 1}{5x - 2}
\]

\[
\text{(fog)}(x) = f[g(x)] = f\left(\frac{2x + 1}{5x - 2}\right)
\]

\[
= 2\left(\frac{2x + 1}{5x - 2}\right) + 1 - 2
\]

\[
= \frac{4x + 2 + 5x - 2}{10x - 4} = \frac{9x}{9} = x
\]

21. If \( f(x) = \frac{x + 3}{4x - 5} \), \( x \neq \frac{5}{4} \),

\[
g(x) = \frac{3 + 5x}{4x - 1} = x
\]

show that \( (fog)(x) = x \).

**Solution:**

\[
f(x) = \frac{x + 3}{4x - 5}, \quad g(x) = \frac{3 + 5x}{4x - 1}
\]

\[
\therefore (fog)(x) = f[g(x)] = f\left(\frac{3 + 5x}{4x - 1}\right) = \frac{3 + 5x + 3}{4x - 1} = \frac{3 + 5x + 12x - 3}{4x - 1} = \frac{17x}{17} = x
\]

**Additional Problems for Practice**

**Based on Exercise 1.1**

1. Describe the following sets in Roster form:
   i. \( \{x / x \text{ is a letter of the word ‘APPLE’}\} \)
   ii. \( \{x / x \text{ is an integer and } -2 < x < \frac{3}{2}\} \)
   iii. \( \{x / x = 2n - 1, n \in \mathbb{N}\} \)

2. Describe the following sets in set-builder form:
   i. \( \{0, 1, 2, 3, \ldots\} \)
   ii. \( \{2, 3, 5, 7, 11, 13\} \)
   iii. \( \left\{1, 1, 1, 1\right\} \)

3. If \( A = \{x / 2x^2 + x - 6 = 0\} \),
   \( B = \{x / x^2 - 4 = 0\} \),
   \( C = \{x / x^2 - 3x - 10 = 0\} \),

   then find
   i. \( A \cup B \cup C \)
   ii. \( A \cap B \cap C \)

4. If \( A, B, C \) are the sets of the letters in the words ‘language’, ‘luggage’ and ‘drainage’ respectively, then verify that \([A - (B \cup C)] = [(A - B) \cap (A - C)]\).
5. If \( A = \{a, b, c, d\} \), \( B = \{c, d, e, f\} \), \( C = \{f, g, h, i\} \) and universal set \( U = \{a, b, c, d, e, f, g, h, i, j\} \), then verify the following:

i. \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)  

ii. \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \)  

iii. \((A \cup C)' = A' \cap C'\)  

iv. \((B \cap C)' = B' \cup C'\)  

v. \( C = (A \cap C) \cup (A' \cap C) \)  

vi. \( n(B \cup C) = n(B) + n(C) - n(B \cap C) \)

6. If \( A \) and \( B \) are subsets of the universal set \( X \) and \( n(X) = 100 \), \( n(A) = 80 \), \( n(B) = 40 \), \( n(A' \cap B') = 10 \), then find

i. \( n(A \cup B) \)  

ii. \( n(A \cap B) \)  

iii. \( n(A' \cap B) \)  

iv. \( n(A \cap B') \)

7. In a survey of 75 students of a class, 40 like apple juice, 35 like orange juice, 30 like pineapple juice, 15 like both apple and orange juices, 12 like both apple and pineapple juices, 10 like both orange and pineapple juices and 4 like all three juices. Find the number of students who

i. did not like any juice.  

ii. like orange juice or pineapple juice.

8. From amongst 800 individuals using internet, 55\% use Facebook, 40\% use Whatsapp and 35\% use both sites. Find the number of individuals who use

i. at least one of the sites.  

ii. neither Facebook nor Whatsapp.  

iii. only one of the sites.

9. In a class, 50 students study Marathi, 40 students study English, 30 students study Hindi, 20 students study both English and Marathi, 16 students study both English and Hindi. None of them study Marathi and Hindi both. If every student study at least one subject, find the number of students in the class.

