Chapter 18: Atoms, Molecules and Nuclei

1. We know 
\[ N = N_0 e^{-\lambda t} \]

For \( X_1 \), \( \lambda = 5 \lambda \)
\[ \therefore N_1 = N_0 e^{-5\lambda t} \] \hspace{1cm} \text{(i)}

For \( X_2 \),
\[ \therefore N_2 = N_0 e^{-\lambda t} \] \hspace{1cm} \text{(ii)}

\[ \frac{N_1}{N_2} = e^{-4\lambda t} = e^{-4t} \]

Given that, \( \frac{N_1}{N_2} = e \)
\[ \therefore \frac{1}{e} = e^{-4t} \text{ or } 4\lambda t = 1 \implies t = \frac{1}{4\lambda} \]

2. According to Bohr’s postulate
\[ mvr = \frac{nh}{2\pi} = \frac{h}{2\pi} \] \hspace{1cm} \text{...(i)}

or \( v = \frac{h}{2\pi mr} \) \hspace{1cm} \text{...(ii)}

We know that the rate of flow of charge is current.

Hence, \( i = \frac{e}{t} = e\left(\frac{v}{2\pi}\right) = \frac{e}{2\pi} \times v \)
\[ = \frac{e}{2\pi} \times \frac{h}{2\pi mr} = \frac{eh}{4\pi^2 m^2 r^2} \] \hspace{1cm} \text{...(ii)}

Magnetic dipole moment, \( M = i \times A \)
\[ \therefore M = \frac{eh}{4\pi^2 m^2 r^2} \times \pi r^2 \]
\[ M = \frac{eh}{4\pi m} \] \hspace{1cm} \text{...(iii)}

Torque, \( \tau = M \times B \)
or \( \tau = MB \sin 60^\circ \)
\[ \therefore \tau = \frac{eh}{4\pi m} \times B \times \frac{\sqrt{3}}{2} \]
\[ \therefore \tau = \frac{ehB}{8\pi m} \sqrt{3} \]

3. We know that, de-Broglie wavelength
\[ \lambda = \frac{h}{mv} \text{ and } E = \frac{1}{2}mv^2 \]
\[ \therefore \lambda = \frac{h}{\sqrt{2mE}} \]

In first case,
\[ 200 \times 10^{-12} = \frac{h}{\sqrt{2mE_1}} \] \hspace{1cm} \text{...(i)}

In second case,
\[ 100 \times 10^{-12} = \frac{h}{\sqrt{2mE_2}} \] \hspace{1cm} \text{...(ii)}

Dividing Equation (i) by Equation (ii), we get
\[ 2 = \left(\frac{E_2}{E_1}\right) \]

Or \( E_2 = 4E_1 \)
So, energy to be added = \( 4E_1 - E_1 = 3E_1 \)

Now, \( \frac{h}{\sqrt{2mE_1}} = 200 \times 10^{-12} \)

or \( \sqrt{2mE_1} = \frac{6.63 \times 10^{-34}}{2 \times 10^{-10}} \)

or \( \sqrt{2mE_2} = 3.315 \times 10^{-24} \)

or \( E_1 = \frac{(3.315 \times 10^{-24})^2}{2(9.1 \times 10^{-31})} = 0.6038 \times 10^{-17} \)

\[ \therefore \text{Energy added} = 3E_1 \]
\[ = 3 \times 0.6038 \times 10^{-17} \]
\[ = \frac{1.8114 \times 10^{-17}}{(1.6 \times 10^{-19})} \] eV
\[ = 113 \text{ eV} \]

4. Power to be obtained from power house
= 200 megawatt
\[ \therefore \text{Energy obtained per hour} \]
= \( 200 \text{ megawatt} \times 1 \text{ hour} \)
= \( (200 \times 10^6 \text{ watt}) \times (3600 \text{ s}) \)
\[ = 72 \times 10^{10} \text{ J} \]