10. If \( A = \{1, 2, 3, 4\} \), write down the set of all possible subsets of \( A \), i.e., the power set of \( A \).

11. Write the following intervals in set-builder form:

i. \((−2, 0]\)  

ii. \([2, 6] \)  

iii. \([-2, 5] \)  

iv. \([-1, 1] \)

12. Using Venn diagrams represent:

i. \((A \cup B \cup C)'\)  

ii. \( A \cap (B \cup C) \)

13. In the Venn-diagram below, shade

i. \( B' \cup (A \cap C) \)  

ii. \((B \cap C) \cup (A' \cap B') \)

---

Based on Exercise 1.2

1. If \( (x + 3, y - 1) = (4, 1) \), find the values of \( x \) and \( y \).

2. If \( \left(\frac{x}{4}, \frac{1}{2}\right) = \left(\frac{5}{2}, 2\right) \), find \( x \) and \( y \).

3. If \( A = \{1, 2, 3\} \), \( B = \{x, y\} \), then find \( A \times B \), \( B \times A \) and \( A \times B \).

4. If \( A = \{1, 2, 3\} \), \( B = \{3, 4\} \), \( C = \{4, 5\} \), then find

i. \((A \times B) \cup (A \times C)\)  

ii. \((A \times B) \cap (A \times C) \)

5. Express \( A = \{(x, y) / x^2 + y^2 = 25, x, y \in W\} \) as a set of ordered pairs.

6. Write the domain and range of the following relations:

i. \{(a, b) / a, b \in N, a < 3 \text{ and } b < 2\}  

ii. \{(a, b) / a, b \in N, a + b = 5\}  

7. Let \( A = \{1, 2, 3\} \), \( B = \{4, 10\} \) and \( R = \{(a, b) / a \in A, b \in B, a.b \text{ is odd}\} \). Show that \( R \) is an empty relation from \( A \) to \( B \).

8. Write the following relation in Roster form and hence find its domain and range:

\[ R = \left\{\left\{\frac{1}{a+1}\right\} / a \in N, 0 < a < 4\right\} \]

9. Write the following relations as sets of ordered pairs:

i. \{(x, y) / x + y = 5, x, y \in \{1, 2, 3, 4\}\}  

ii. \{(x, y) / x > y + 1, x = 4, 6, 8 \text{ and } y = 1, 3\}  

10. Find the domain and range of the following functions:

i. \( f(x) = \sqrt{x - 2} \)  

ii. \( f(x) = \sqrt{25 - x^2} \)

11. Find the range of each of the following functions:

i. \( f(x) = x^2 - 8x + 19 \), for all \( x \in R \)  

ii. \( f(x) = 5x + 4 \), \(-2 \leq x \leq 3\)
12. If \( f(x) = (x + 5)(3x - 1) \), then find \( f(1), f(-2) \).

13. If \( f(x) = 3x^2 + 2x + 5 \), then find \( f(x + 1) \).

14. Which of the following relations are functions? If it is a function, determine its domain and range. Also find the function by formula (if possible).
   i. \( \{(1, 2), (2, 3), (3, 4), (4, 5)\} \)
   ii. \( \{(1, 2), (1, 4), (2, 4), (3, 6)\} \)

15. If \( f(x) = ax - 10 \) and \( f(1) = 6 \), then find \( a \).

16. If \( f(x) = f(2x + 1) \) for \( f(x) = x^2 - 5x + 7 \), then find \( x \).

17. Let \( A = \{1, 2, 3, 4, 5\} \) and \( Z \) be the set of integers. If \( f: A \to Z \) is defined by \( f(x) = 2x + 1 \), then show that \( f \) is a function from \( A \) to \( Z \). Also find the range of \( f \).

18. Find whether the following functions are one-one, onto or not:
   i. \( f: R \to R \) given by \( f(x) = x^2 \)
   ii. \( f: R \to R \) given by \( f(x) = 2x + 3 \).

19. Find which of the following functions are one-one onto, many-one onto, one-one into, many-one into.
   i. \( f = \{(1, 3), (2, 6), (3, 11), (4, 18)\} \) defined from \( A \) to \( B \), where \( A = \{1, 2, 3, 4\} \) and \( B = \{3, 6, 11, 18, 27\} \)
   ii. \( f: R \to R \) given by \( f(x) = 2x^2 + 7 \) for all \( x \in R \).

Based on Exercise 1.3

1. Let \( f \) and \( g \) be two real valued functions defined by \( f(x) = 3x - 1 \) and \( g(x) = x - 2 \). Find
   i. \( f + g \) ii. \( f - g \) iii. \( \frac{f}{g} \)

2. Find \( \text{gof} \) and \( \text{fog} \), where \( f(x) = \frac{x}{2} \) and \( g(x) = \frac{x + 1}{x - 1} \).

3. If \( f(x) = \frac{5x + 1}{x - 5} \), then prove that \( f \) is an identity function.

4. If \( f(x) = \frac{7x + 4}{5x - 3} \) and \( g(x) = \frac{3x + 4}{5x - 7} \), then show that \( (\text{gof})(x) = (\text{gof})(x) = x \).

5. If \( f = \{(1, 3), (2, 4), (3, 5), (4, 6), (5, 7)\} \), \( g = \{(3, 9), (4, 16), (5, 25), (6, 36), (7, 49)\} \), find \( (\text{gof}) \).

6. Show that \( f: R \to R \) given by \( f(x) = 4x + 7 \) is one-one onto. Find its inverse function. Also find \( f^{-1}(19) \) and \( f^{-1}(-5) \).

Based on Miscellaneous Exercise - 1

1. Write down the following sets in set-builder form:
   i. \( \{1, 4, 9, 16, 25, 36\} \)
   ii. \( \{0, \pm2, \pm4, \pm6\} \)

2. If \( U = \{x / x \in I, -2 \leq x \leq 10\} \), \( A = \{-2, 0, 2, 4, 6, 8\} \), \( B = \{1, 2, 3, 9, 10\} \), \( C = \{3, 6, 9\} \), write down the following sets:
   i. \( A \cup C \)
   ii. \( A \cap B \)
   iii. \( A' \cap C \)
   iv. \( (A \cap B) \cap C' \)

3. In a class of 180 students, 95 like English, 110 like Hindi and 30 like both subjects. How many students neither like English nor like Hindi?

4. Out of 40 players participating in Cricket and Football, 18 play Cricket and 9 play both Cricket and Football. How many players play Football?

5. If \( A = \{1, 3, 4\} \) and \( B = \{3, 5\} \), find \( A \times B \), \( B \times B \), \( B \times A \), \( (A \times B) \cap (B \times A) \).

6. If \( X = \{a, b, c\} \) and \( Y = \{p, q\} \), which of the following are relations from \( X \) to \( Y \):
   i. \( R_1 = \{(a, p), (a, q), (b, p), (c, q)\} \)
   ii. \( R_2 = \{(p, a), (a, p), (a, q), (b, q), (q, c)\} \)

7. Determine the domain and range of the following relation:
   \( R = \{(a, b) / |a| < 2, b = |a + 1|, a \in Z\} \)

8. Which of the following relations are functions? If it is a function, determine its domain and range:
   i. \( \{(1, 1), (2, 8), (3, 27), (4, 64)\} \)
   ii. \( \{(1, 1), (1, -1), (2, 2), (2, -2)\} \)

9. Find whether the following function is one-one or not:
   \( f: R - \{-2\} \to R \) defined by
   \( f(x) = \frac{4x + 3}{x + 2} \) for \( x \in R - \{-2\} \)
10. Find whether the following functions are onto or not:
   i. \( f : \mathbb{R} \rightarrow \mathbb{R} \) defined by \( f(x) = 5x + 11 \) for all \( x \in \mathbb{R} \)
   ii. \( f : \mathbb{Z} \rightarrow \mathbb{Z} \) defined by \( f(x) = 5x - 11 \) for all \( x \in \mathbb{Z} \).

11. A function \( f : \mathbb{R} \rightarrow \mathbb{R} \) defined by \( f(x) = 5 + \frac{x}{6} \), \( x \in \mathbb{R} \). Show that \( f \) is one-one and onto. Hence find \( f^{-1} \).

12. A function \( f \) is defined as follows:
   \[ f(x) = 3x + 7, \quad -3 \leq x \leq 1. \]
   Find the values of \( f(-2), f(1), f(2) \), if they exist.

13. A function \( f \) is defined as follows:
   \[ f(x) = 3 + x \text{ for } -2 < x < 2. \]
   Find the values of \( x \) such that \( f(x) = 2 \) and \( f(x) = 4 \).

14. If \( f(x) = 2x^3 - 3x + 11 \), find \( f(x + 1) \).

15. If \( f(x) = 2x + a \) and \( f(2) = 9 \), find \( a \) and \( f(3) \).

16. If \( f(x) = ax^2 + bx + 5 \) and \( f(1) = 12, f(2) = 21 \), find \( a \) and \( b \).

17. Find composite of \( f \) and \( g \) and express it by formula:
   \( f = \{(1, 4), (2, 5), (3, 6), (4, 7)\} \)
   \( g = \{(4, 9), (5, 11), (6, 13), (7, 15)\} \)

18. If \( f(x) = 16x^2 \) and \( g(x) = \sqrt{x} \), find \( f \circ g \) and \( g \circ f \).

19. If \( f(x) = \frac{3x+1}{5x-3}, x \neq \frac{3}{5} \), then show that \( (f \circ f)(x) = x \).

20. If \( f(x) = \frac{x+2}{3x-7} \) and \( g(x) = \frac{2+7x}{3x-1} \), then show that \( (f \circ g)(x) = x \).

**Multiple Choice Questions**

1. The set of intelligent students in a class is
   (A) A null set
   (B) A singleton set
   (C) A finite set
   (D) Not a well defined collection.