Here only 10% of output is utilized. In order to obtain \( 72 \times 10^{10} \text{ J} \) of useful energy, the output energy from the power house
\[ = \left(72 \times 10^{10}\right) \times 100 \]
\[ = 72 \times 10^{11} \text{ J} \]

Let this energy be obtained from a mass-loss of \( \Delta m \) kg. Then
\[ (\Delta m)c^2 = 72 \times 10^{11} \]

Or \( \Delta m = \frac{72 \times 10^{11}}{(3 \times 10^8)^2} = 8 \times 10^{-5} \text{ kg} \)
\[ \Delta m = 0.08 \text{ g} \]
Since 0.90 milligram (= 0.90 \times 10^{-3} g) mass is lost in 1 g uranium, hence for a mass loss of 0.08 g the uranium required
\[= \frac{1 \times 0.08}{0.90 \times 10^{-3}}\]
\[= 88.89 \approx 89 \, g\]
Thus, to run the power house, 89 g uranium is required per hour.

5. Lyman series belongs to the ultraviolet region.

6. K.E. = \frac{13.6}{n^2} \text{ eV}, P.E. = \frac{-2(13.6)}{n^2} \text{ eV}

For Hydrogen, Z = 1
\[\therefore \Delta K.E = K.E_i - K.E_f\]
\[\Delta K.E. = 13.6 \left[ \frac{1}{(2)^2} - \frac{1}{(1)^2} \right]\]
\[= -10.2 \, \text{eV} \, (\text{decrease})\]
\[\Delta P.E. = -2(13.6) \left[ \frac{1}{(2)^2} - \frac{1}{(1)^2} \right]\]
\[= 20.4 \, \text{eV} \, (\text{increase})\]
Angular momentum, \(L = \frac{nh}{2\pi}\)
\[\therefore \Delta L = \frac{\hbar}{2\pi} \left( 2 - 1 \right) = \frac{\hbar}{2\pi}\]
\[= 1.05 \times 10^{-34} \, \text{J-s} \, (\text{increase})\]

7. For Lyman series, \(n_i = 1\) and \(n_f = 2\) and \(Z = 2\) (He)
\[\Delta E = -13.6 \left( Z^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \right)\]
\[= -13.6 \times (2)^2 \times \left( -\frac{3}{4} \right) = 13.6 \times 3\]
\[\therefore \text{Total available energy} = 3 \times 13.6 \, \text{Joule}\]

8. Orbital frequency,
\[f = \frac{v_n}{2\pi r_n}\]
\[v_n = \frac{2.2 \times 10^6 Z}{n} \text{ m/s} = \frac{2.2 \times 10^6 (1)}{2}\]
\[= 1.1 \times 10^6 \, \text{ms}^{-1}\]
Now radius,
\[r_n = 0.53 \times 10^{-10} \frac{n^2}{Z} = 4 \times 0.53 \times 10^{-10} \, \text{m}\]
\[\therefore = 2.12 \times 10^{-10} \, \text{m}\]
\[f = \text{number of revolution in one second}\]
\[= \frac{N}{t} = \frac{v_n}{2\pi r_n}\]
\[\therefore \text{Number of revolutions},\]
\[N = f \times t = \frac{1.1 \times 10^6}{2\pi \times 2.12 \times 10^{-10}} \times 10^{-8}\]
\[= 8.2 \times 10^6 \, \text{revolutions}\]
\[\therefore \text{Period} = \frac{1}{8.2 \times 10^6} = 1.2 \times 10^{-7} \, \text{s}\]

9. \(^{1}\text{H}_2 + ^{1}\text{H}_2 \rightarrow ^{2}\text{He}_4 + \text{Energy}\)

Binding energy (B.E.) of \(^{1}\text{H}_2 = 2 \times 1.1 \, \text{MeV}\]
\[= 2.2 \, \text{MeV}\]
\[\therefore \text{B.E. of two} ^{1}\text{H}_2 = 2 \times 2.2 = 4.4 \, \text{MeV}\]
B.E. of \(^{2}\text{He}_4\) nucleus = 4 \times 7.1 = 28.4 \, \text{MeV}\]
\[\therefore \text{Energy released when two} ^{1}\text{H}_2 \text{ fuse to form} ^{2}\text{He}_4 = 28.4 - 4.4 = 24 \, \text{MeV}\]