2. Which of the following is the empty set?
   (A) \( \{x / x \text{ is a real number and } x^2 - 1 = 0\} \)
   (B) \( \{x / x \text{ is a real number and } x^2 + 1 = 0\} \)
   (C) \( \{x / x \text{ is a real number and } x^2 - 9 = 0\} \)
   (D) \( \{x / x \text{ is a real number and } x^2 = x + 2\} \)

3. If a set \( A \) has \( n \) elements, then the total number of subsets of \( A \) is
   (A) \( n \)
   (B) \( n^2 \)
   (C) \( 2^n \)
   (D) \( 2n \)

4. If \( A = \{1, 2, 3\}, B = \{3, 4\} \), then \( A \cup (B \cap C) \) is
   (A) \( \{1, 2, 3\} \)
   (B) \( \{1, 2, 3, 4\} \)
   (C) \( \{1, 2, 4, 5\} \)
   (D) \( \{1, 2, 3, 4, 5, 6\} \)

5. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is
   (A) 80 percent
   (B) 40 percent
   (C) 60 percent
   (D) 70 percent

6. If \( A, B \) and \( C \) are any three sets, then \( A \times (B \cup C) \) is equal to
   (A) \( (A \times B) \cup (A \times C) \)
   (B) \( (A \cup B) \times (A \cup C) \)
   (C) \( (A \times B) \cap (A \times C) \)
   (D) None of these

7. If \( A = \{2, 4, 5\}, B = \{7, 8, 9\} \), then \( n(A \times B) \) is
   (A) 6
   (B) 9
   (C) 3
   (D) 0

8. Which set is the subset of all given sets
   (A) \( \{1, 2, 3, 4, \ldots\} \)
   (B) \( \{1\} \)
   (C) \( \{0\} \)
   (D) None of these

9. The smallest set \( A \) such that
   \( A \cup \{1, 2\} = \{1, 2, 3, 5, 9\} \) is
   (A) \( \{2, 3, 5\} \)
   (B) \( \{3, 5, 9\} \)
   (C) \( \{1, 2, 5, 9\} \)
   (D) None of these

10. Let \( U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \), \( A = \{1, 2, 5\}, B = \{6, 7\} \), then \( A \cap B' \) is
    (A) \( B' \)
    (B) \( A \)
    (C) \( A' \)
    (D) \( B \)

11. The shaded region in the given figure is
    (A) \( A \cap (B \cup C) \)
    (B) \( A \cup (B \cap C) \)
    (C) \( A \cap (B \cap C) \)
    (D) \( A - (B \cup C) \)
12. Of the members of three athletic teams in a school 21 are in the cricket team, 26 are in the hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and football, and 12 play football and cricket. Eight play all the three games. The total number of members in the three athletic teams is
(A) 43 (B) 76
(C) 49 (D) None of these

13. If A, B, C are three sets, then $A \cap (B \cup C)$ is equal to
(A) $(A \cup B) \cap (A \cup C)$
(B) $(A \cap B) \cup (A \cap C)$
(C) $(A \cup B) \cup (A \cap C)$
(D) None of these

14. If $A = \{x, y\}$, then the power set of $A$ is
(A) $\{x, y\}$
(B) $\{\phi, x, y\}$
(C) $\{\phi, \{x\}, \{y\}\}$
(D) $\{\phi, \{x\}, \{y\}, \{x, y\}\}$

15. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is/are not relations from $X$ to $Y$?
(A) $R_1 = \{(x, y)| y = 2 + x, x \in X, y \in Y\}$
(B) $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$
(C) $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$
(D) $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$

16. Let $A = \{a, b, c\}$ and $B = \{1, 2\}$. Consider a relation $R$ defined from set $A$ to set $B$. Then $R$ is equal to set
(A) $A$ (B) $B$
(C) $A \times B$ (D) $B \times A$

17. Let $R$ be a relation on $N$ defined by $x + 2y = 8$. The domain of $R$ is
(A) $\{2, 4, 8\}$ (B) $\{2, 4, 6, 8\}$
(C) $\{2, 4, 6\}$ (D) $\{1, 2, 3, 4\}$

18. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 6, 8, 11, 15\}$ which of the following are functions from $A$ to $B$

i. $f: A \to B$ defined by
   $f(1) = 1$, $f(2) = 6$, $f(3) = 8$, $f(4) = 8$

ii. $f(1) = 1$, $f(2) = 6$, $f(3) = 15$

iii. $f(1) = 6$, $f(2) = 6$, $f(3) = 6$, $f(4) = 6$

(A) (i) & (iii) (B) (i) & (ii)
(C) (ii) (D) (i) & (iii)

19. If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, then the value of $f[f(5)]$ is
(A) 111 (B) 110
(C) 109 (D) 101

20. The domain of the function:
$$\frac{1}{(2x - 3)(x + 1)}$$

(A) $R - \{-1\}$ (B) $R - \left[\frac{3}{2}\right]$

(C) $R - \left\{-\frac{3}{2}\right\}$ (D) $R$

21. If $f(x) = \frac{3x + 2}{4x - 3}$ for $x \neq \frac{3}{4}$, then $f(4)$ is
(A) $17x$ (B) $3x$
(C) $4x$ (D) $x$

22. If $f(x) = x^2 + 5x + 7$, then the value of $x$ for which $f(x) = f(x + 1)$ is
(A) $3$ (B) $-6$
(C) $-3$ (D) $6$

23. The domain of the function $\frac{x}{2 + x^2}$ is
(A) $(1, \infty)$ (B) $(-\infty, 1)$
(C) $(1,1)$ (D) $(-\infty, \infty)$

24. The range of the function $\sqrt{4 - x^2}$ is
(A) $[-3, 2]$ (B) $[0, 2]$
(C) $(0, 2)$ (D) $(-2, 2)$

25. If $A = \{1, 2, 3, 4\}$; $B = \{a, b\}$ and $f$ is a mapping such that $f: A \to B$, then $A \times B$ is
(A) $\{(a, 1), (3, b)\}$
(B) $\{(a, 2), (4, b)\}$
(C) $\{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b), (4, a), (4, b)\}$
(D) None of these

Answers to Additional Practice Problems

Based on Exercise 1.1

1. i. $\{A, P, L, E\}$
   ii. $\{-1, 0, 1\}$
   iii. $\{1, 3, 5, 7, \ldots\}$

2. i. $\{x / x \in W\}$
   ii. $\{x / x$ is a prime number, $x < 14\}$
   iii. $\left\{x / x = \frac{1}{2n - 1}, n \in N, 1 \leq n \leq 5\right\}$

3. i. $\left\{-2, \frac{3}{2}, 2, 5\right\}$
   ii. $\{-2\}$

6. i. $90$
   ii. $30$
   iii. $10$
   iv. $50$
7. i. 3  
   ii. 55

8. i. 480  
   ii. 320  
   iii. 200

9. 84

10. \( P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\} \)

11. i. \((-2, 0) = \{x / x \in \mathbb{R}, -2 < x < 0\}\)
   ii. \((2, 6] = \{x / x \in \mathbb{R}, 2 < x \leq 6\}\)
   iii. \([-2, 5) = \{x / x \in \mathbb{R}, -2 \leq x < 5\}\)
   iv. \([-1, 1] = \{x / x \in \mathbb{R}, -1 \leq x \leq 1\}\)

12. i. \(A \cap (B \cup C)\)
    ![Venn Diagram](image)

   ii. \(A \cup B \cup C\)'
    ![Venn Diagram](image)

13. i. \(A \cap (B \cup C)\)
    ![Venn Diagram](image)

   ii. 
    ![Venn Diagram](image)

Based on Exercise 1.2

1. \(x = \frac{1}{4}, y = 3\)

2. \(x = \frac{1}{4}, y = 3\)

3. \(A \times B = \{(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)\}\)
   
   \(B \times A = \{(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)\}\)
   
   \(A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}\)

4. i. \(\{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}\)
   ii. \(\{(1, 4), (2, 4), (3, 4)\}\)

5. \(A = \{(0, 5), (3, 4), (4, 3), (5, 0)\}\)

6. i. \(\text{Domain} = \{1, 2\}, \text{Range} = \{1\}\)
   ii. \(\text{Domain} = \{1, 2, 3, 4\}, \text{Range} = \{4, 3, 2, 1\}\)