10. Total energy of \(\text{C}_{12}\) atom
\[= \text{Number of Nucleons} \times 7.68\]
\[= 12 \times 7.68 = 92.16 \, \text{MeV}\]

Similarly, energy for \(\text{C}_{13}\) atom
\[= 13 \times 7.47 = 97.11 \, \text{MeV}\]

Energy required to remove 1 neutron from \(\text{C}_{13}\) = (97.11 - 92.16) = 4.95 \, \text{MeV}\]

11. Using, \(\frac{N}{N_0} = \left( \frac{1}{2} \right)^{\frac{t}{T}}\)
\[\therefore \text{For 33\% decay,} \quad \frac{N}{N_0} = \frac{67}{100}\]
\[\therefore \left( \frac{67}{100} \right) = \left( \frac{1}{2} \right)^{\frac{t}{T}} \quad \ldots (i)\]
For 67\% decay, \(\frac{N}{N_0} = \frac{33}{100}\)
\[\therefore \frac{33}{100} = \left( \frac{1}{2} \right)^{\frac{t}{T}} \quad \ldots (ii)\]
Dividing equation (ii) by equation (i) we get,
\[
\frac{33}{67} = \left(\frac{1}{2}\right)^{(t_2-t_1)/10}
\]
\[
\approx \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{(t_2-t_1)/10}
\]
or \(t_2 - t_1 = 10\)