8. \(R = \left[\begin{array}{c}
1 \\
1 \\
\frac{1}{2} \\
\frac{1}{3} \\
\frac{1}{4}
\end{array}\right], \quad \text{Domain} = \{1, 2, 3\}, \text{Range} = \left[\begin{array}{c}
1 \\
1 \\
\frac{1}{2} \\
\frac{1}{3} \\
\frac{1}{4}
\end{array}\right] \)

9. i. \(\{(1, 4), (2, 3), (3, 2), (4, 1)\}\)
   ii. \(\{(4, 1), (6, 1), (8, 1), (6, 3), (8, 3)\}\)

10. i. \(\text{Domain} = \{2, 4\}\)
    \(\text{Range} = [-1, 1]\)
   ii. \(\text{Domain} = [-5, 5]\)
    \(\text{Range} = [-5, 5]\)

11. i. \(\text{Range} = [3, \infty]\)
   ii. \(\text{Range} = [-6, 19]\)

12. \(f(1) = 12, f(-2) = -21\)

13. \(3x^2 + 8x + 10\)

14. i. \(\text{It is a function.} \)
    \(\text{Domain} = \{1, 2, 3, 4\}\)
    \(\text{Range} = \{2, 3, 4, 5\}\)
    \(f(x) = x + 1\)
   ii. \(\text{Not a function.}\)

15. 16

16. \(-1, \frac{4}{3}\)
17. \( f = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\} \)
   Range of \( f = \{3, 5, 7, 9, 11\} \)

18. i. Not one-one, not onto
    ii. One-one, onto

19. i. One-one into
    ii. Many-one into

Based on Exercise 1.3

1. i. \( 4x - 3 \)  
   ii. \( 2x + 1 \) 
   iii. \( \frac{3x - 1}{x - 2} \)

2. \((gof)(x) = \frac{x + 2}{x - 2}, (fog)(x) = \frac{x + 1}{2(x - 1)} \)

5. \((gof) = \{(1, 9), (2, 16), (3, 25), (4, 36), (5, 49)\} \)

6. \( f^{-1}(x) = \frac{x - 7}{4}, f^{-1}(19) = 3, f^{-1}(-5) = -3 \)

Based on Miscellaneous Exercise – 1

1. i. \( \{x / x = n^2, n \in \mathbb{N}, n \leq 6\} \)
   ii. \( \{x / x = 2n, n \in \mathbb{Z}, -3 \leq n \leq 3\} \)

2. i. \( \{-2, 0, 2, 3, 4, 6, 8, 9\} \)
   ii. \( \{2\} \)
   iii. \( \{3, 9\} \)
   iv. \( \{2\} \)

3. 5

4. 31

5. \( A \times B = \{(1,3), (1,5), (3, 3), (3, 5), (4, 3), (4, 5)\} \)
   \( B \times B = \{(3, 3), (3, 5), (5, 3), (5, 5)\} \)
   \( B \times A = \{(3,1), (3,3), (3, 4), (5, 1), (5, 3), (5, 4)\} \)
   \( (A \times B) \cap (B \times A) = \{(3, 3)\} \)

6. i. \( R_1 \) is a relation.
   ii. \( R_2 \) is not a relation.

7. Domain = \(-1, 0, 1\)
   Range = \(\{0, 1, 2\} \)

8. i. It is a function.
    Domain = \(\{1, 2, 3, 4\} \)
    Range = \(\{1, 8, 27, 64\} \)
   ii. Not a function.

9. One-one

10. i. Onto
    ii. Not onto

11. \( f^{-1}(x) = 6(x - 5) \)

12. \( f(-2) = 1, f(1) = 10, f(2) \) does not exist.

13. \(-1, 1\)

14. \( 2x^3 + 6x^2 + 3x + 10 \)

15. \( a = 5, f(3) = 11 \)

16. \( a = 1, b = 6 \)

17. \( \text{gof} = \{(1, 9), (2, 11), (3, 13), (4, 15)\} \)
   \( \text{gof}(x) = 2x + 7 \)

18. \( \text{gof}(x) = 16x, \text{gof}(x) = 4x \)

Answers to Multiple Choice Questions

1. (D) 2. (B) 3. (C) 4. (B) 5. (C) 6. (A) 7. (B) 8. (D) 9. (B) 10. (B) 11. (D) 12. (A) 13. (B) 14. (D) 15. (D) 16. (C) 17. (C) 18. (D) 19. (B) 20. (C) 21. (D) 22. (C) 23. (D) 24. (B) 25. (C)