12. From law of conservation of momentum,
\[
m_u = 2mv \text{ or } v = \frac{u}{2}
\]
Excitation energy,
\[
\Delta E = \frac{1}{2}m u^2 - 2 \times \frac{1}{2}m \left(\frac{u}{2}\right)^2 = \frac{1}{4}m u^2
\]
Minimum excitation energy
\[
= 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \text{ eV}
\]
\[
= \frac{3}{4} \times 13.6 = 10.2 \text{ eV}
\]
\[
\therefore (10.2)(1.6 \times 10^{-19}J) = \frac{1}{4}(1.0078)(1.66 \times 10^{-27})u^2
\]
\[
\therefore u = 6.25 \times 10^4 \text{ m/s}
\]
13. Using magnetic moment,
\[
\mu = \text{current} \times \text{area} = \frac{q \times A}{t}
\]
\[
\therefore \mu = \frac{\omega}{2\pi} \times q \times \pi r^2 = \frac{1}{2} \omega qr^2
\]
But orbital angular momentum,
\[
L = mr^2 = h
\]
\[
\text{[quantum unit} = \frac{h}{2\pi}]
\]
i.e., \(or^2 = h/2\pi m\)
\[
\therefore \mu = \frac{1}{4}\pi \frac{qh}{m}
\]
\[
= (1.6 \times 10^{-19})(1.05 \times 10^{-24})
\]
\[
= 9.2 \times 10^{-34} \text{ Am}^2
\]
14. A photon is emitted when hydrogen atom comes to first excited state i.e., \(n = 2\)
\[
\therefore \text{Energy transferred}
\]
\[
= -13.6\left(\frac{1}{2^2} - \frac{1}{1^2}\right)
\]
\[
= \frac{3}{4} \times 13.6 \text{ eV}
\]
\[
= 10.2 \text{ eV}
\]
By conservation of momentum,
\[
\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + 10.2
\]
or \(v_1^2 - v_1 + 10.2 = 0\) \[\text{[eliminating} \ v_2\]
\[
\therefore \ v_1 \text{ is real} \Rightarrow v^2 \geq 4 \times 10.2
\]
or \(v_{\text{min}} = \frac{\sqrt{4 \times 10.2}}{m}
\]
\[
\therefore \ K.E_{\text{min}} = \frac{1}{2}m (v_{\text{min}})^2
\]
\[
= \frac{1}{2}m \frac{4 \times 10.2}{m}
\]
\[
= 20.4 \text{ eV}
\]
15. Sum of masses of deuteron and lithium nuclei before disintegration
\[
= 2.0147 + 6.0169
\]
\[
= 8.0316 \text{ amu}
\]
Mass of \(\alpha\) particles
\[
= 2 \times 4.0039
\]
\[
= 8.0078 \text{ amu}
\]
Difference of mass
\[
= 8.0316 - 8.0078
\]
\[
= 0.0238 \text{ amu}
\]
Mass converted into energy
\[
= 0.0238 \times 931.3 \text{ MeV}
\]
Energy given to each \(\alpha\) particle
\[
= \frac{0.0238 \times 931.3}{2}
\]
\[
= 11.08 \text{ MeV}
\]
16. For C\(^{14}\), \(\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730}
\]
\[
\Rightarrow \lambda = 1.21 \times 10^{-4} \text{ yr}^{-1} \text{ since } A = 0.144 \text{ Bq and } A_0 = 0.28 \text{ Bq}
\]
Using, \(A = A_0 e^{-\lambda t} \text{ or } t = \frac{1}{\lambda} \ln \left(\frac{A_0}{A}\right)
\]
\[
t = \frac{1}{1.21 \times 10^{-4}} \ln \left(\frac{0.28}{0.144}\right)
\]
\[
\approx 5500 \text{ years}
\]
17. Assertion is false, reason is true. The reduced mass of atomic deuterium is greater than that of atomic hydrogen as
\[
\mu = \frac{m_e m_n}{m_e + m_n}
\]
where \(m_e\) = mass of electron and \(m_n\) = mass of nucleus.
18. For three energy levels, the possible transition are as shown in the diagram.
It is given, \( \lambda_1 < \lambda_2 < \lambda_3 \Rightarrow \nu_1 > \nu_2 > \nu_3 \).
The largest gap will correspond to \( \nu_1 \) or \( h\nu_1 \)
\[ h\nu_1 = h\nu_2 + h\nu_3 \] or \[ \frac{h}{\lambda_1} = \frac{h}{\lambda_2} + \frac{h}{\lambda_3} \]
\[ \Rightarrow \frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3} \]

19. Angular momentum of nth orbit = \( \frac{nh}{2\pi} \).
Again, \( mvr = \frac{nh}{2\pi} \)
\[ \therefore v = \frac{nh}{2\pi mr} \] ....(i)
The time taken for completing an orbit
\[ T = \frac{2\pi}{v} = \frac{2\pi r(2\pi mr)}{nh} \]
Or \[ T = \frac{4\pi^2 mr^2}{nh} \] ....(ii)
Now, \( r = r_0 n^2 \) ....[\( \because r \propto n^2 \)]
\[ \therefore T = \frac{4\pi^2 mr^2 n^4}{nh} = \frac{4\pi^2 mr^2 n^3}{h} \]
Number of orbits completed in \( 10^{-6} \) s = \( \frac{10^{-6}}{T} \)
\[ = \frac{10^{-6} \times h}{4\pi^2 mr^2 n^3} \]
\[ = \frac{10^{-6} \times (6.63 \times 10^{-34})}{4(3.14)^2(9.1 \times 10^{-31})(5.3 \times 10^{-11})^2(2)} \]
\[ = 8.22 \times 10^{8} \]

20. To ionize the H atom in ground state minimum K.E. of photoelectron needed = 13.6 eV.
\[ \therefore \phi = 1.9 \text{ eV} \]
\[ \therefore \text{Minimum energy (or maximum wavelength)} \]
incident = 13.6 + 1.9 \approx 16 \text{ eV}
\[ \therefore \lambda_\text{max} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{16 \times 1.6 \times 10^{-19}} = 77.3 \text{ nm} \approx 77 \text{ nm} \]